Testing cosmic inflation with gravitational wave experiments

Gianmassimo Tasinato

Swansea University and University of Bologna

Thanks to many collaborators within LISA CosWG
and collaborators in Swansea:
Marisol Cruz, Aya Ghaleb, Ameek Malhotra, Alisha Marriott-Best, Ivonne Zavala

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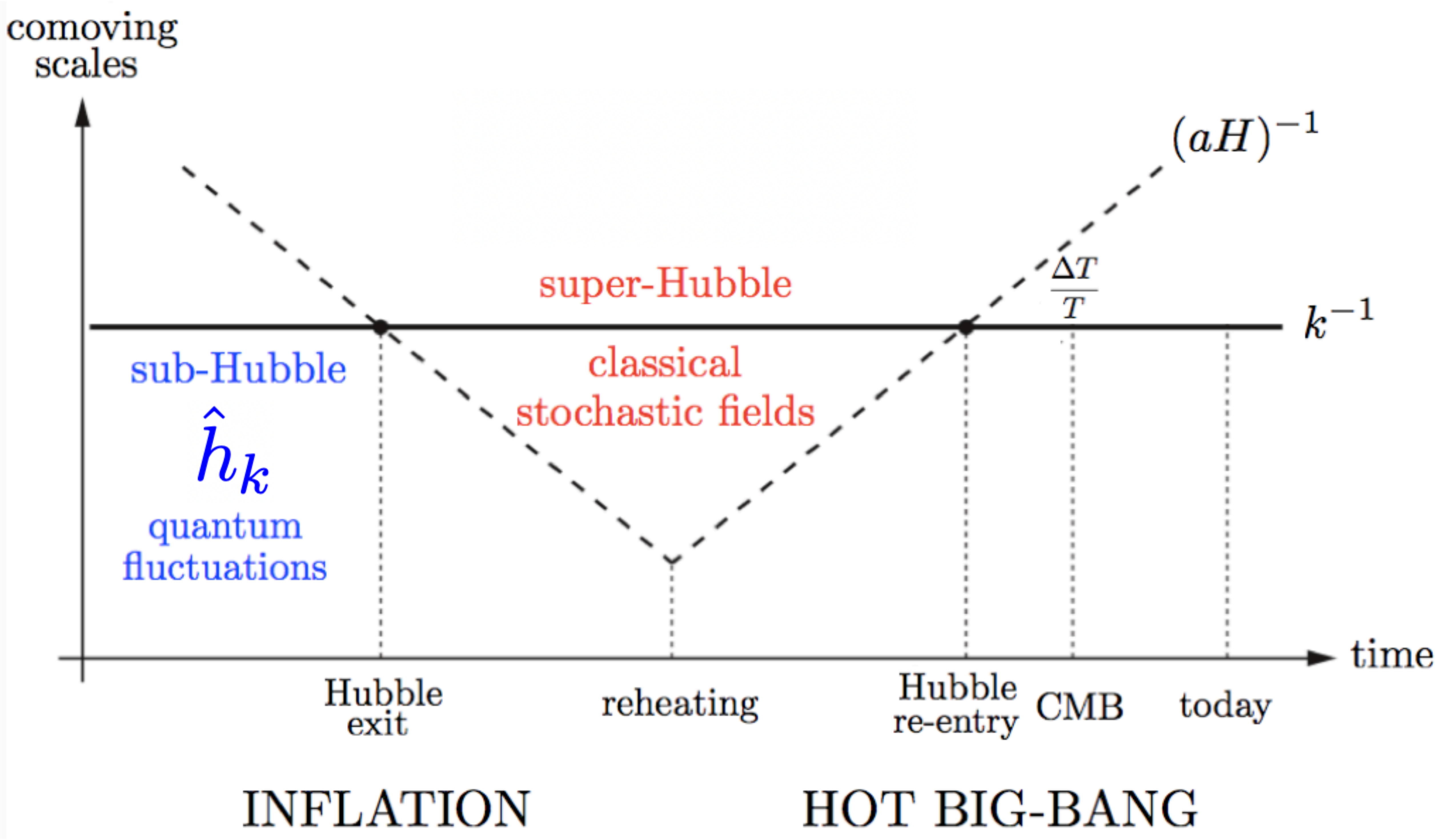
Introduction

Early Universe phenomena source stochastic gravitational wave backgrounds (SGWB)

- **Cosmological Inflation**

  QM production of GW from a brief phase of quasi-exponential expansion

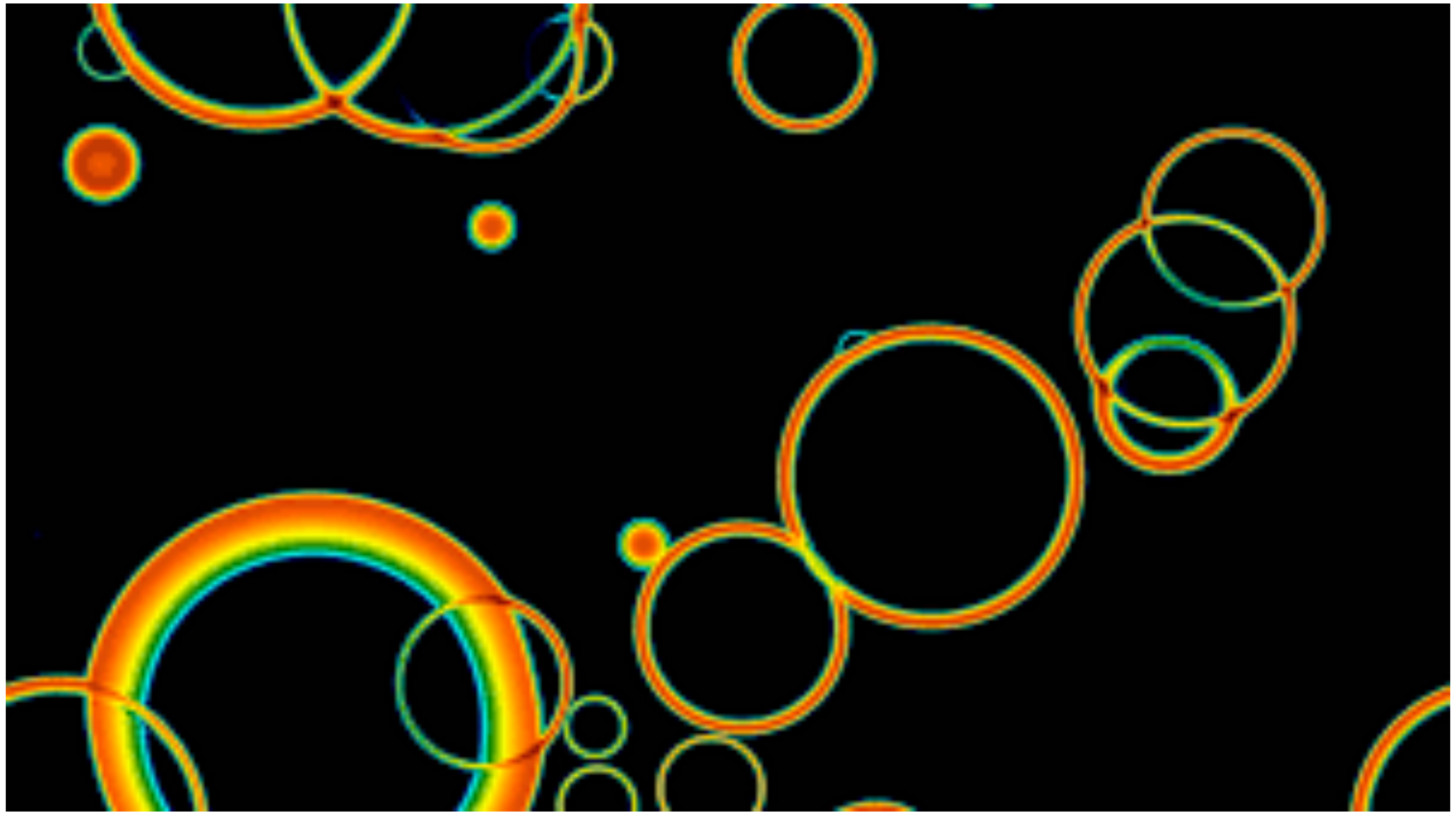


Introduction

Early Universe phenomena source stochastic gravitational wave backgrounds (SGWB)

- **First-order cosmological phase transitions**

  Collisions of bubbles generate GW

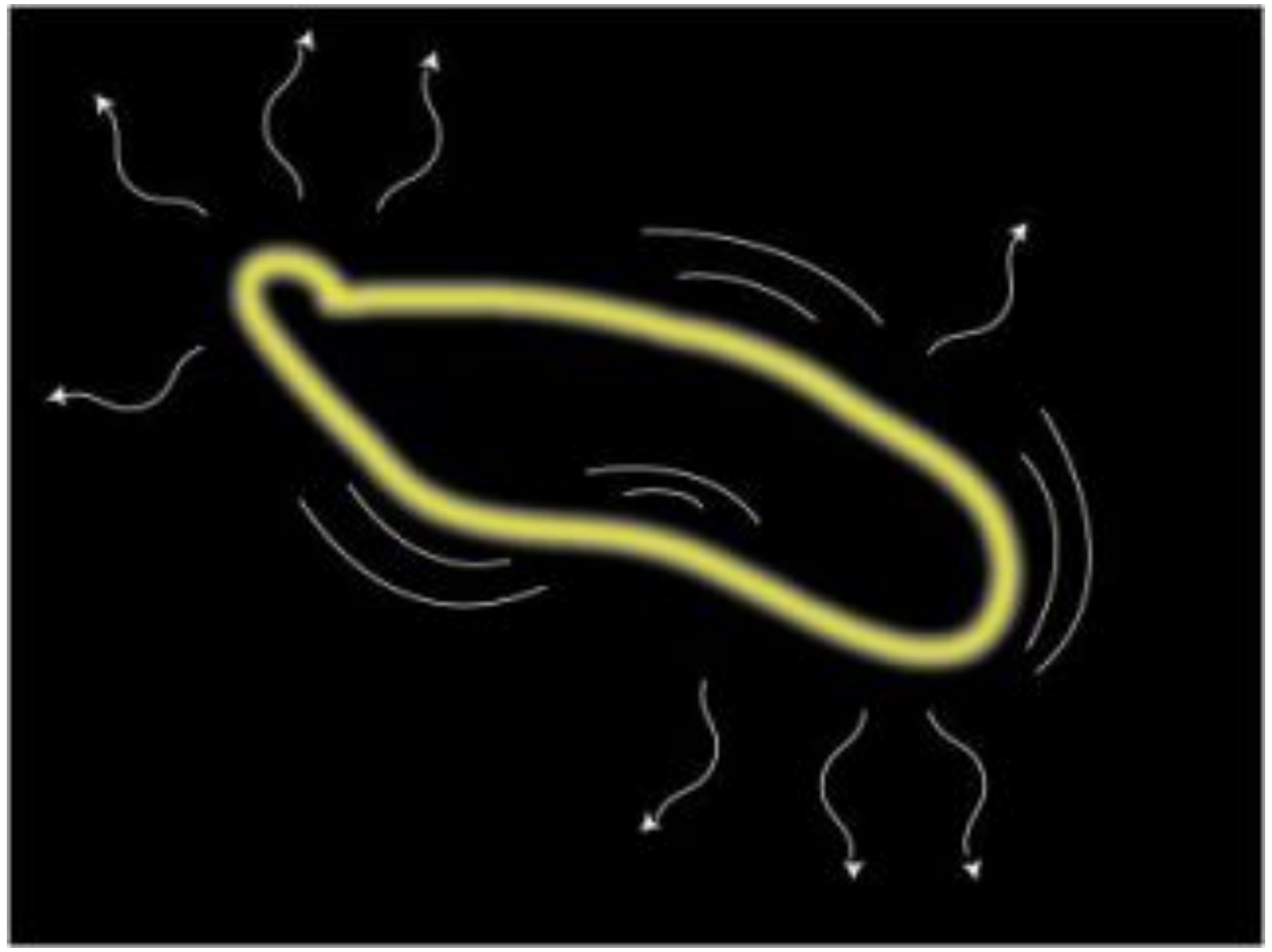


Introduction

Early Universe phenomena source stochastic gravitational wave backgrounds (SGWB)

▷ **Cosmic strings**

Dynamics and interactions of strings generate GW

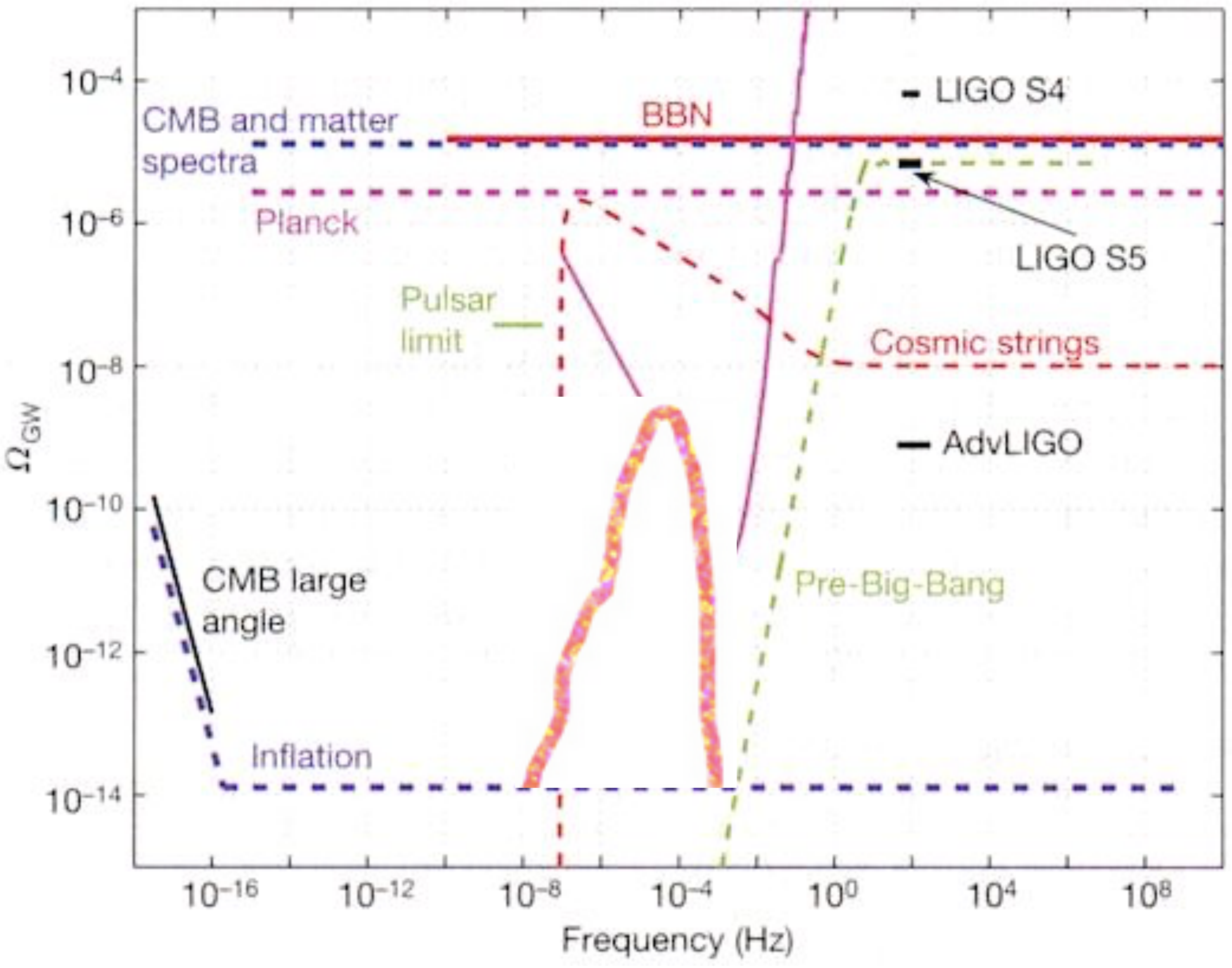


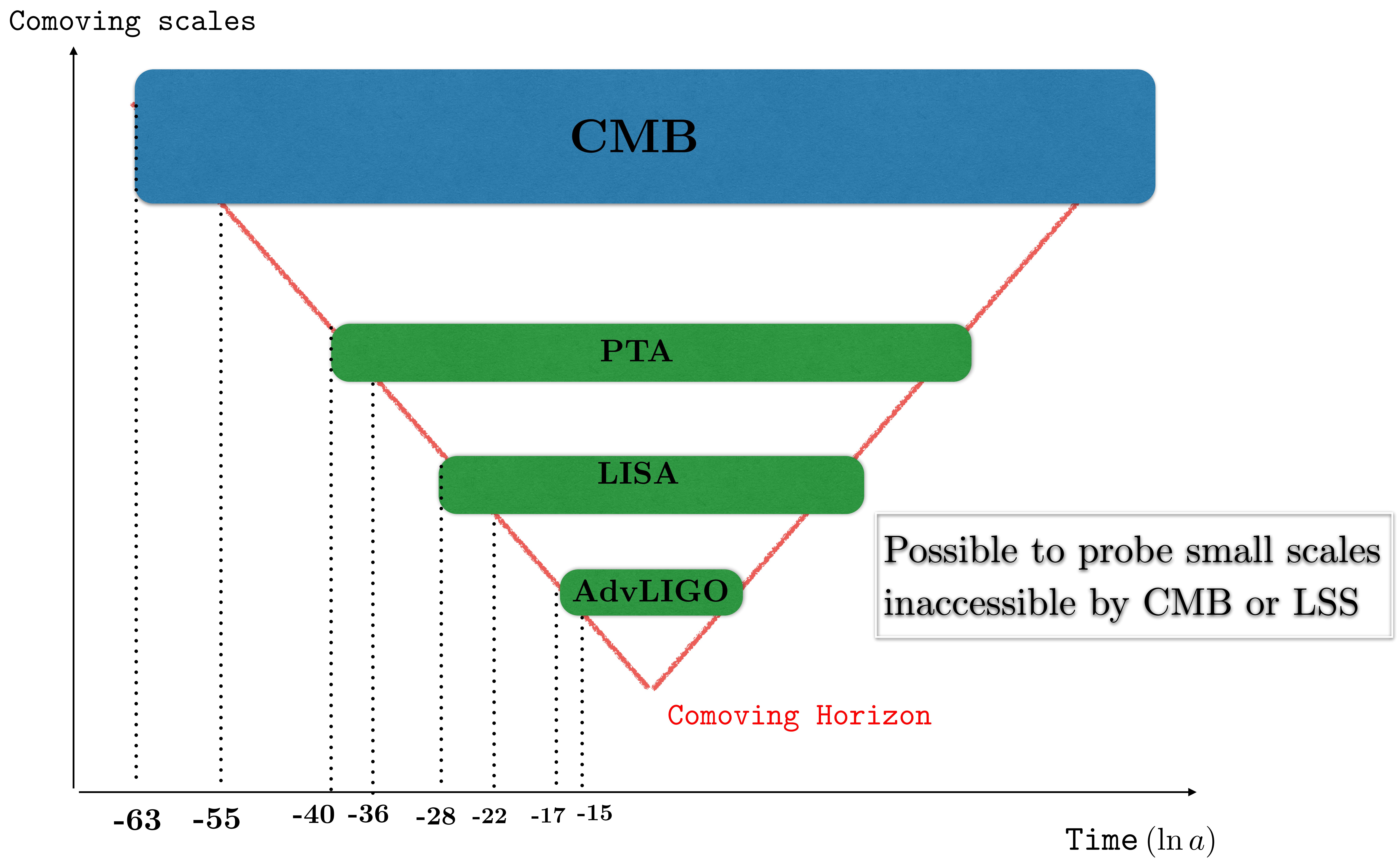
Introduction

Early Universe phenomena source stochastic gravitational wave backgrounds (SGWB)

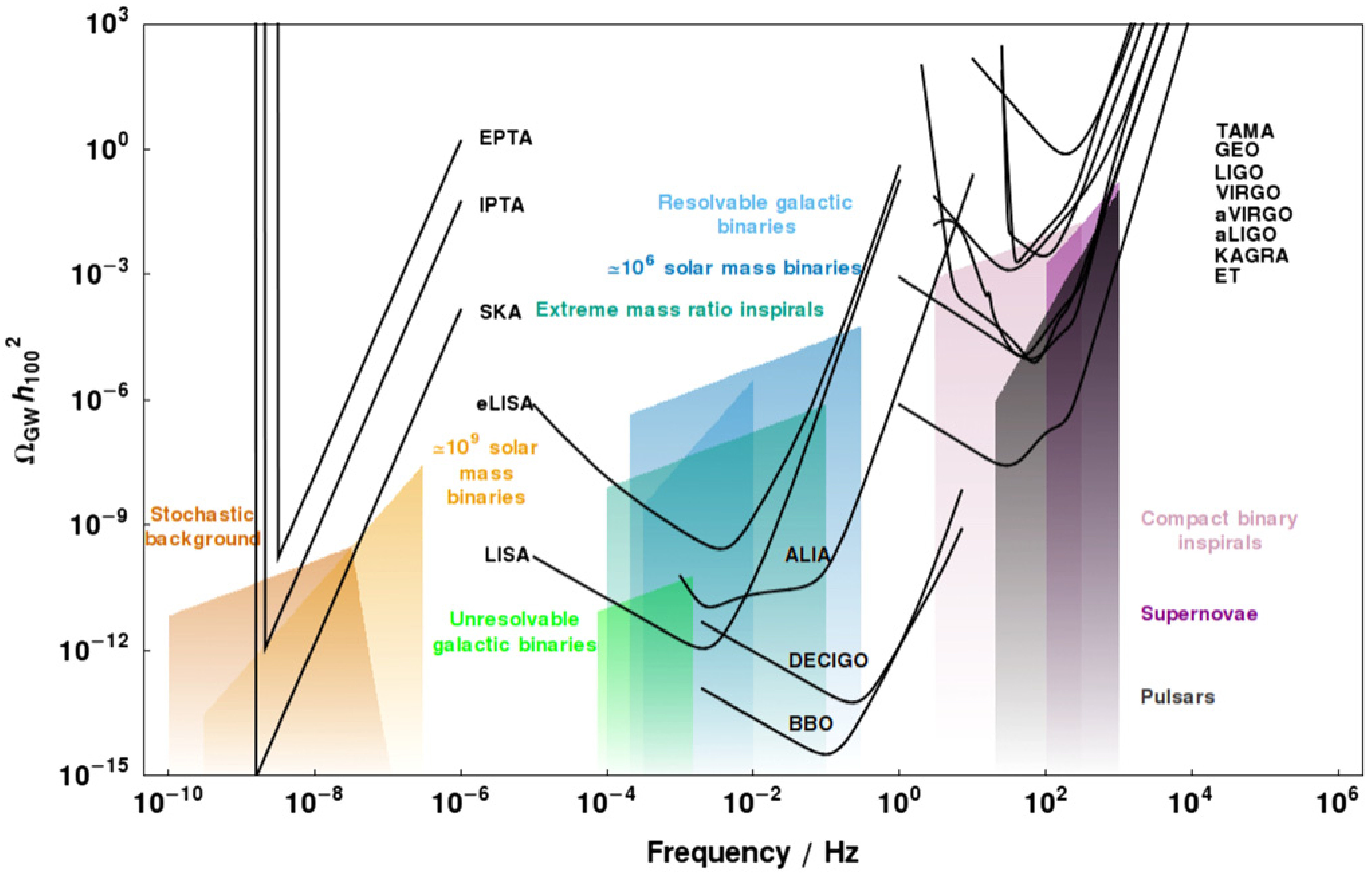
If suitable conditions are satisfied, the SGWB signal can be loud enough to be detectable with GW experiments

We focus on this case in what Follows





Comoving scales
CMB
PTA
LISA
AdvLIGO
Possible to probe small scales inaccessible by CMB or LSS
Comoving Horizon
-63
-55
-40
-36
-28
-22
-17
-15
Time $(\ln a)$



These experiments will take place

Let's see if we can use them for probing the early universe

How to characterize the SGWB?

- Frequency dependence
- Anisotropies
- Circular polarization

Theory → GW Exp

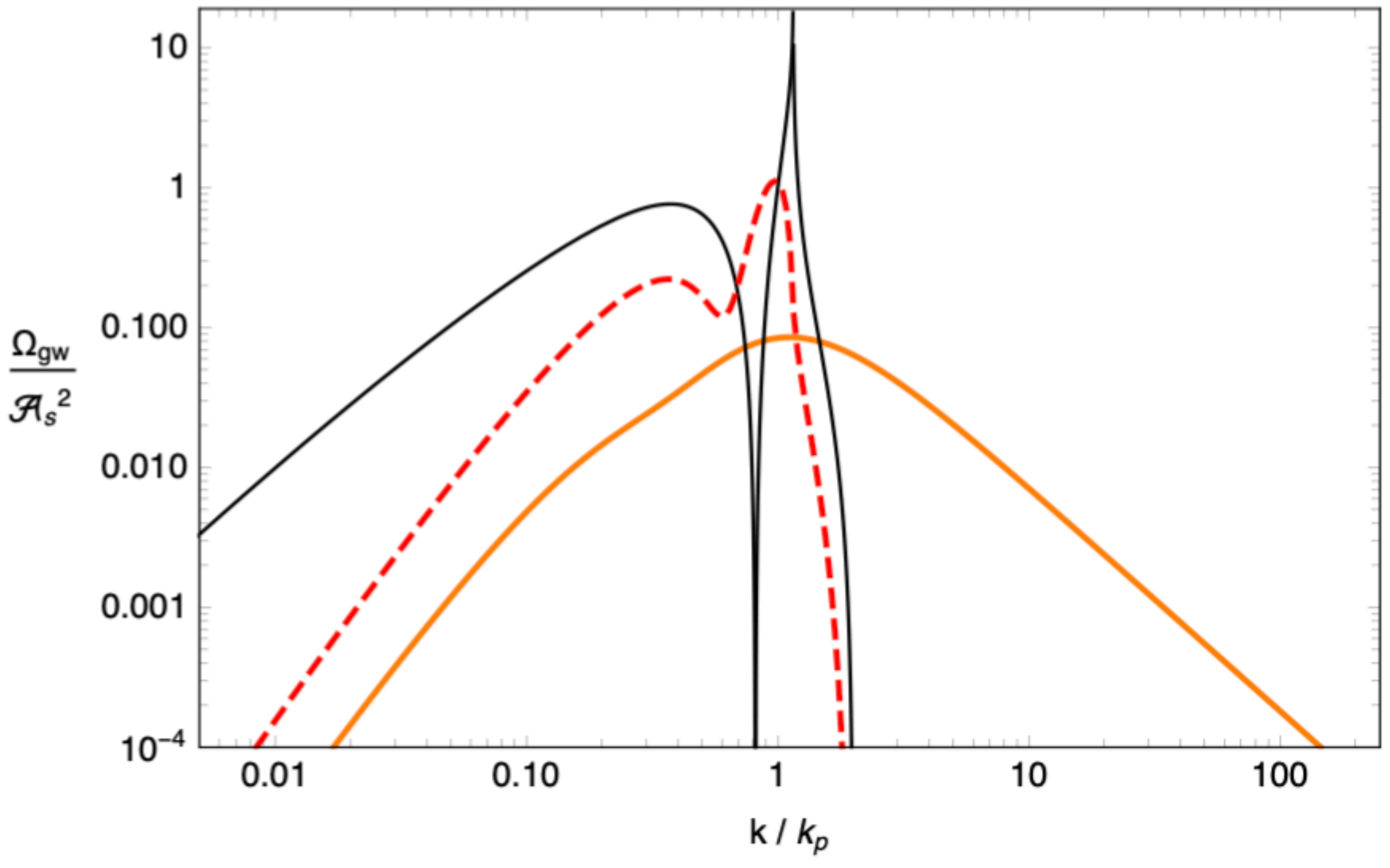
Frequency dependence of SGWB

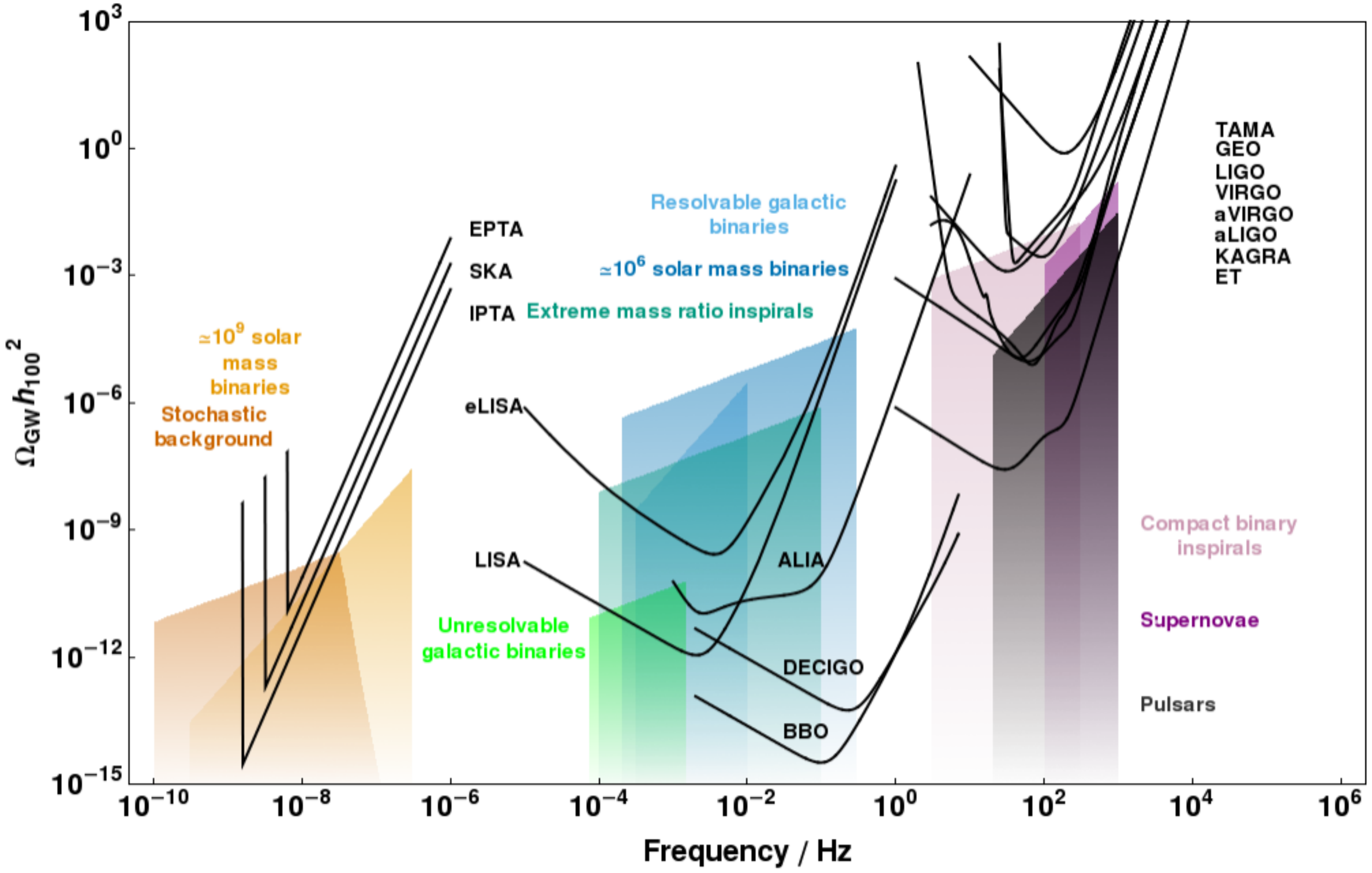
Frequency dependence of SGWB

SCALAR INDUCED GW

▷ Rich frequency profile of SGWB energy parameter

$$\Omega_{\mathrm{GW}} = \frac{1}{\rho_{\mathrm{cr}}} \frac{d\rho_{\mathrm{GW}}}{d\ln f}$$





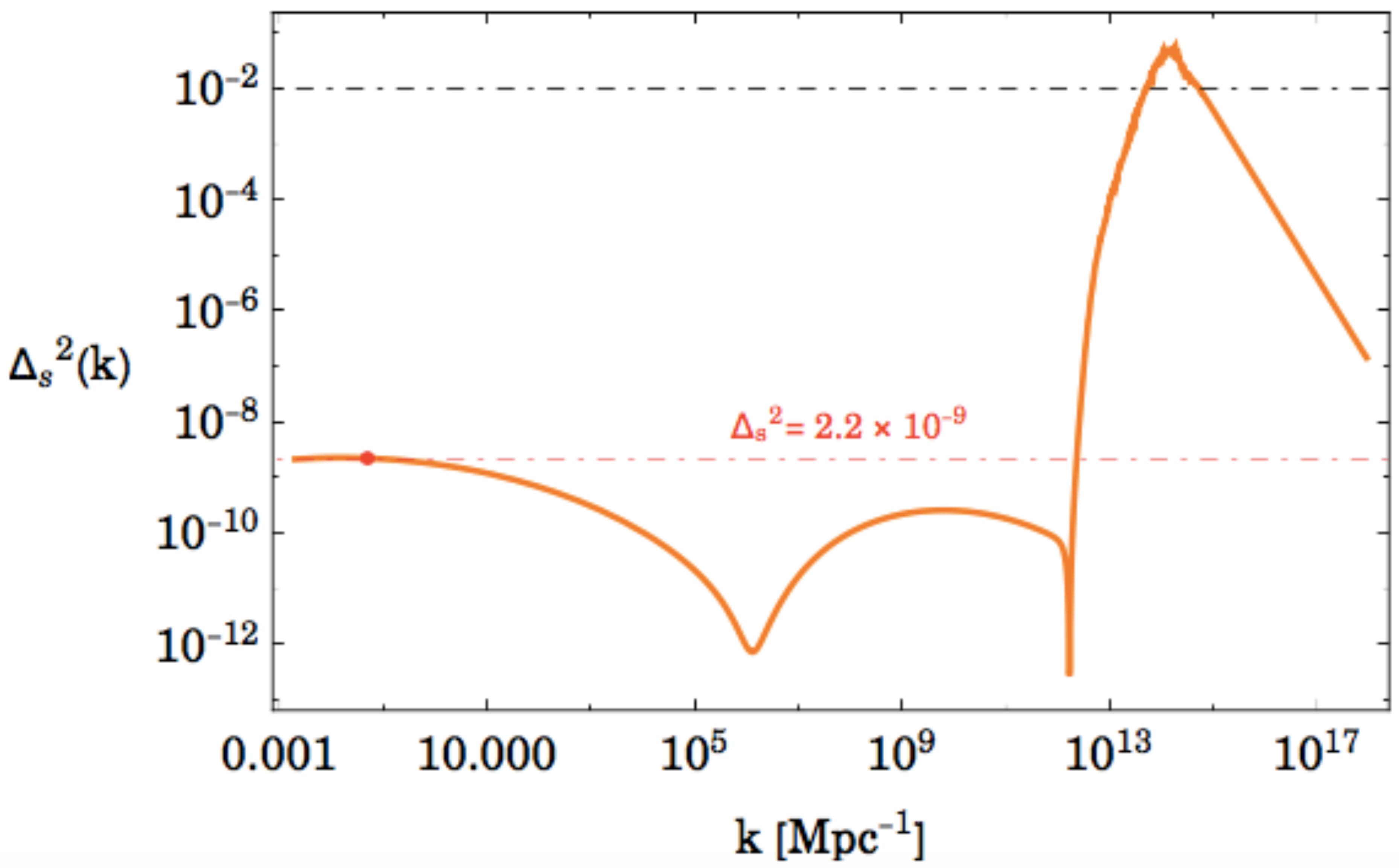
Frequency dependence of SGWB

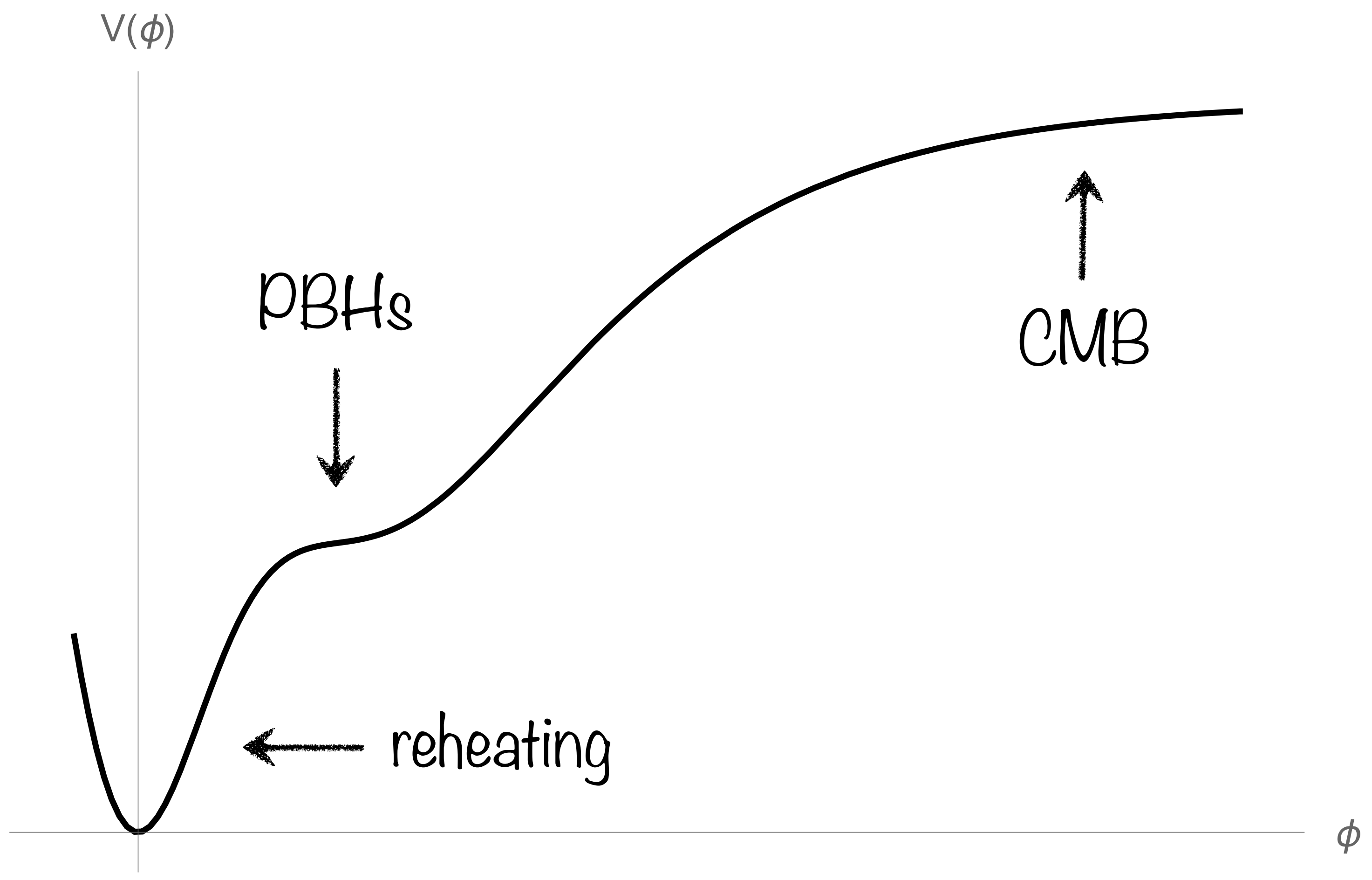
SCALAR INDUCED GW

▷ **Rich frequency profile** of `SGWB energy parameter`

Inflationary models with inflection points

$\rightarrow$ rapid growth of primordial $\mathcal{P}_\zeta$



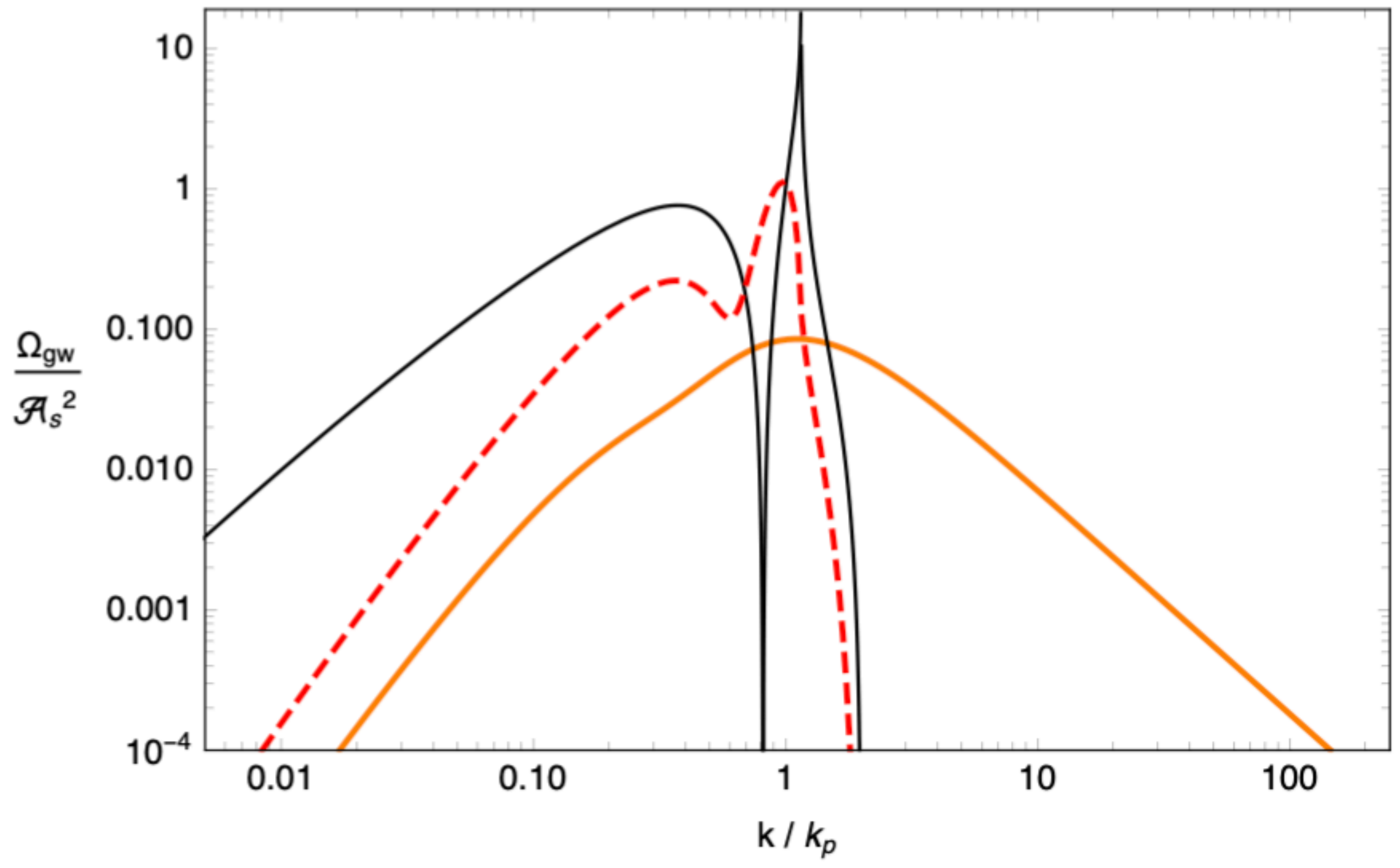


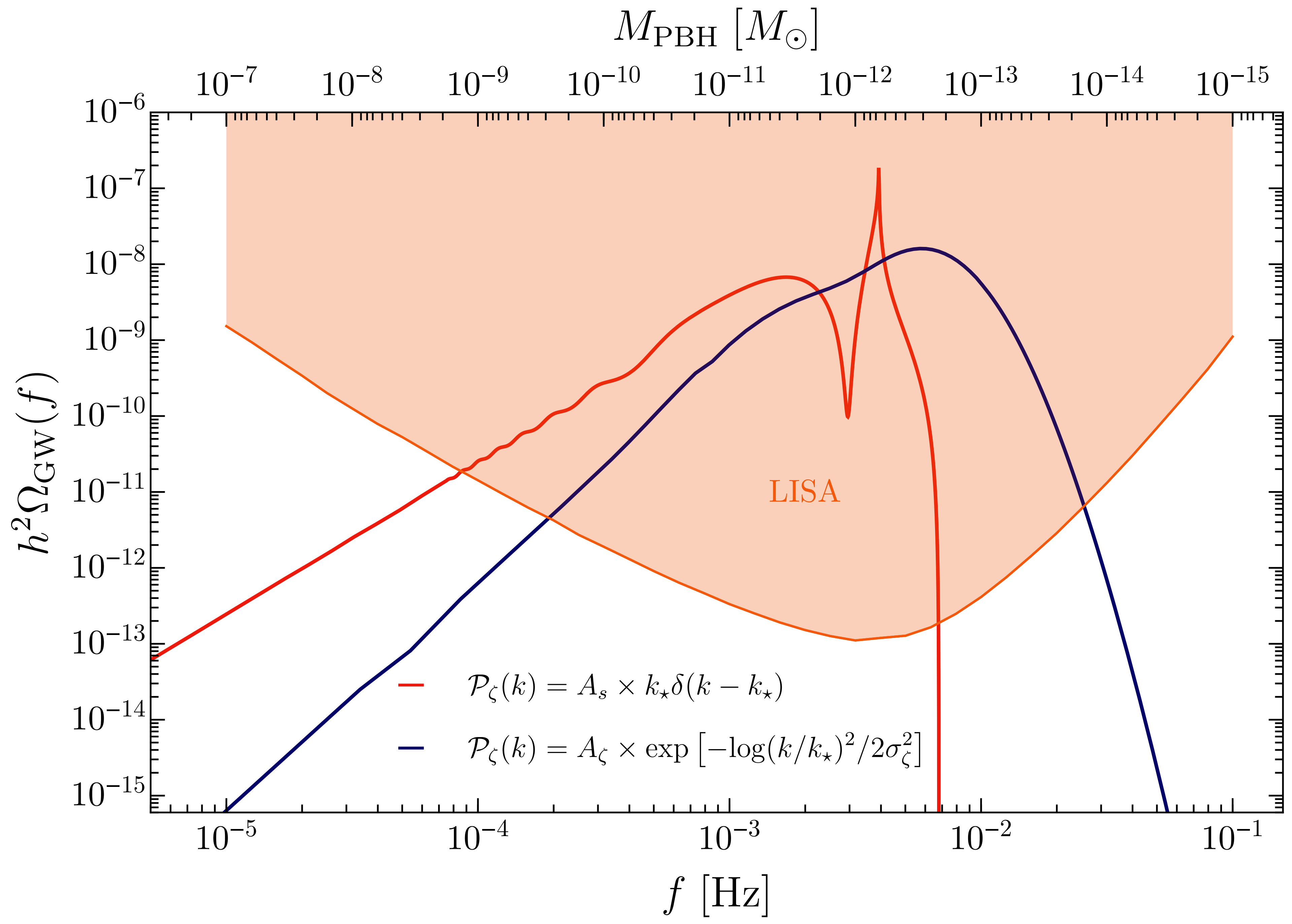
[Kinney et al], [Garcia-Bellido et al]

Frequency dependence of SGWB

SCALAR INDUCED GW

SGWB produced at 2$^{\text{nd}}$ order in perturbations during RD or MD



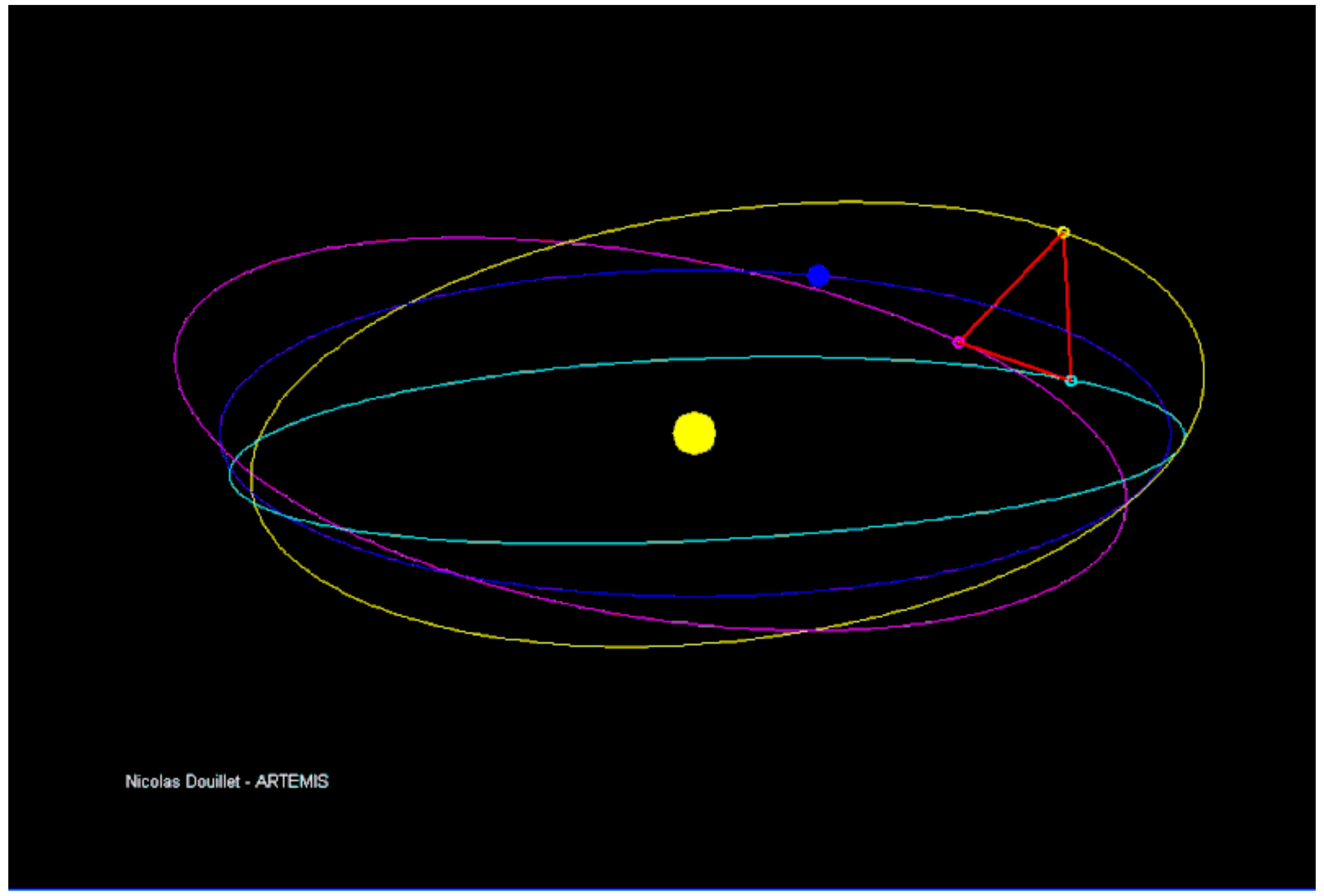


Smoking gun of cosmological sources, since astro SGWB have power-law profiles.

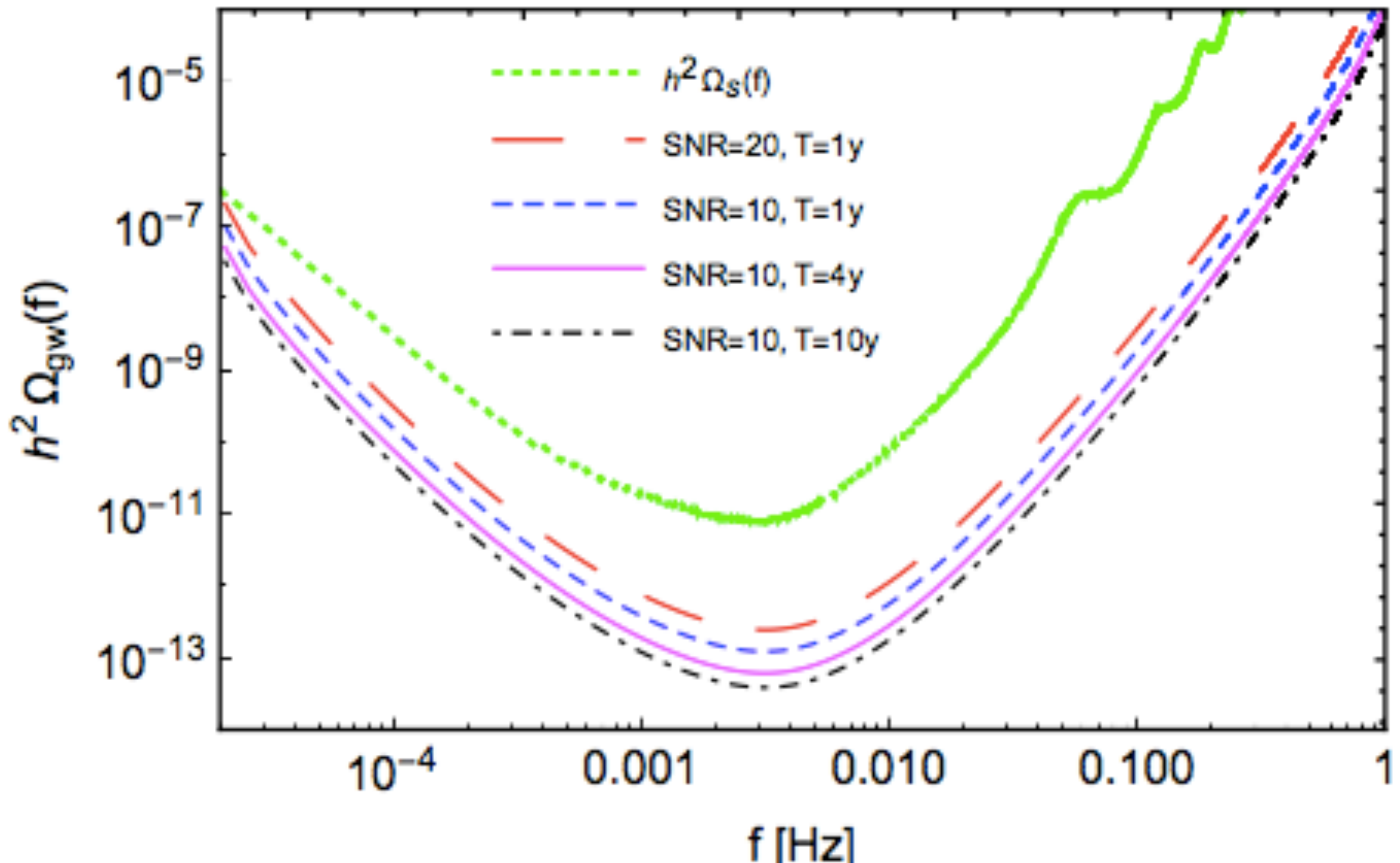
Plots from [Bartolo et al.]

How to measure ? The case of LISA

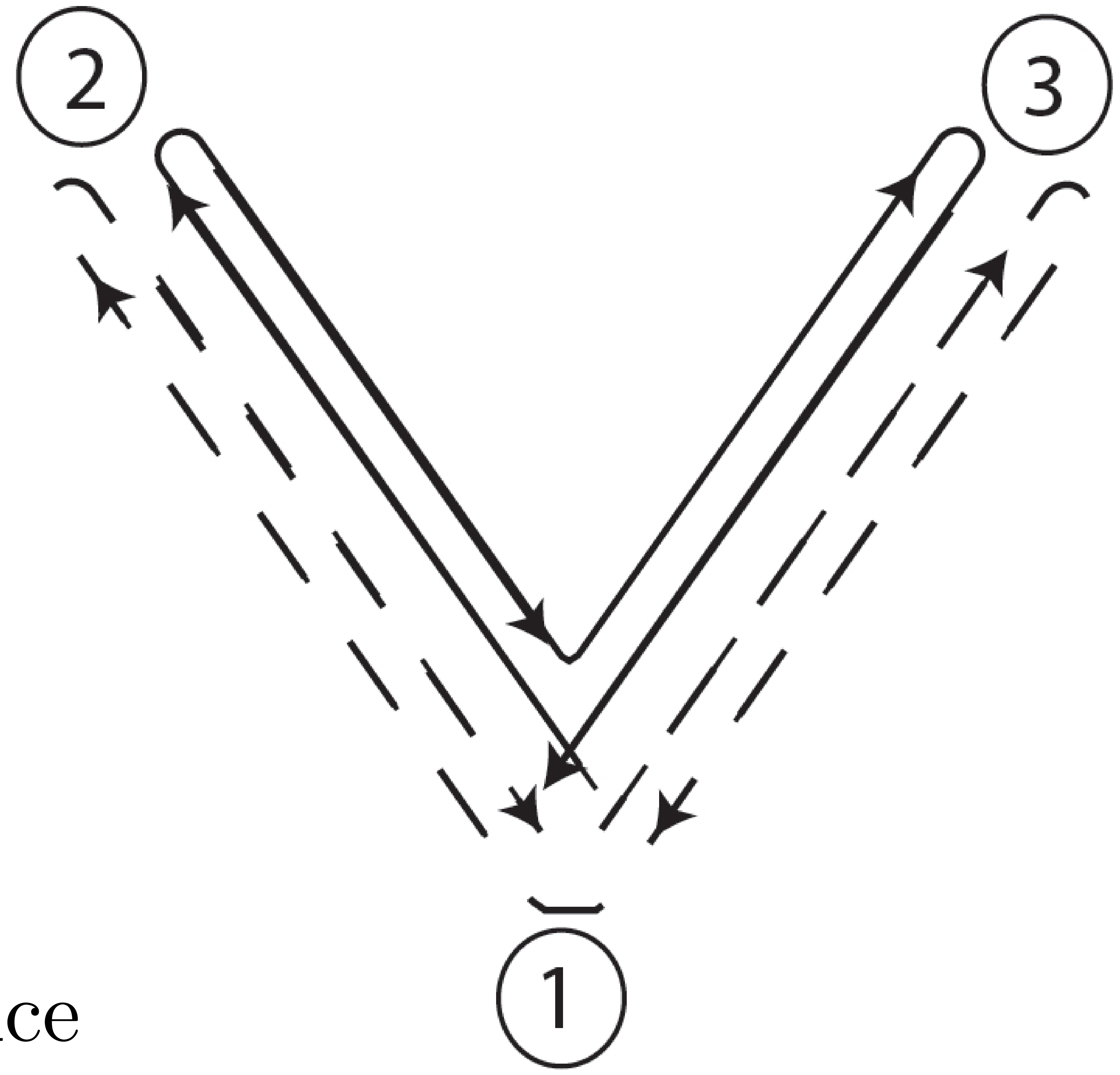
LISA **The Laser Interferometer Space Antenna** Observing gravitational waves from space	
Cosmic Vision Themes	The Gravitational Universe
Primary goals	Observing low-frequency gravitational waves (from 0.1 mHz to 0.1 Hz) and studying their various sources from across the cosmos
Orbit	Three spacecraft in an Earth-trailing heliocentric orbit about 50 million km from Earth (inter-spacecraft separation of 2.5 million km)
Launch	2037
Lifetime	Four years, with possible six-year extension
Type	L-class mission



It probes a specific GW frequency range and amplitude that cannot be investigated by other means



What does LISA measure?



- Time delays between light signals travelling through interferometer arms
- Essential to identify convenient combination of signals that reduce systematic noise
- **Time-delay interferometry**

How to measure ? The case of LISA

1. First measure shape of the signal: single peak, two peaks, etc
2. Match with theoretical template $\rightarrow$ Fit parameters with data

First: characterize the shape of signal

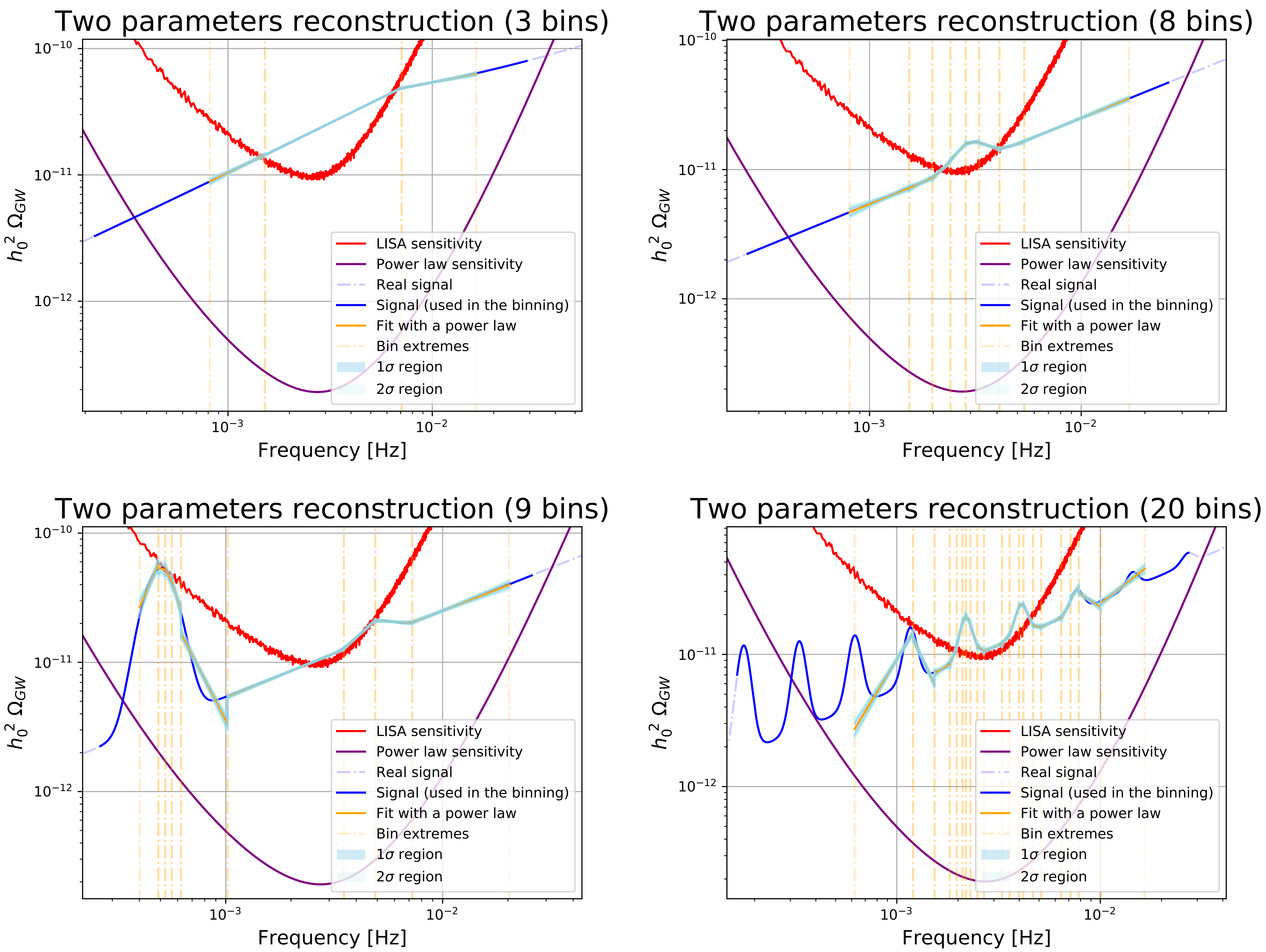
GWbinner code:
divide LISA band
in small intervals
reconstructing signal
in each bin

arXiv:1906.09244

Algorithm

– Break the LISA band into bins reconstruct signal as power-law in each bin

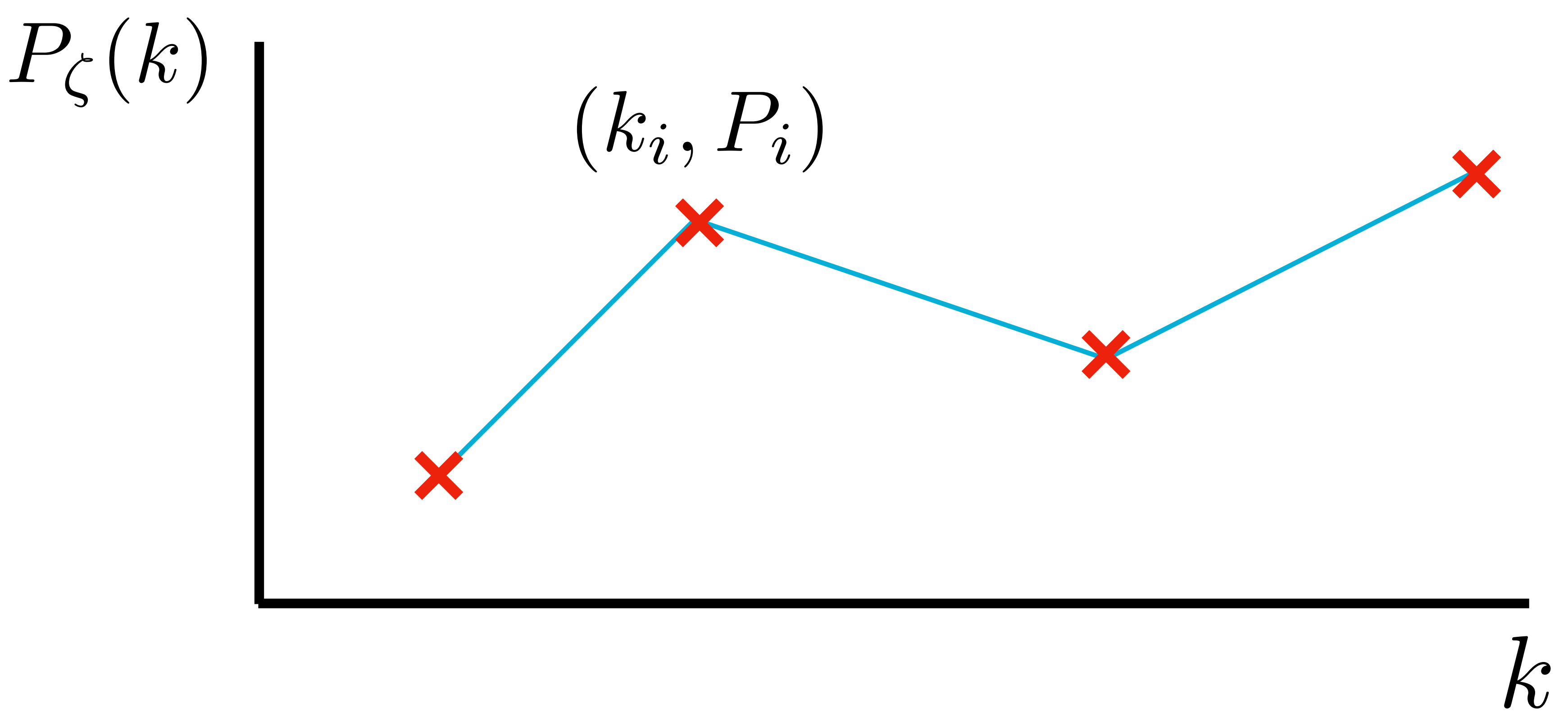
– Join continuously the result

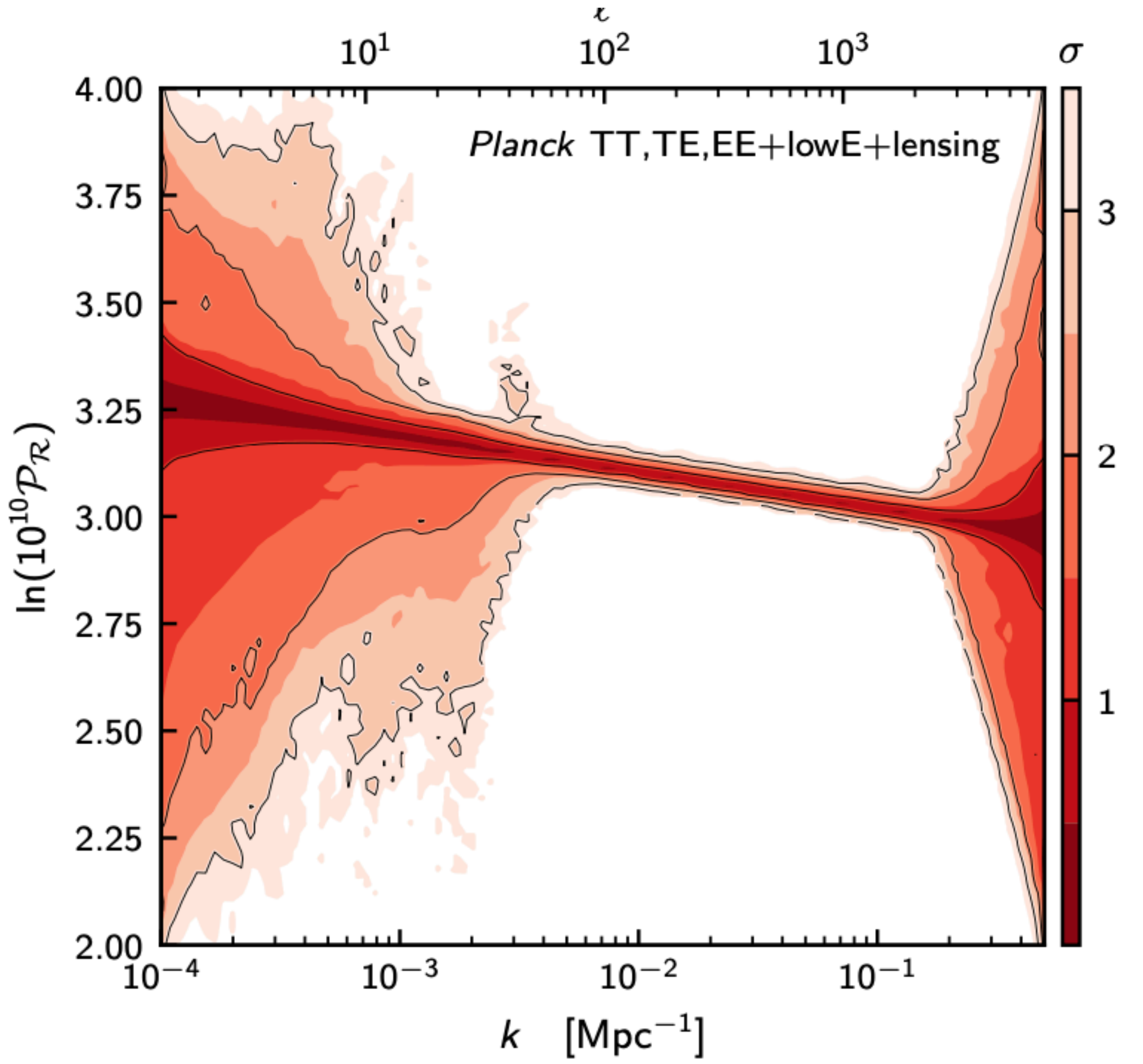


Reconstructing curvature power spectrum

Our method

Reconstruct P_ζ using linear interpolating splines (other possibilities as well) — no strong assumptions regarding functional form of spectrum





Similar methods have been previously used for the CMB

Planck 2018: Constraints on Inflation

Reconstructing curvature power spectrum

Our method

Let the data determine node positions and amplitudes. Use Bayesian evidence for relative probabilities of spline models with different number of nodes.

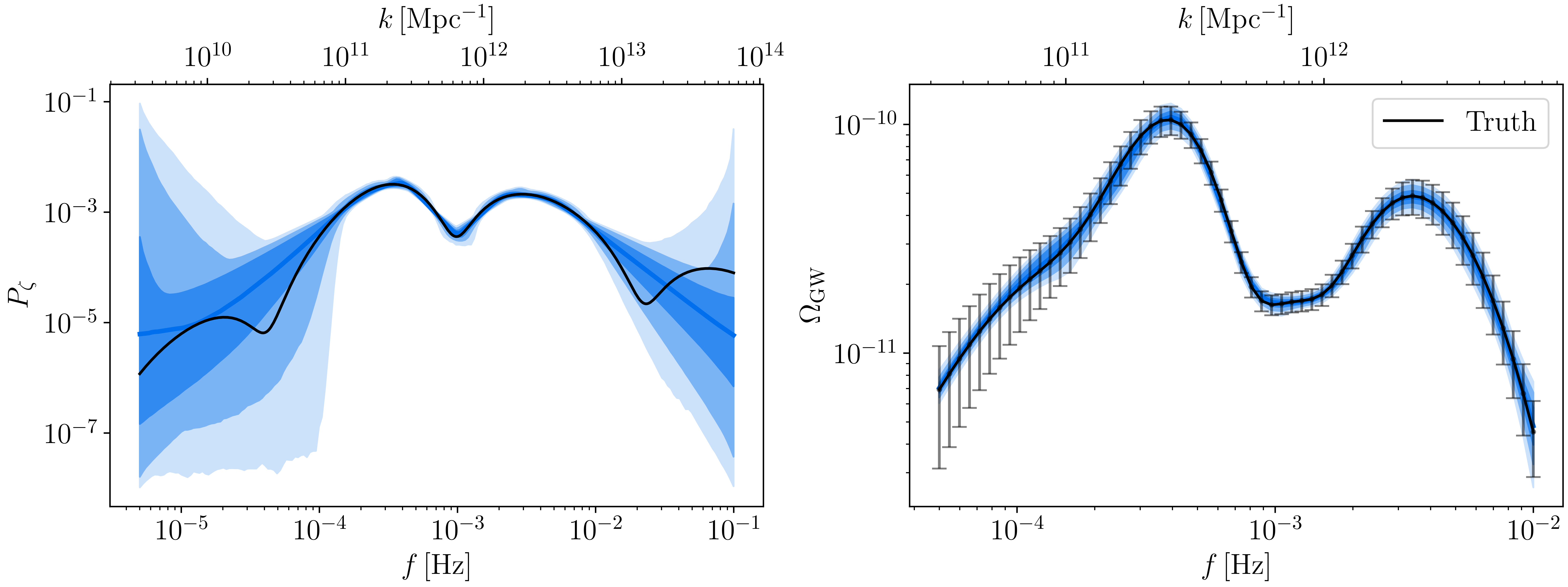
At the end, reconstruct distribution of $P_\zeta(k)$ and $\Omega_{\rm GW}(k)$ from the samples after marginalising over the reconstructions with different number of nodes

$$\text{e.g.} \quad \mathbb{E}[P_{\zeta,k}] = \frac{\sum_n \mathbb{E}[P_{\zeta,k}|\mathcal{M}_n]\mathcal{Z}_n}{\sum_n \mathcal{Z}_n}.$$

Relative probability/weight of model with n nodes

Reconstructing curvature power spectrum

Example: Radiation domination



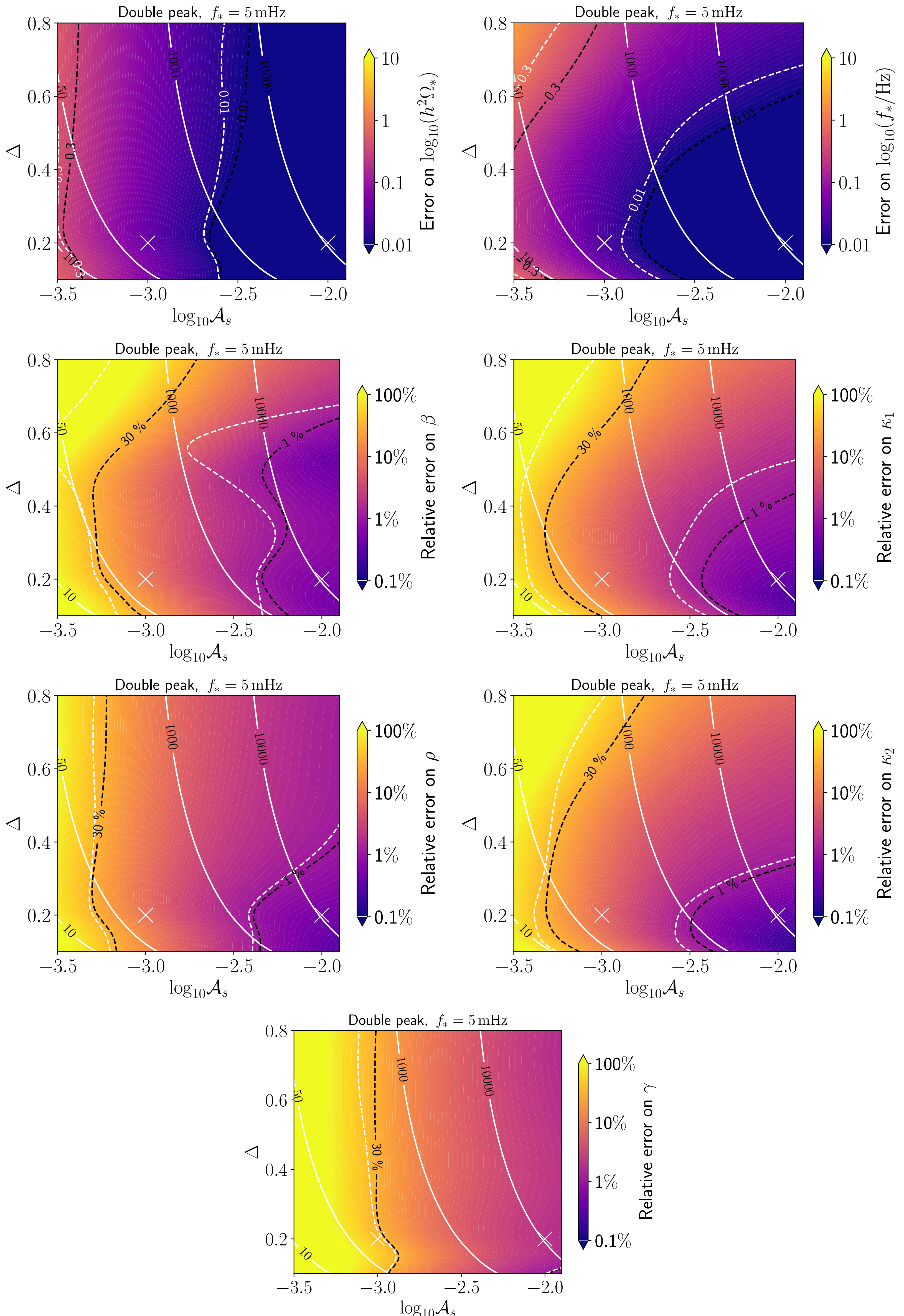
Match precisely with theoretical template

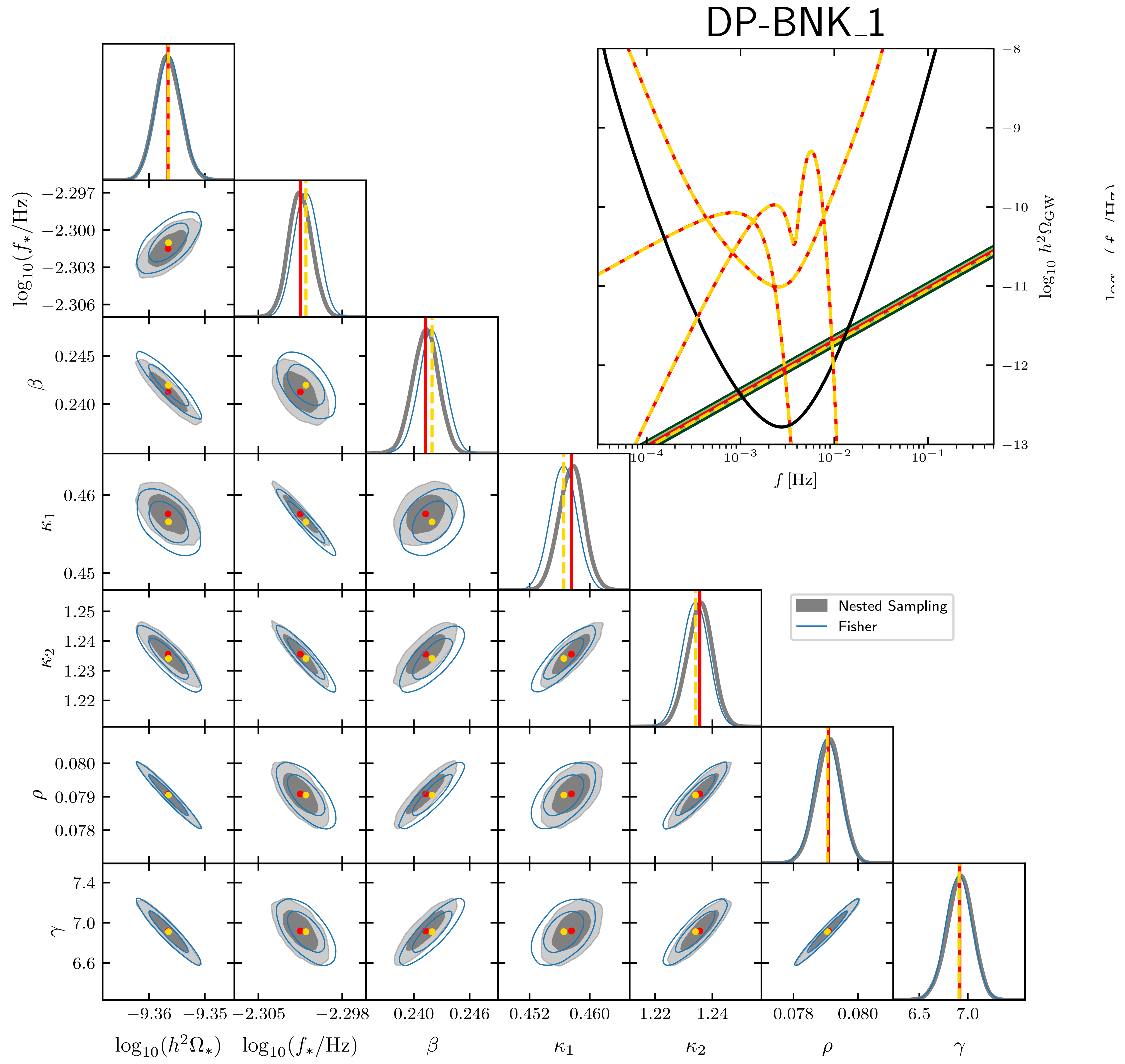
Scalar-induced GW

LISA CosWG

Braglia et al

El Gammal et al

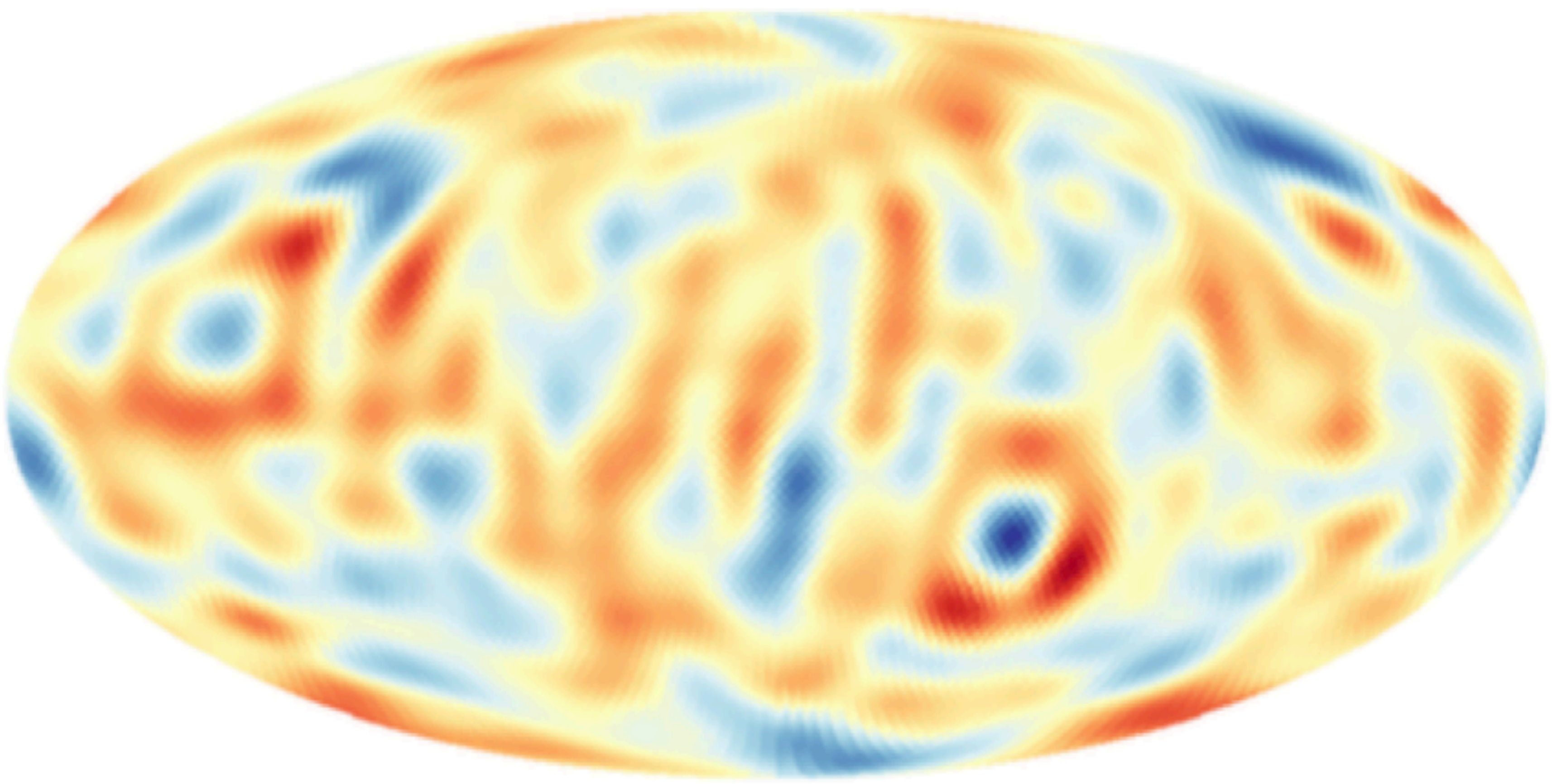




Anisotropies of the SGWB

Anisotropies of the SGWB

- Observable offering **new opportunities** to distinguish among sources of SGWB
- Relatively unexplored: **much work needed** for their characterisation both from a **theoretical** and an **experimental** point of view

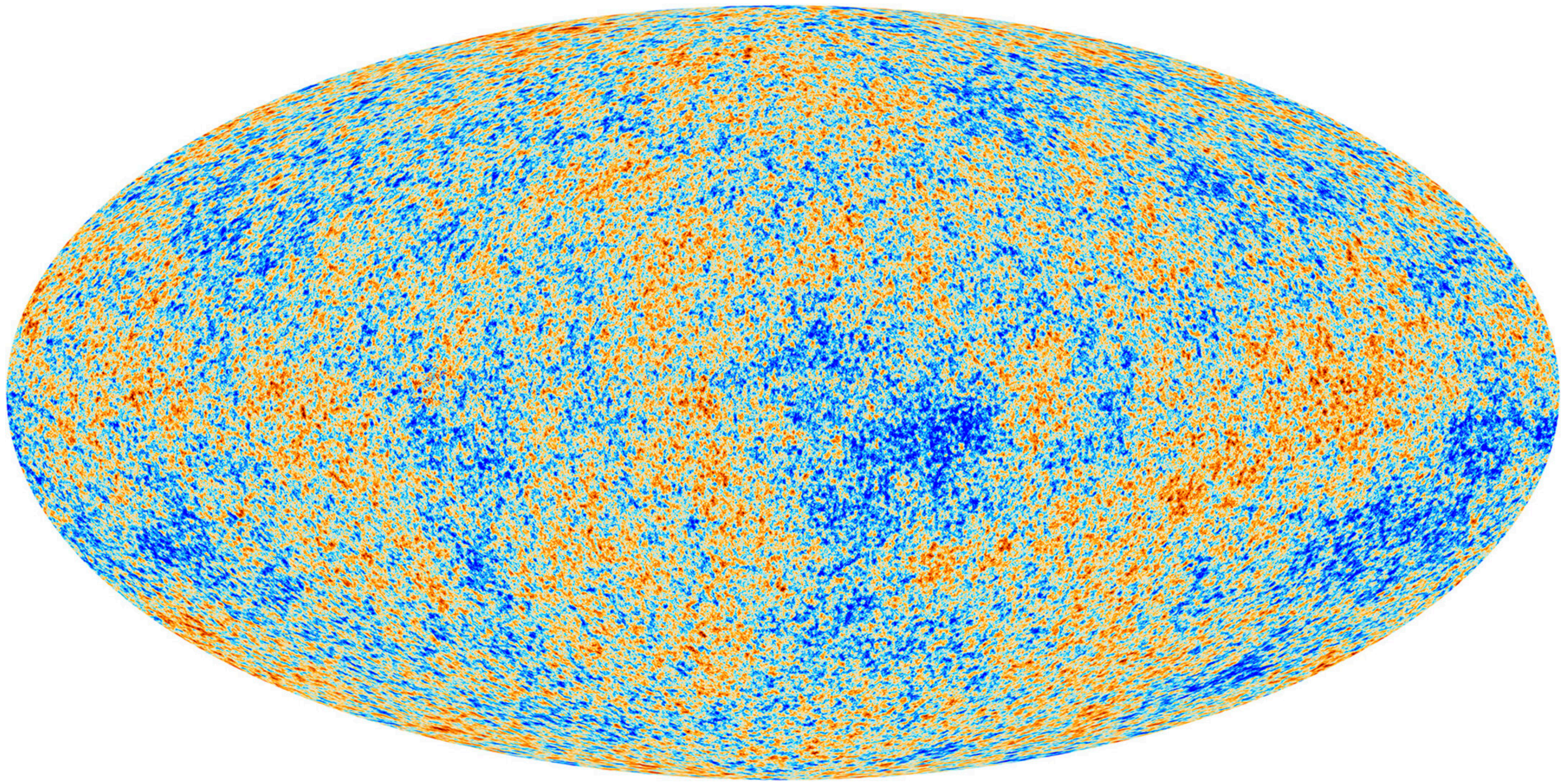


[Renzini, Contaldi]

Anisotropies of the SGWB

Theoretical framework

Explore parallelism with physics of CMB anisotropies



Boltzmann approach [Contaldi, Bartolo et al] following [Dodelson]

Anisotropies of the SGWB

Theoretical framework

Explore parallelism with physics of CMB anisotropies

Boltzmann approach [Contaldi, Bartolo et al] following [Dodelson]

Geometrical optics/short wavelength approx

GW: Stream of collisionless gravitons following null geodesics through perturbed metric

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)\, d\eta^2 + (1-2\Psi)\,\delta_{ij}\, dx^i dx^j\right]$$

Anisotropies of the SGWB

Theoretical framework

Explore parallelism with physics of CMB anisotropies

Boltzmann approach [Contaldi, Bartolo et al] following [Dodelson]

Geometrical optics/short wavelength approx

GW: Stream of collisionless gravitons following null geodesics through perturbed metric

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)\,d\eta^2 + (1-2\Psi)\,\delta_{ij}\,dx^i dx^j\right]$$

GW distribution function $\Delta(\eta,\, \vec{x},\, q,\, \hat{n})$

Direction of propagation $\hat{n} = \vec{p}/|\vec{p}|$

Conformal time

Magnitude $q = |\vec{p}|a$

of comoving momentum.

Frequency $f = |\vec{p}|/(2\pi)$

Anisotropies of the SGWB

Boltzmann equation

$$\frac{d\,\Delta}{d\,\eta} = \mathcal{C}\,[\Delta] + \mathcal{I}\,[\Delta]$$

Collision term

absent since GW decouple

Injection term

initial conditions

▷ Solve **perturbatively** Boltzmann eq

$$\boxed{\Delta(\eta,\, \vec{x},\, q,\, \hat{n}) \;=\; \bar{\Delta}(q) - \frac{\partial\bar{\Delta}}{\partial \ln q}\,\Gamma(\eta,\, \vec{x},\, q,\, \hat{n})}$$

Equilibrium distribution

$$\delta_{\rm GW}(\eta,\, \vec{x},\, q,\, \hat{n}) \;=\; \frac{\Omega_{\rm GW} - \bar{\Omega}_{\rm GW}}{\bar{\Omega}_{\rm GW}} \;=\; \left[4 - \frac{d\,\ln\bar{\Omega}_{\rm GW}}{d\,\ln q}\right]\,\Gamma$$

Anisotropies of the SGWB

Boltzmann equation

Schematically, the solution of Boltzmann eq for Γ is

$$\Gamma(\eta,\,\vec{x},\,q,\,\hat{n}) \;=\; \Gamma_{\rm initial} + \Phi + \underbrace{\int_{\eta_i}^{\eta} d\tilde{\eta}\,\left(\Phi' + \Psi'\right)} + \text{Doppler} + \text{tensor contributions}$$

Initial conditions

Great opportunity

to directly probe GW sources

Effect of propagation

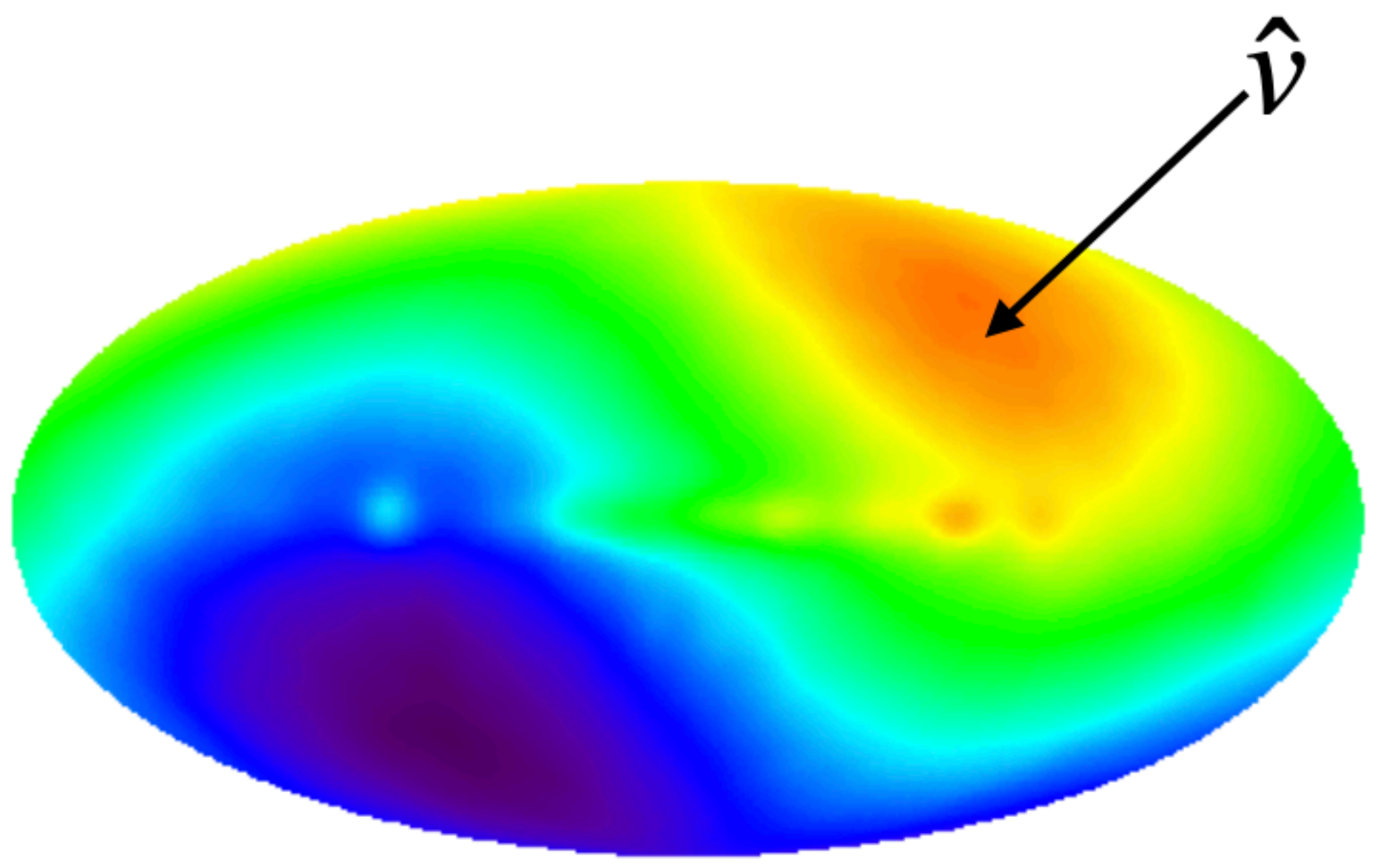
and inhomogeneities

Kinematic effects $\propto\ \hat{n}\vec{v}$

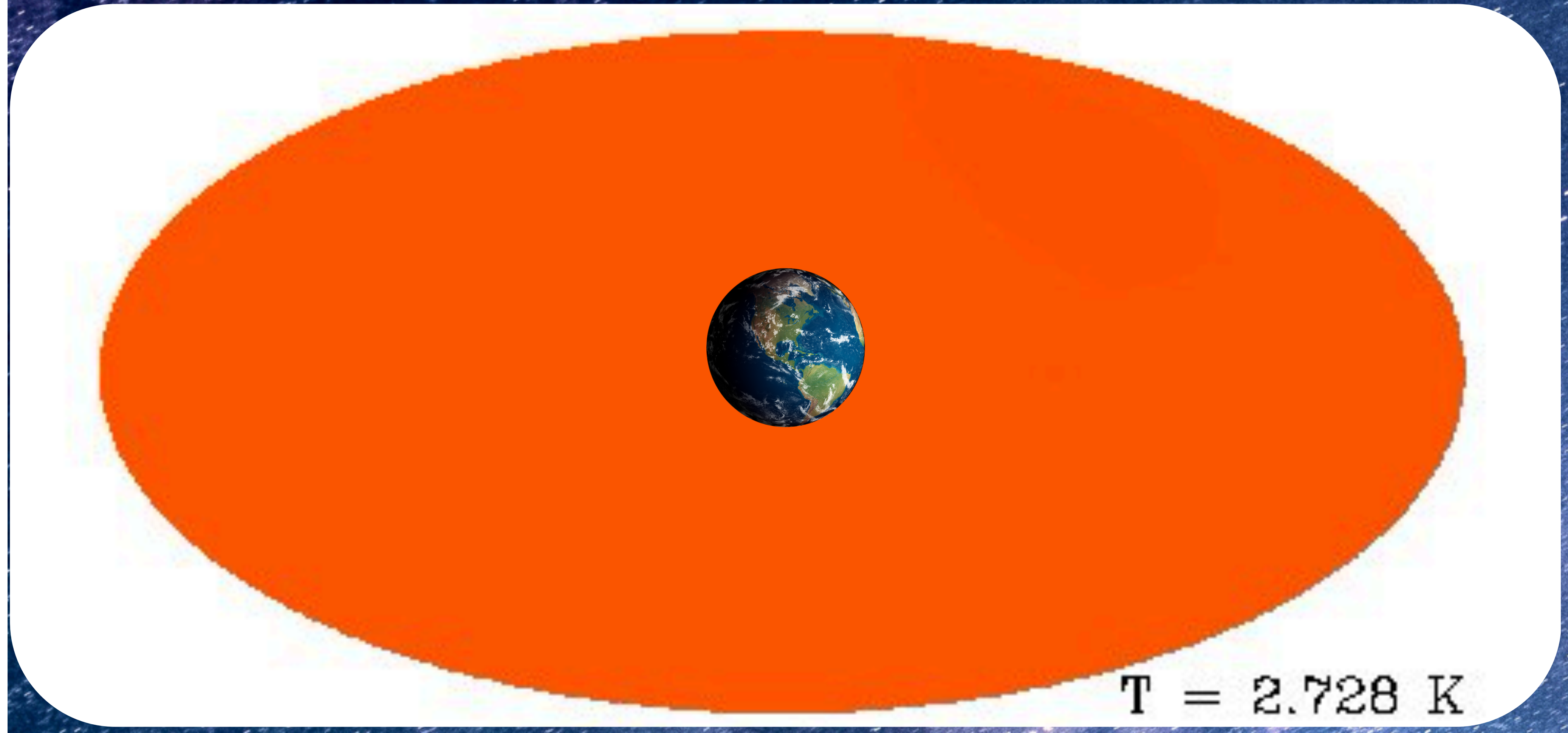
contain precious info on SGWB properties

Kinematic anisotropies

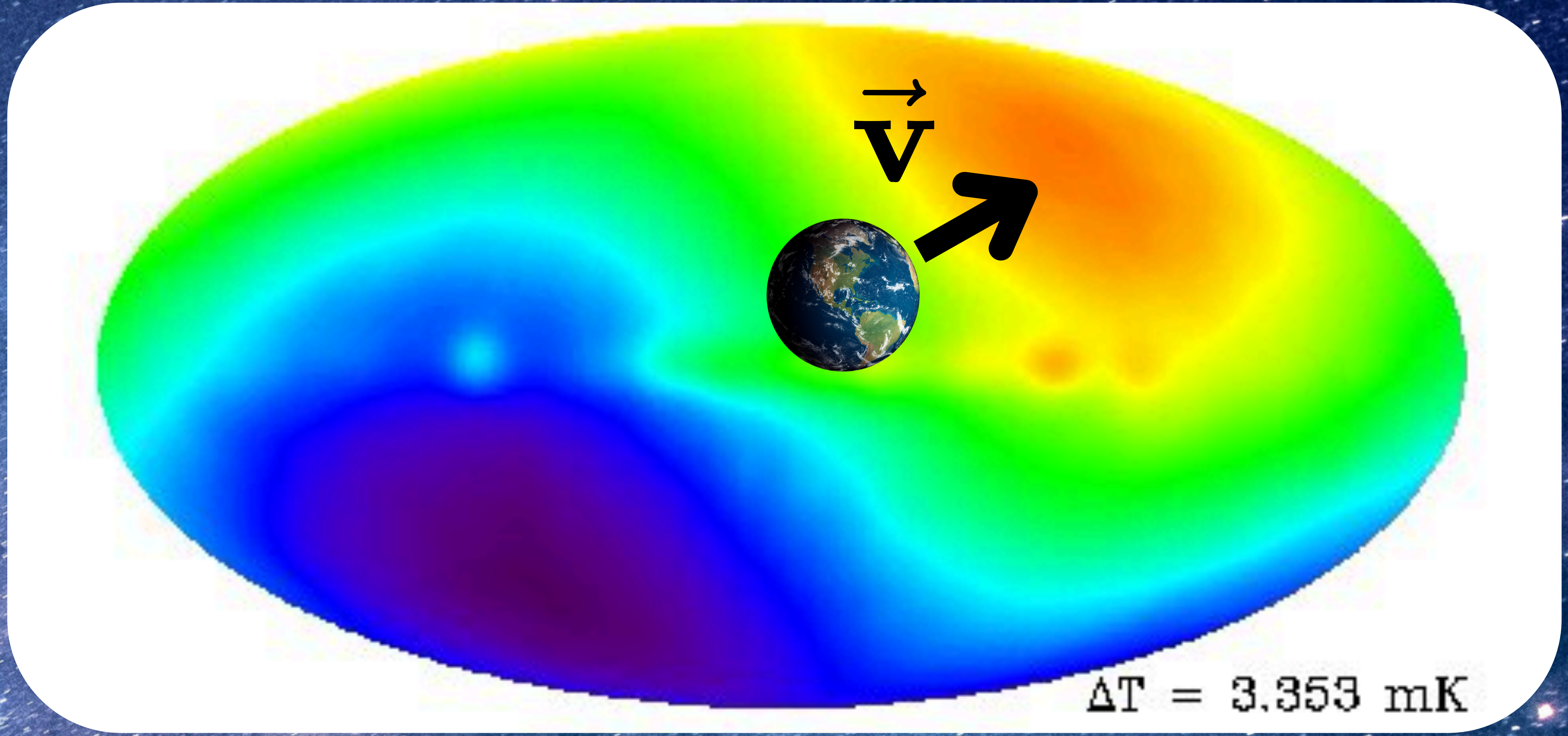
Dominant source of anisotropy for Cosmological SGWB



Kinematic anisotropies



Isotropic signal



Kinematic anisotropies

Already observed in the CMB!

$$\beta = |\vec{\mathbf{v}}|/c = 1.23 \times 10^{-3}$$
$$(l, b) = (264^\circ, 48^\circ)$$

Kinematic anisotropies of the SGWB with PTA

- For cosmo SGWB, we do expect a large **Doppler anisotropy** due to our relative motion wrt SGWB source rest frame

$$\boxed{\left.\frac{\Delta\Omega_{\rm GW}}{\bar{\Omega}_{\rm GW}}\right|_{\rm Doppler} \simeq \mathcal{O}(10^{-3})}$$

`Well-defined, ambitious target`

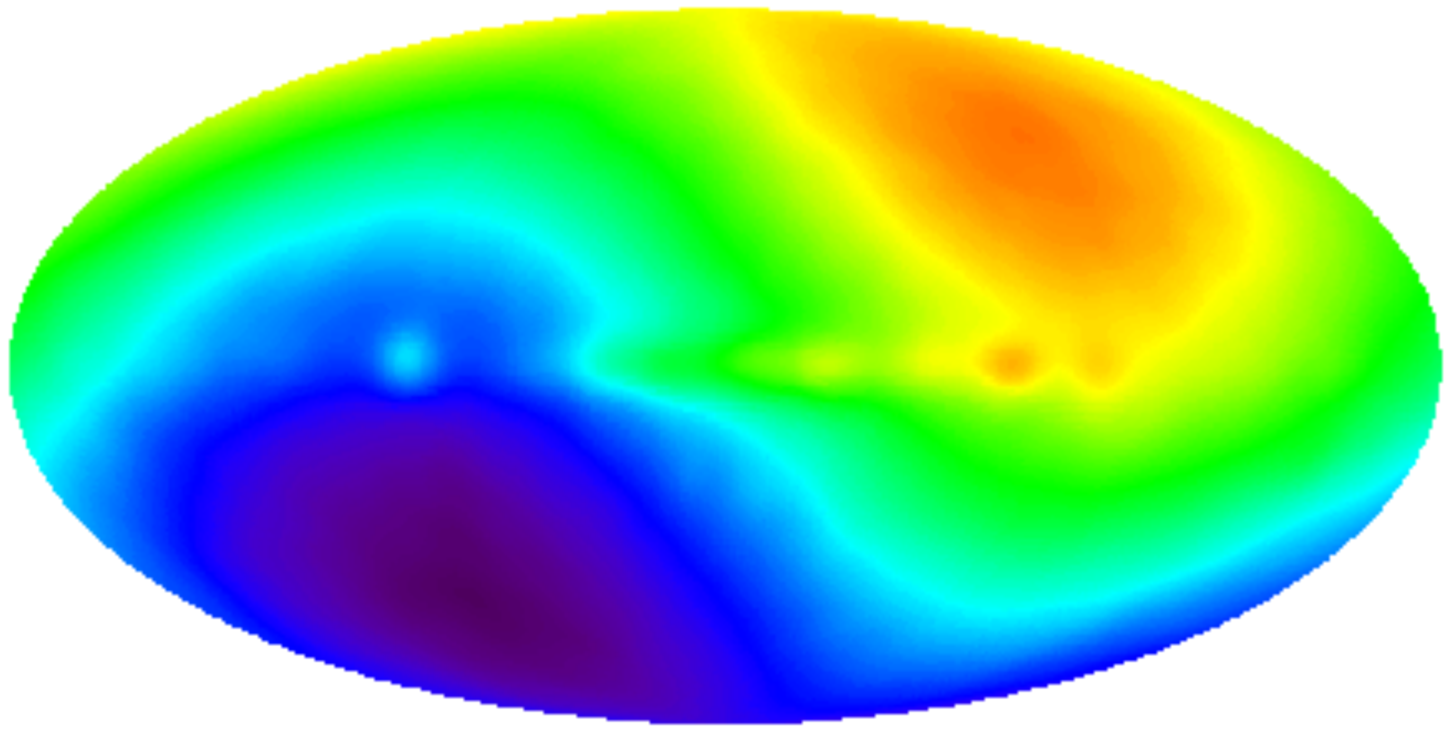
- Especially interesting as **independent probe** of intensity and direction of our speed wrt SGWB rest frame

- Also sensitive to the frequency profile of $\Omega_{\rm GW}(f)$.

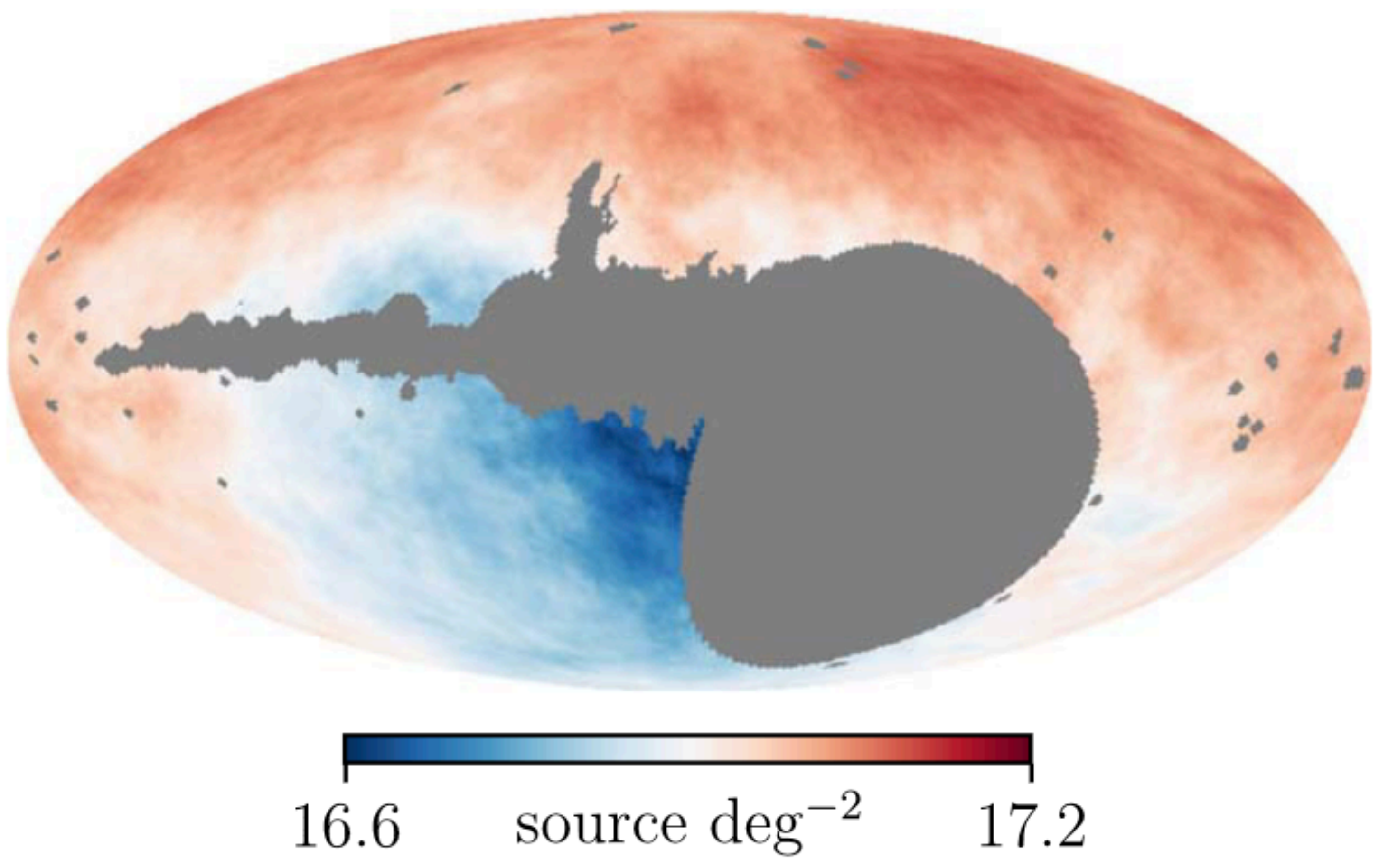
Dipole tension

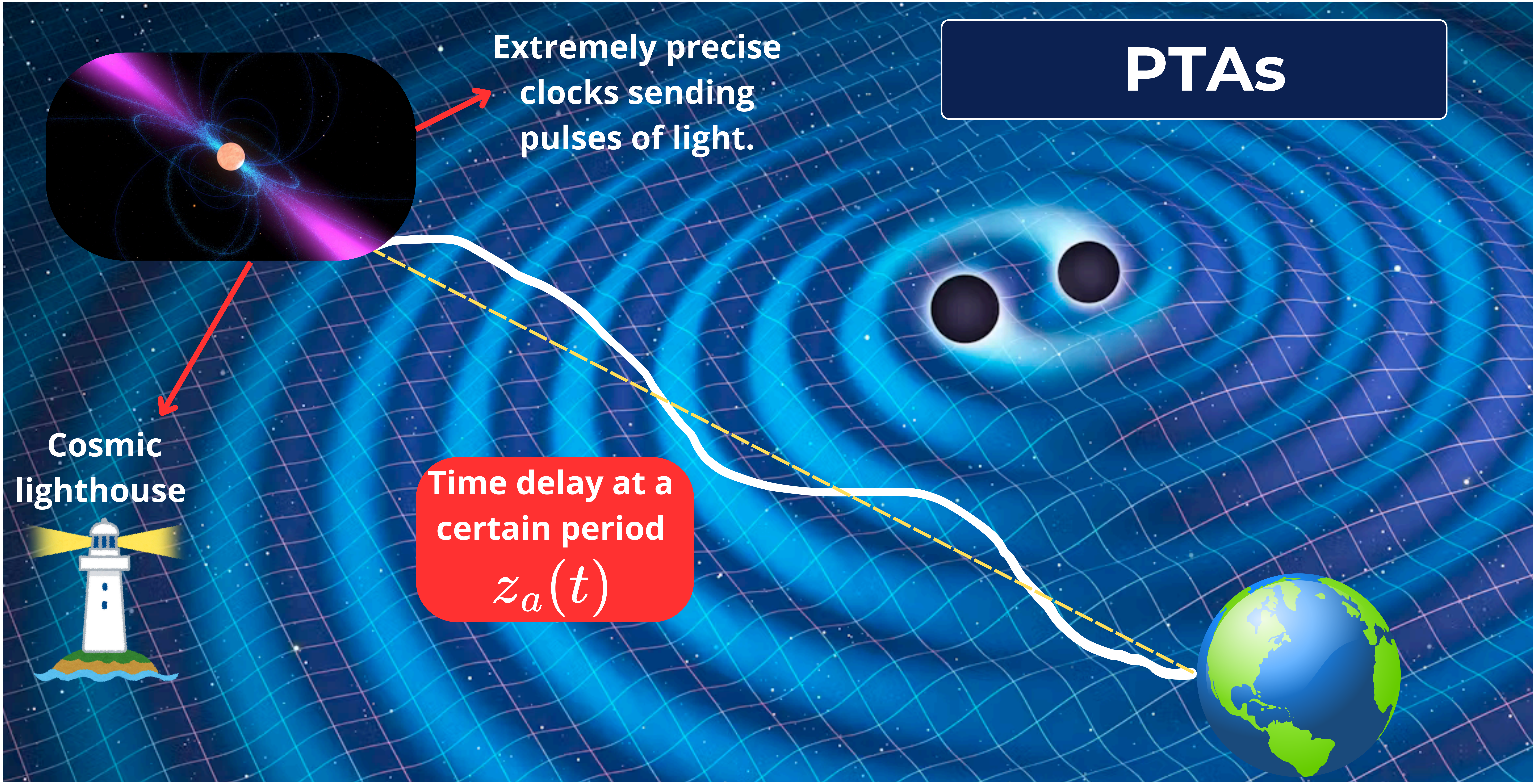
CMB and LSS estimates of β appear to be in tension, $\beta_{\mathrm{LSS}} \approx 2\beta_{\mathrm{CMB}}$

[See Peebles (2022) for a review]



COBE dipole

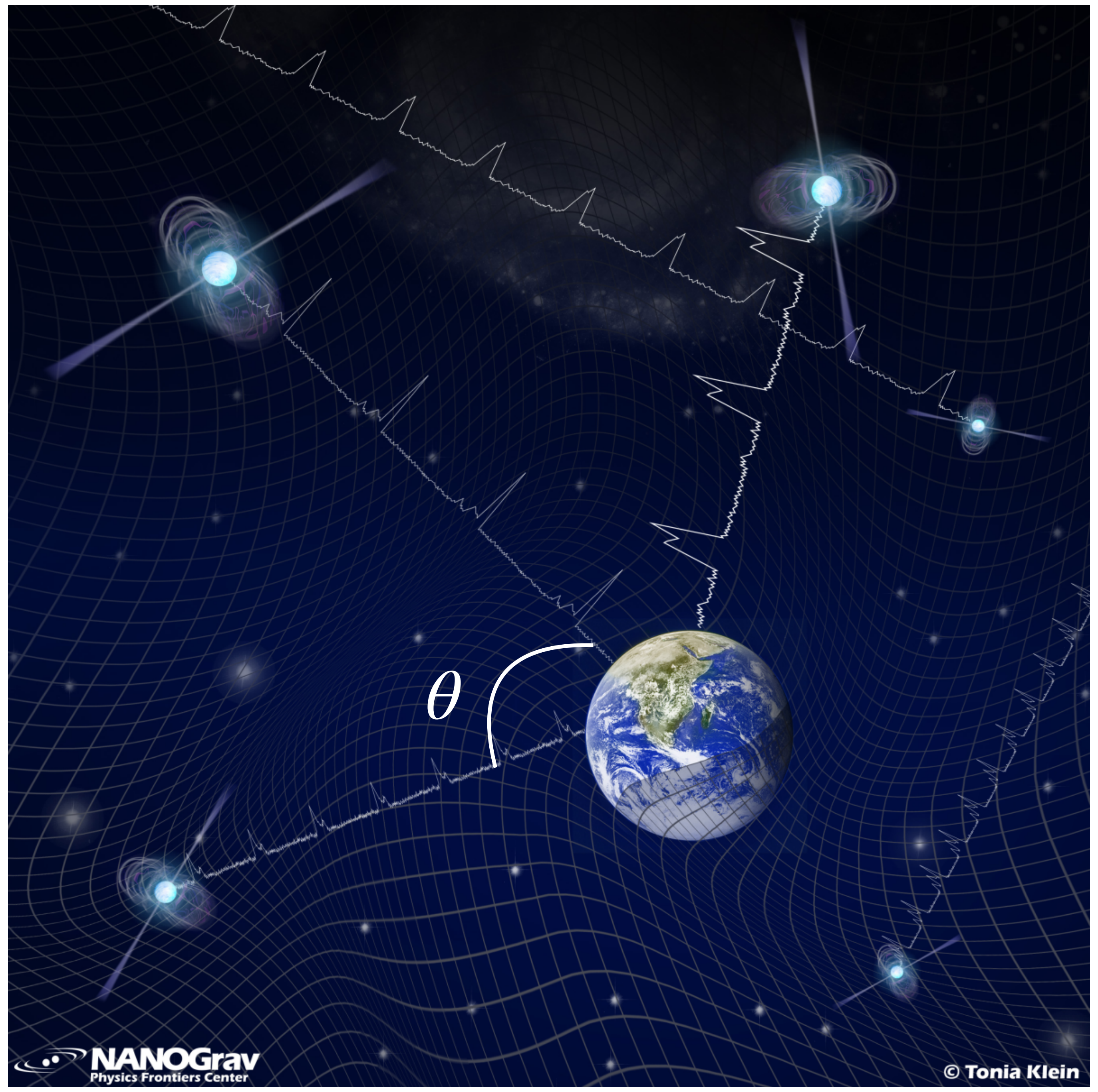


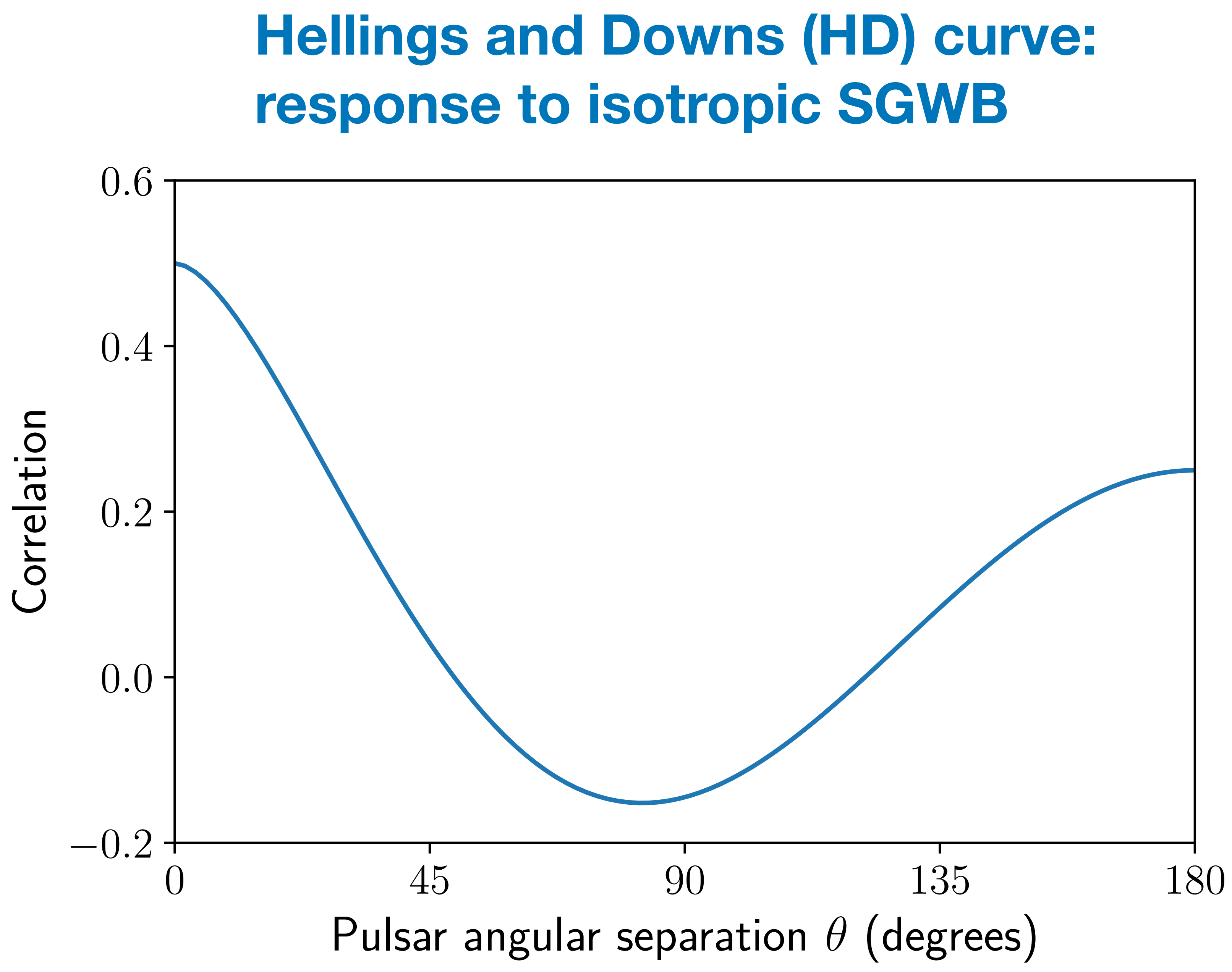

PTAs
Extremely precise clocks sending pulses of light.
Cosmic lighthouse
Time delay at a certain period
$z_a(t)$

Pulsar Timing Arrays and Gravitational Waves

Recently, several PTA collaborations found relatively strong evidence for a signal compatible with stochastic gravitational wave background.

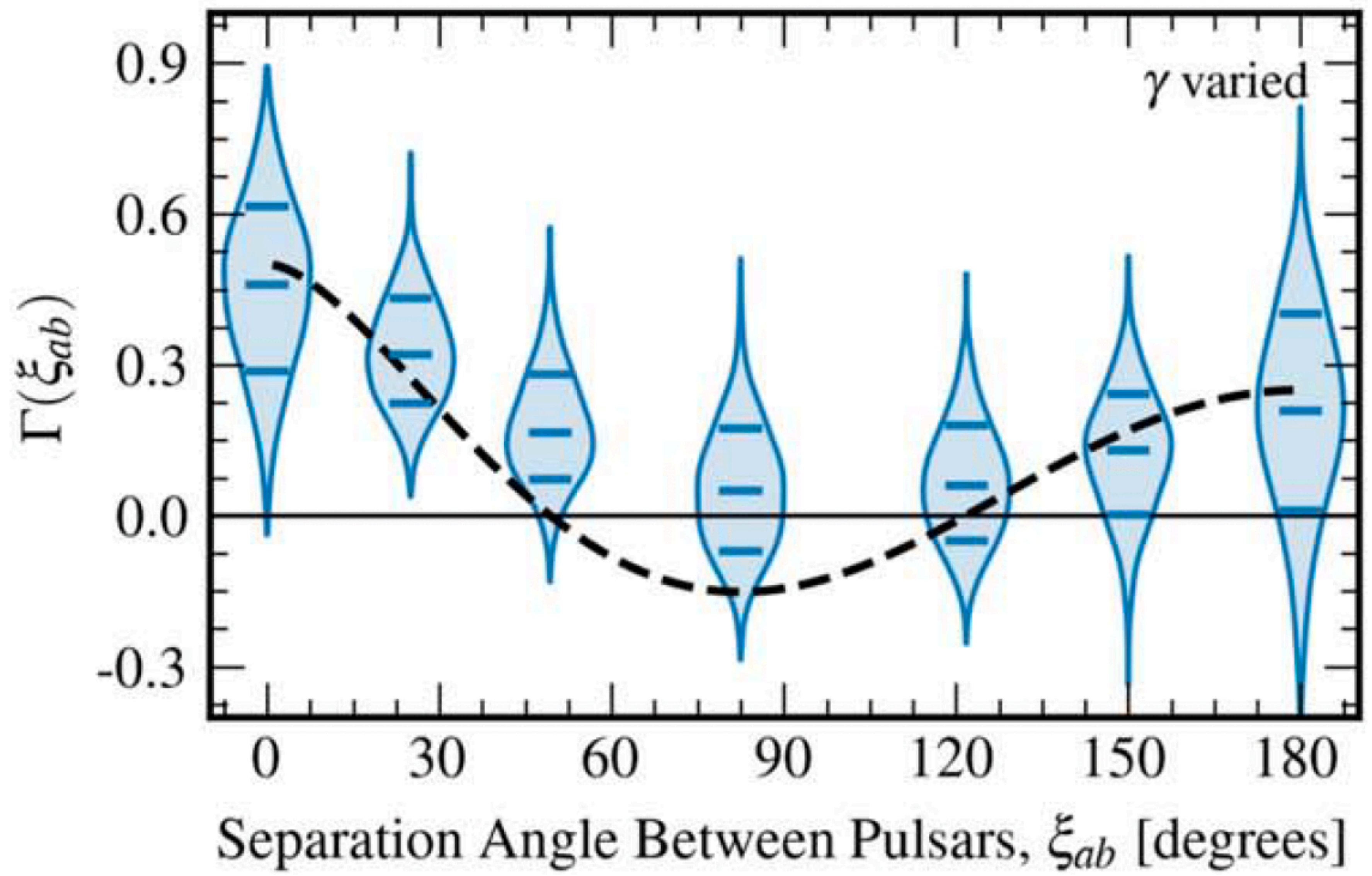
E.g. NANOGrav monitored 67 pulsars for a 15 years period. They found the characteristic angular correlation between signals detected with different pulsars, as predicted by General Relativity. This is called Hellings-Downs curve.





Pulsar Timing Arrays and Gravitational Waves

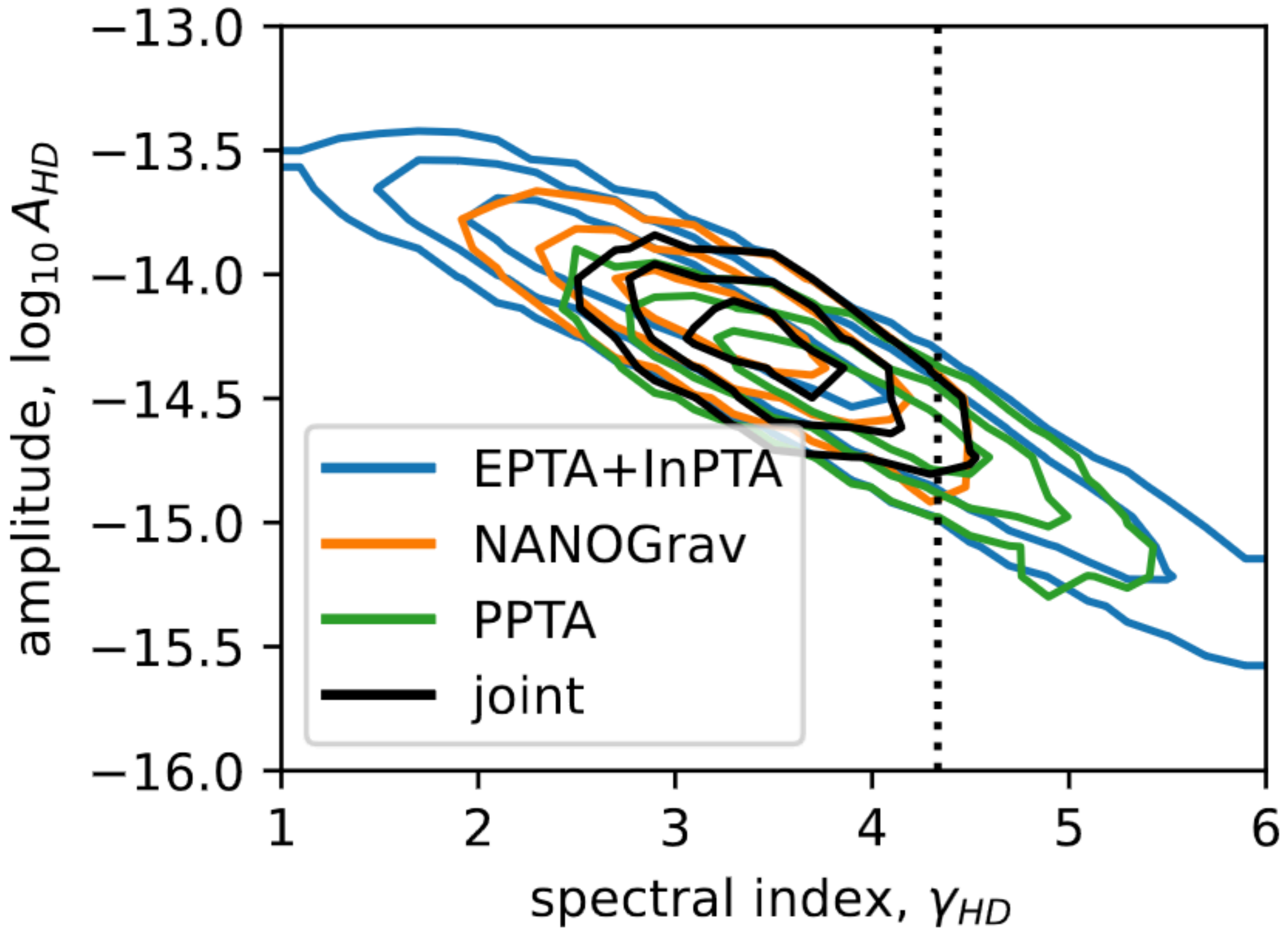
- Common spectrum process detected by NANOGrav, EPTA, PPTA, InPTA, CPTA
- HD correlations detected with ~$3-4\sigma$ significance



NANOGrav 15 year analysis

$$\Omega_{\rm GW}(f) = \frac{2\pi^2 f^2}{3H_0^2} A^2 \left(\frac{f}{f_{\rm ref}}\right)^{2\alpha}$$

$$\gamma \equiv 3 - 2\alpha.$$



arxiv: 2309.00693

$$\Omega_{\rm GW} = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln k} \simeq (5 \pm 2) \times 10^{-9}$$

Kinematic anisotropies of the SGWB with PTA

- First task: Develop the theory. Derive the PTA response functions to kinematic anisotropies.

$\beta = \frac{v}{c} \simeq 10^{-3}$ relative velocity

$$\Gamma_{ab} = \Gamma_{ab}^{(0)} + \beta\,(n_I - 1)\,\Gamma_{ab}^{(1)}$$

Response function

HD

Correction to HD

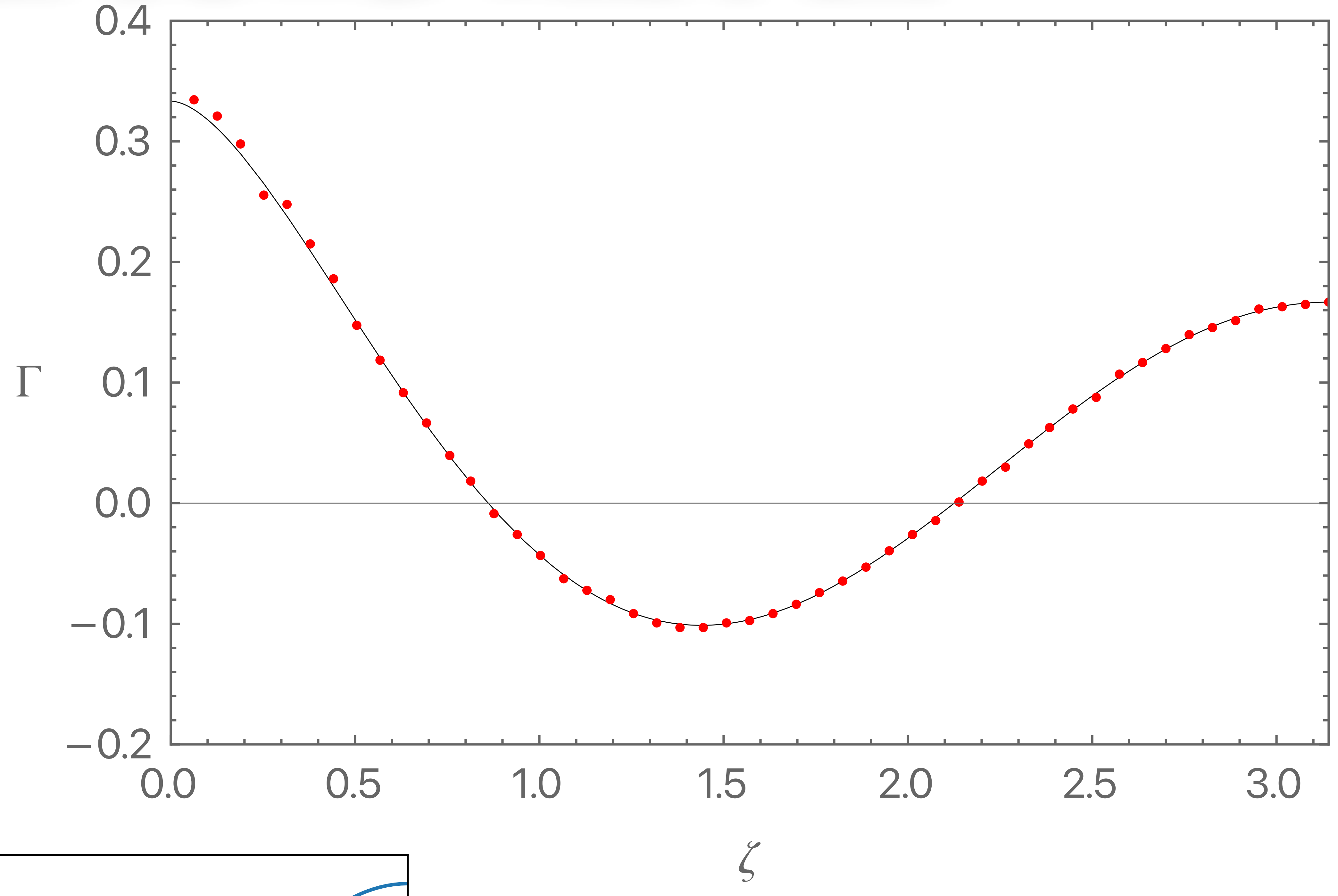
$$n_I = \frac{d\ln I(f)}{d\ln f}$$

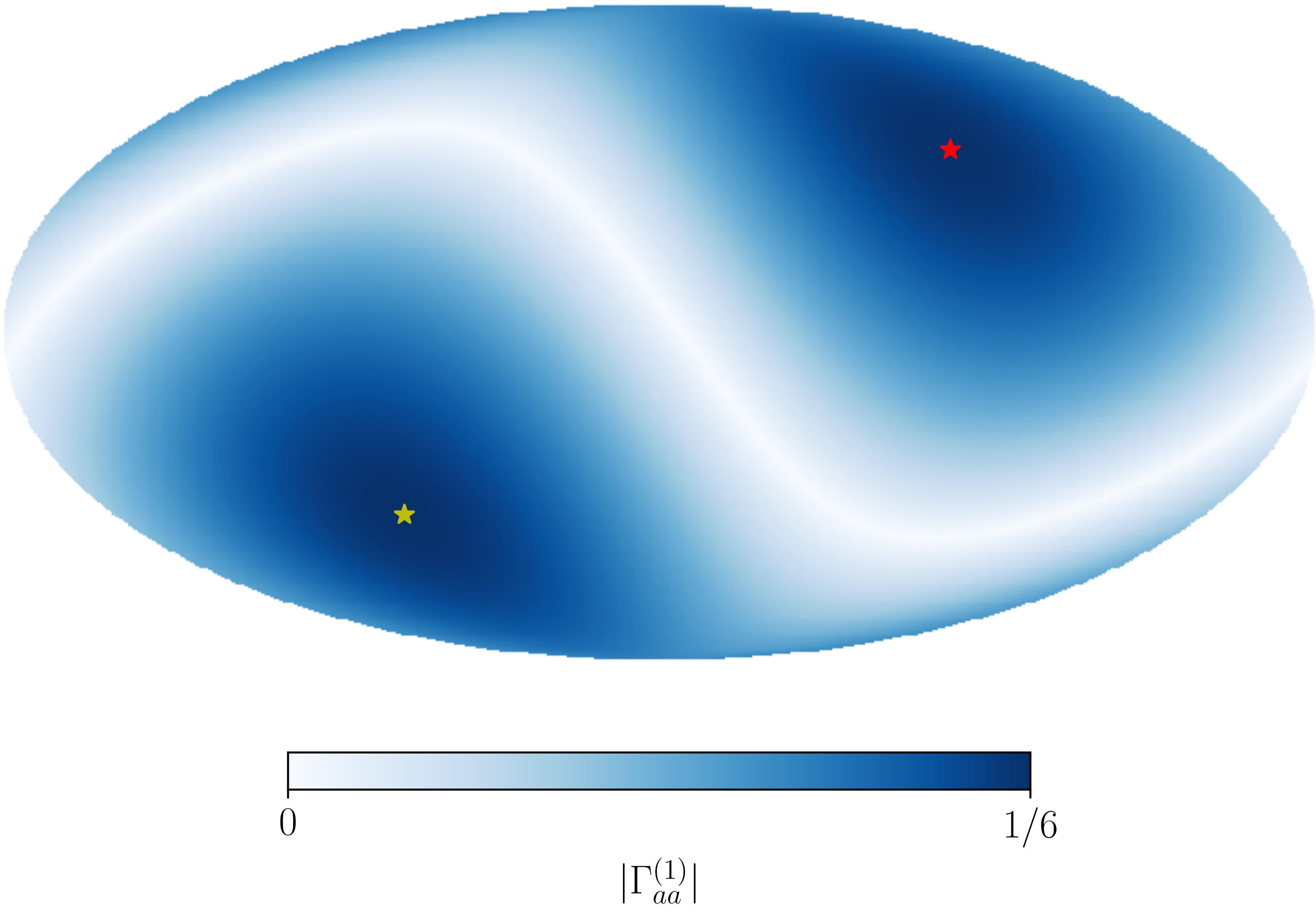
Kinematic anisotropies of the SGWB with PTA

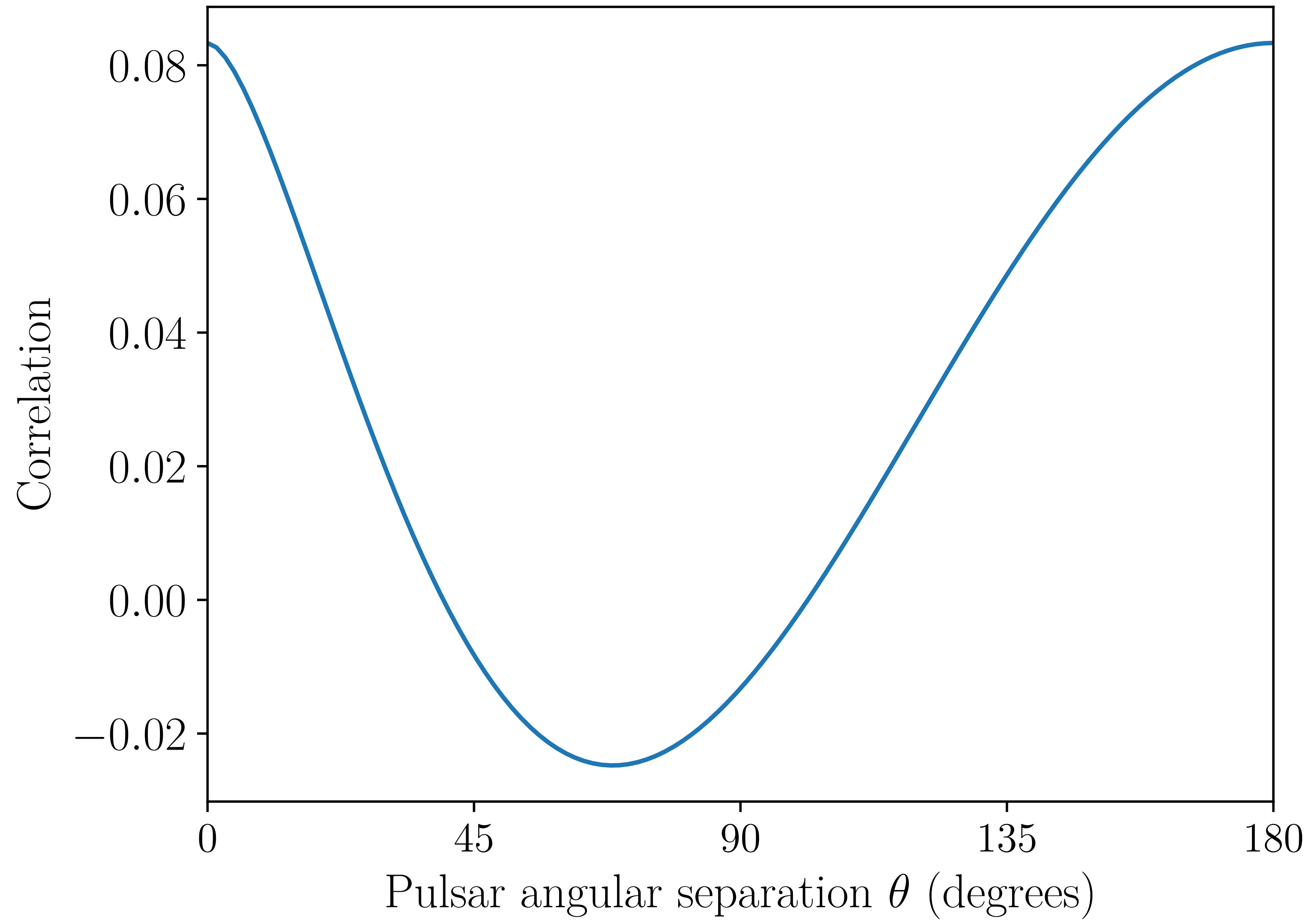
$$\Gamma^{(0)}_{ab} = \frac{1}{3} - \frac{y_{ab}}{6} + y_{ab}\ln y_{ab}$$

$$\Gamma^{(1)}_{ab} = \left(\frac{1}{12} + \frac{y_{ab}}{2} + \frac{y_{ab}\ln y_{ab}}{2(1-y_{ab})}\right)\left[\hat{v}\cdot\hat{x}_a + \hat{v}\cdot\hat{x}_b\right] ,$$

Enhanced when pulsars locations along $\vec{v}$





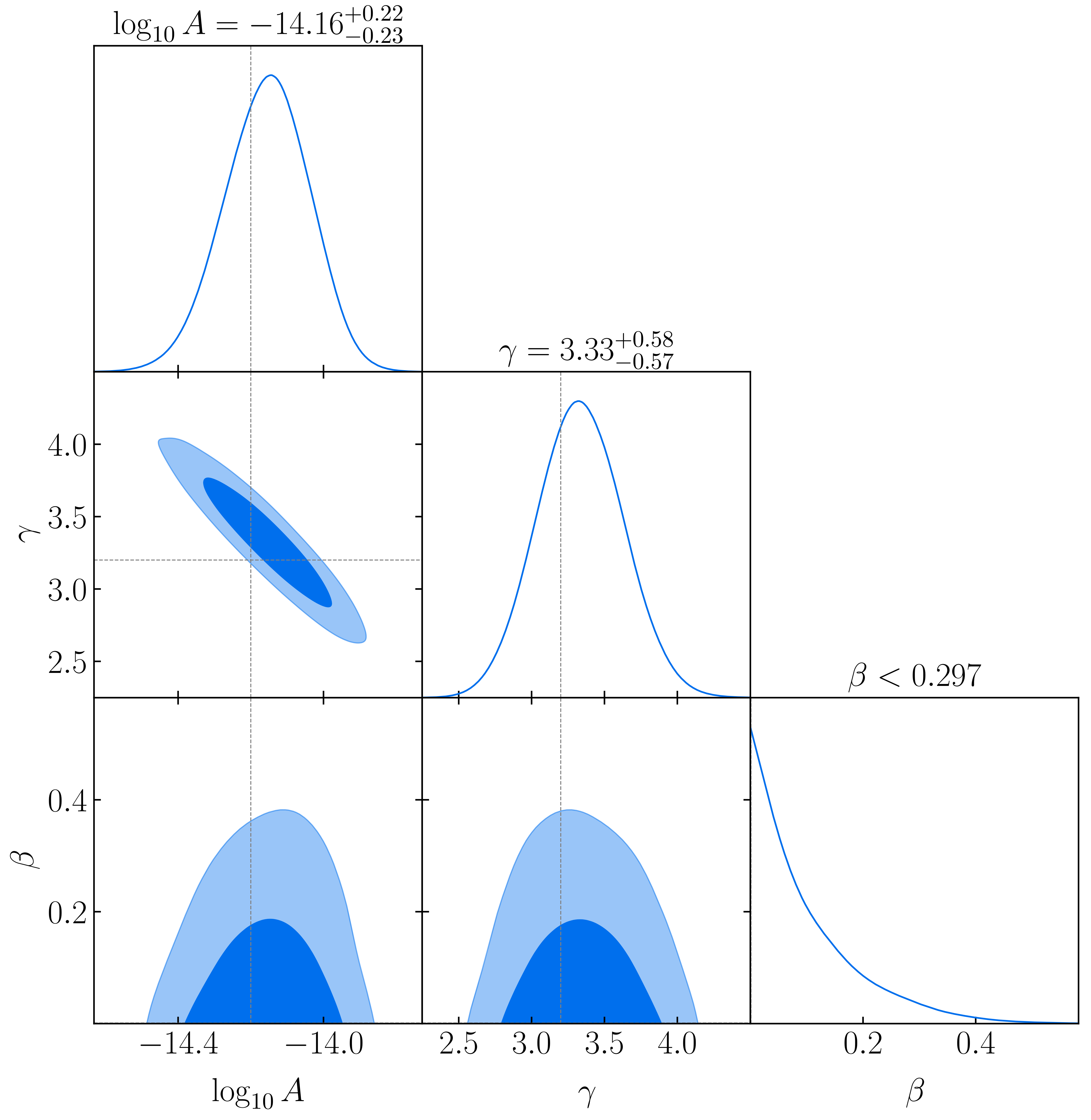


Perspectives for Detecting Kinematic Anisotropies with PTA

- Take existing NANOGrav data, and model signal as power law

$$I(f) = \frac{A^2}{2f}\left(\frac{f}{f_\star}\right)^{3-\gamma}$$

Use NANOGrav likelihood and methods in ENTERPRISE packages.



Perspectives for Detecting Kinematic Anisotropies with PTA

$$\boxed{\mathrm{SNR}_{\mathrm{iso}} \propto \sqrt{T N_{\mathrm{pair}}} \frac{I}{\sigma^2_{\mathrm{noise}}}}$$

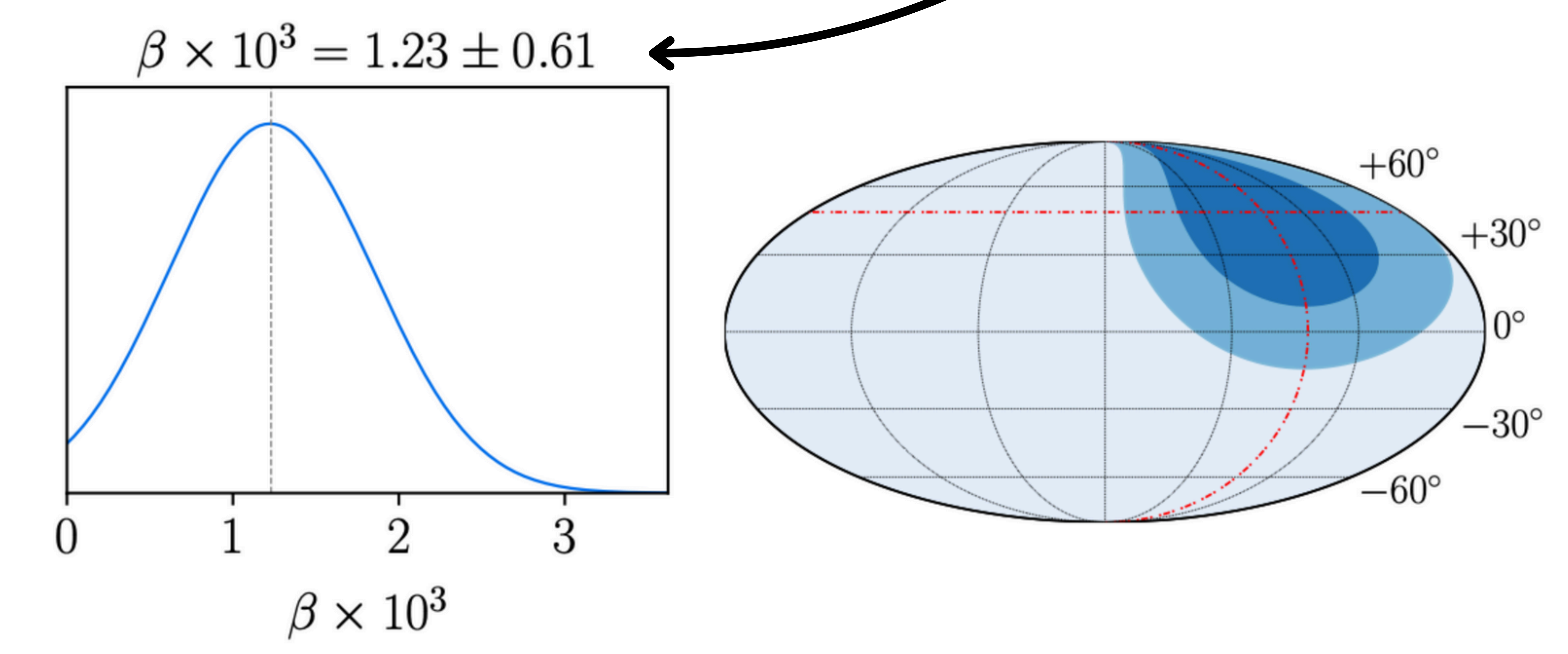
$$N_{\mathrm{pair}} \sim N^2$$

In order to beat $\beta \sim 10^{-3}$

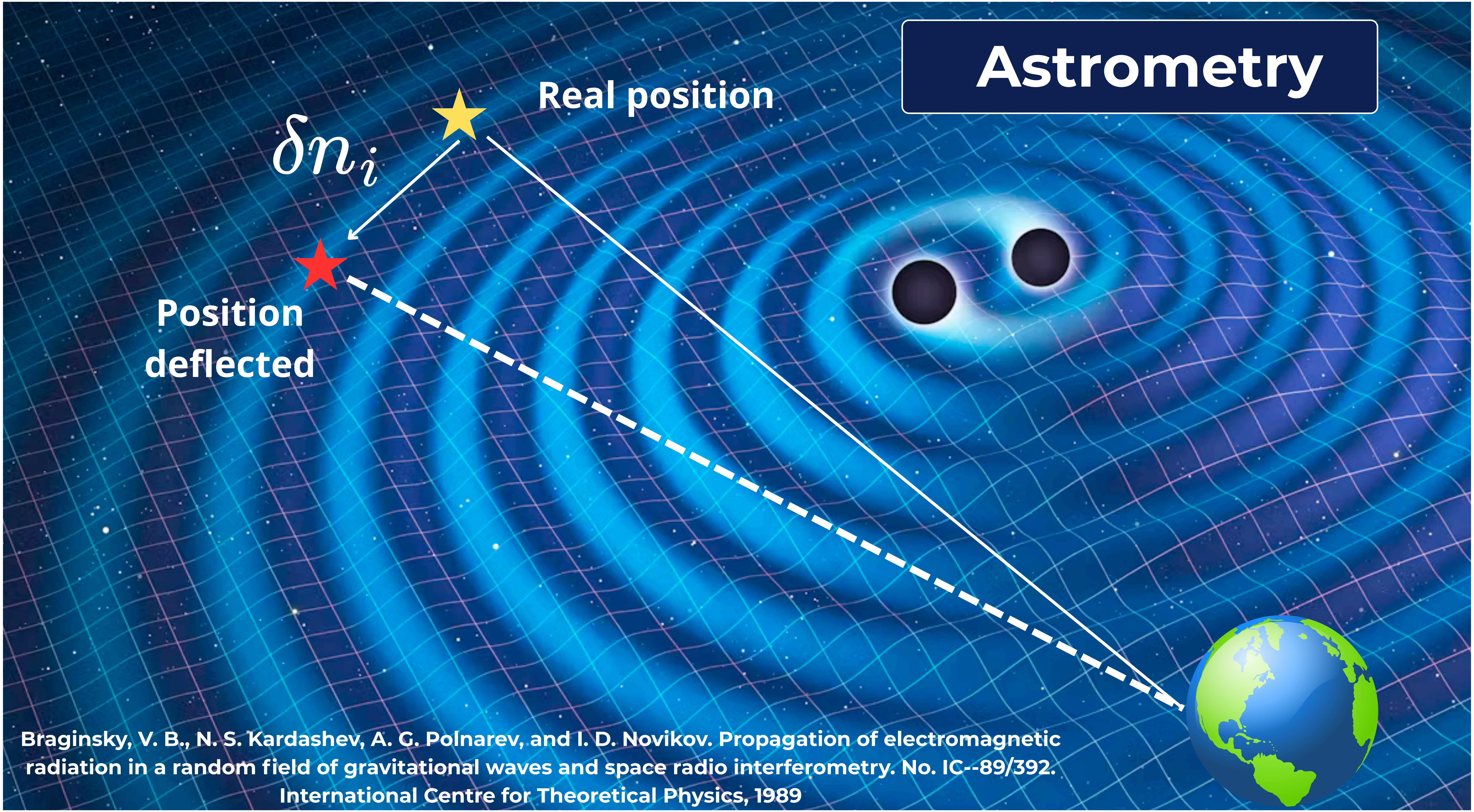
we need $N \sim 10^3$

Kinematic Anisotropies with PTAs

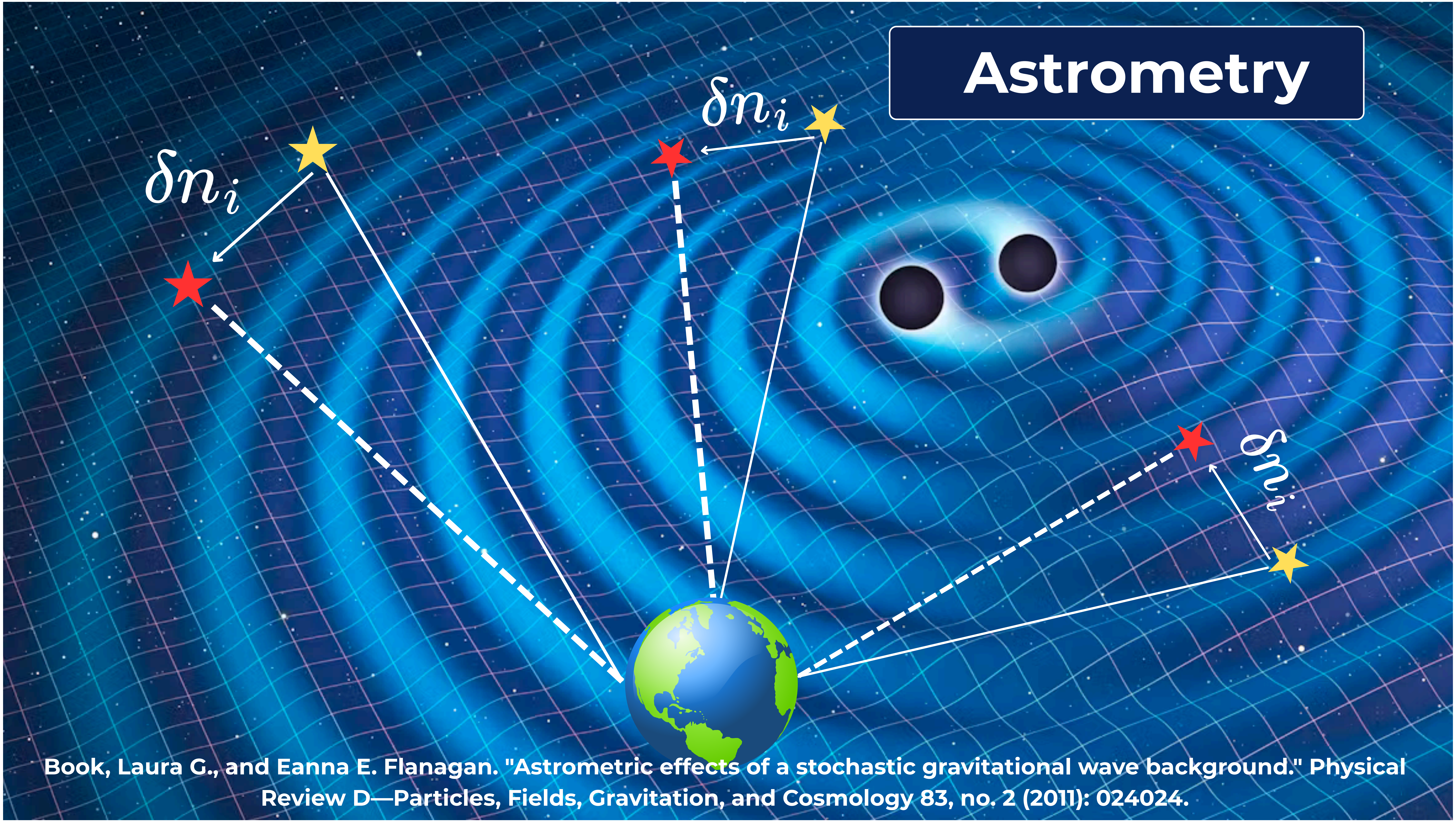
68% C.L. with 4000 identical pulsars and $T_{\rm obs} = 20\,\text{years}$



Large number of pulsars even for future experiments in SKA.



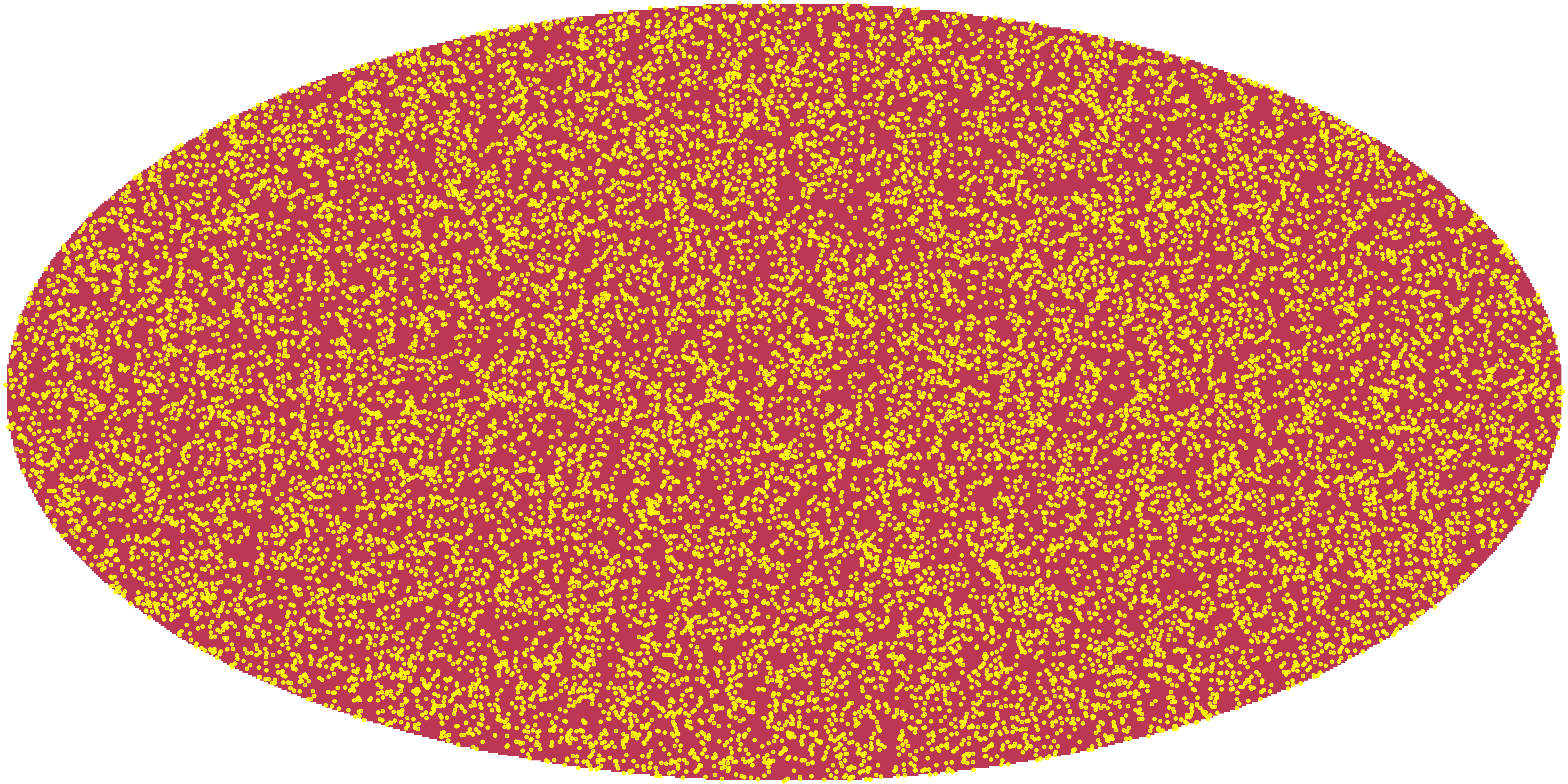
Braginsky, V. B., N. S. Kardashev, A. G. Polnarev, and I. D. Novikov. Propagation of electromagnetic radiation in a random field of gravitational waves and space radio interferometry. No. IC--89/392. International Centre for Theoretical Physics, 1989


Astrometry
δn_i
δn_i
δn_i
Book, Laura G., and Eanna E. Flanagan. "Astrometric effects of a stochastic gravitational wave background." Physical Review D—Particles, Fields, Gravitation, and Cosmology 83, no. 2 (2011): 024024.

Astrometry and SGWB

Precision astrometry with a large number of stars as a SGWB detector

[see Book, Flanagan (2010) for a review]



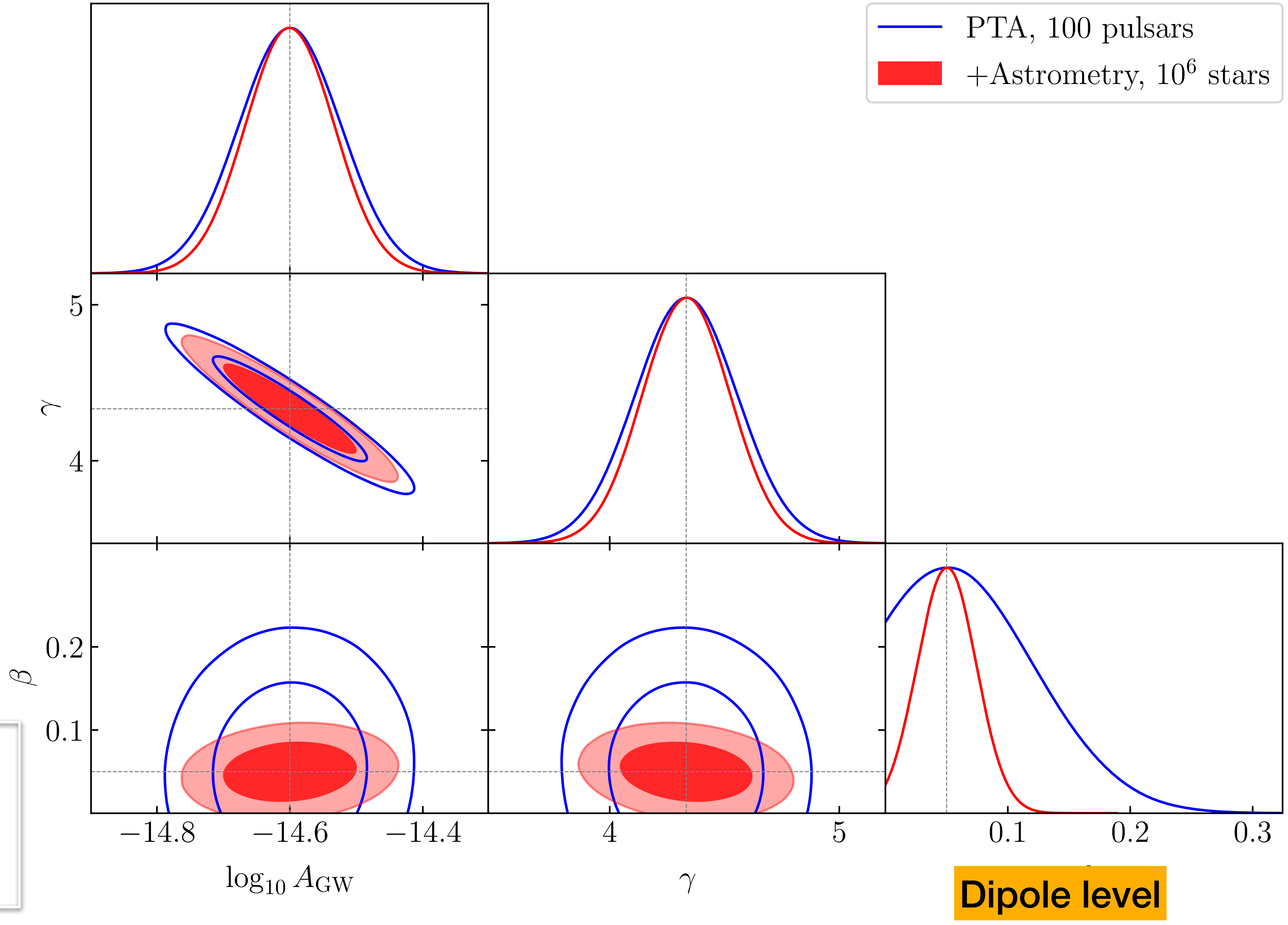
Gaia has $\boxed{N \sim 10^9}$ observed over 10 years with $\mathcal{O}(mas)$ precision. Already used to put constraints on low-frequency SGWB [Darling et al. 2018; Aoyama et al. 2021; Jaraba et al. (2023)]

Astrometry x PTA

Dipole anisotropy

Minimum detectable dipole anisotropy relative to monopole ~ 0.05.

Current PTA level ~ 0.1



ML techniques needed
to fully exploit data sets

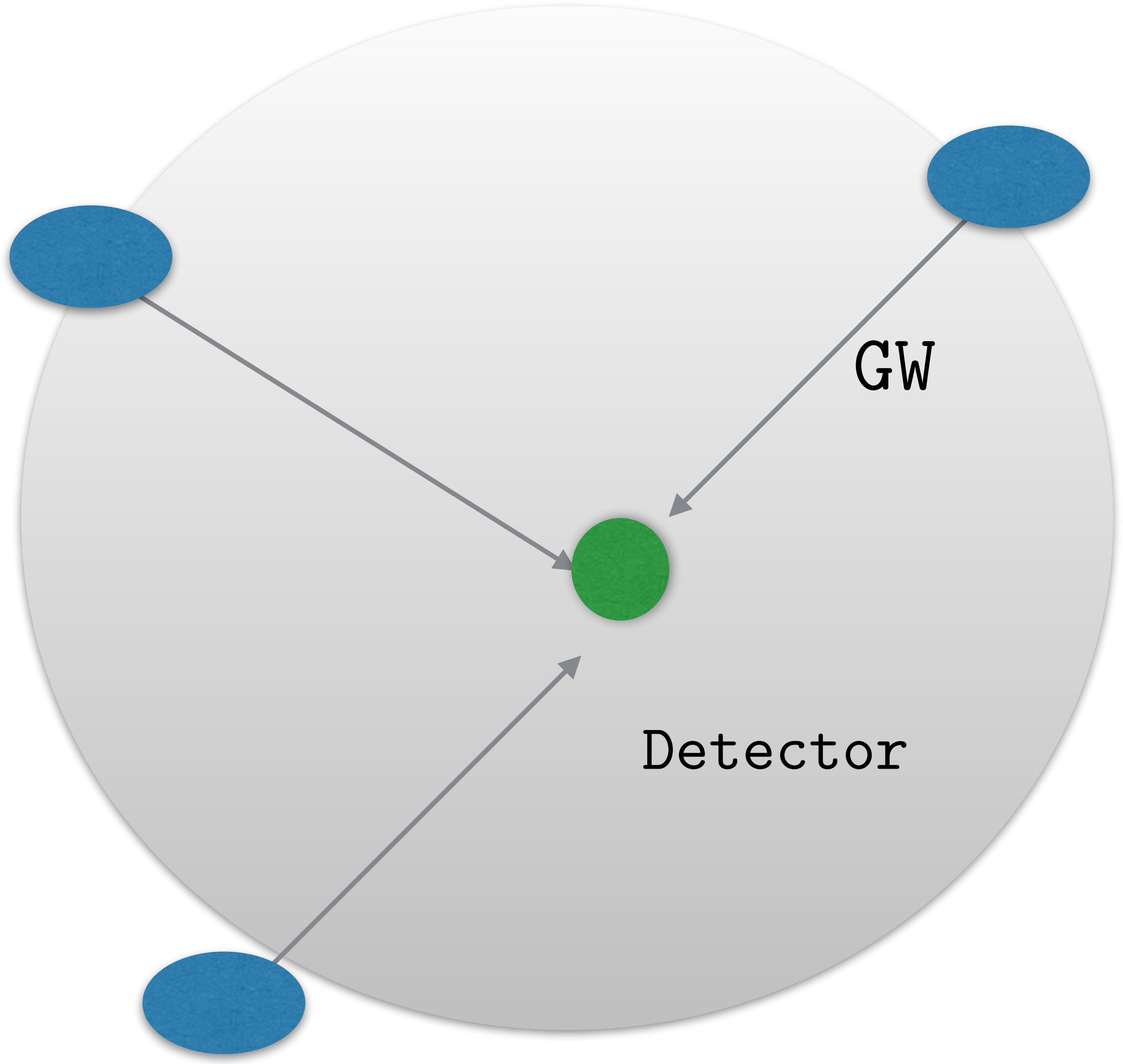
Further applications of anisotropies

Non-Gaussianity

Tensor non-Gaussianity

Problem:
GWs from different directions tend to ‘Gaussianize’ the signal

[Allen], [Adshead, Lim] [Bartolo et al.]



GW emitted from
different patches of sky

Phase decorrelations due to propagation in a perturbed universe

[Bartolo et al.]

Applications of Anisotropies of the SGWB

Multi-Messenger Cosmology

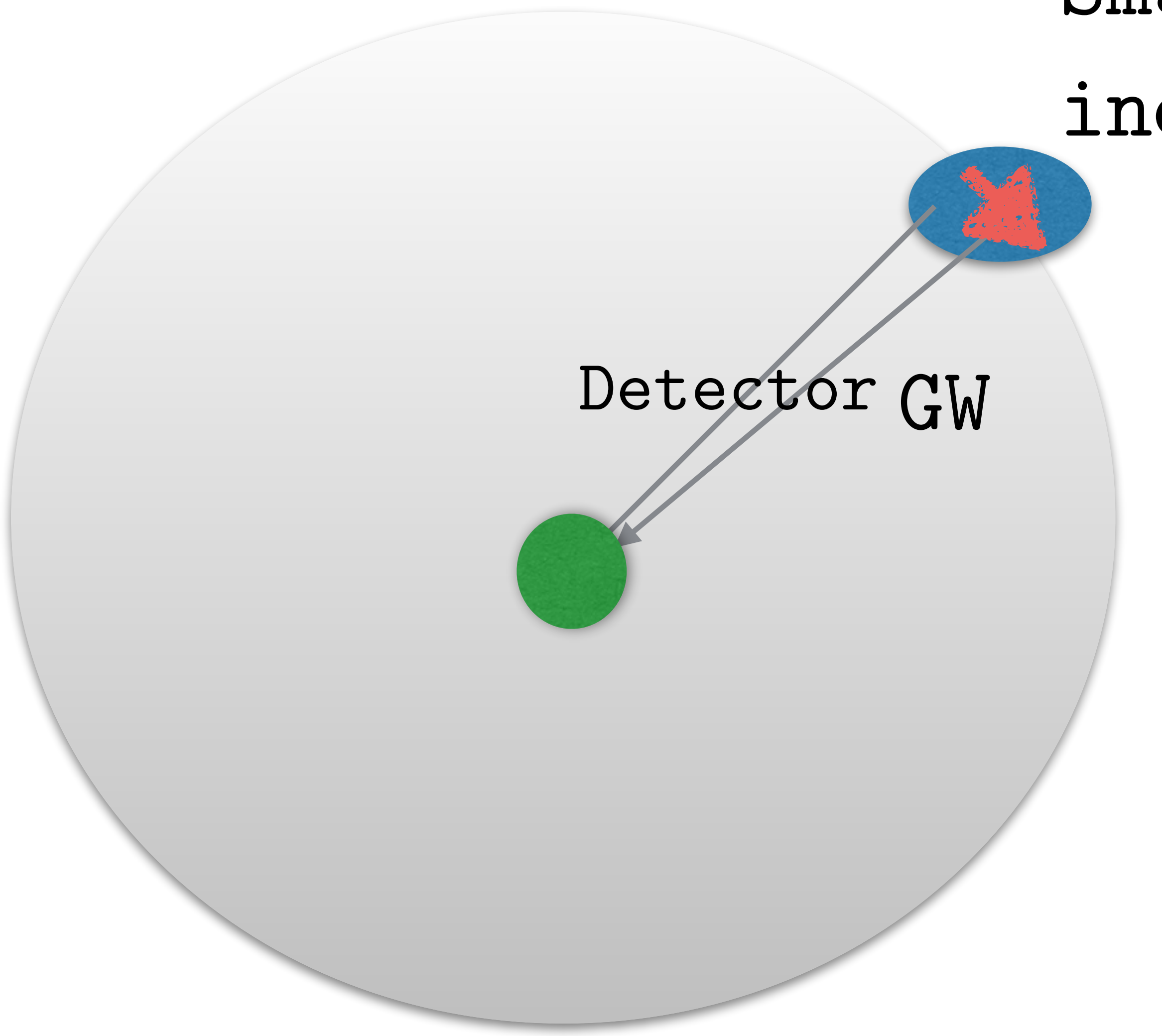
[Adshead et al, Dimastrogiovanni et al]

Recall: we can have $\delta_{\rm GW} \propto \zeta$

Anisotropies induced by propagation effects RD and MD universe

Quadrupolar anisotropies induced by large mixed non-Gaussianity $\langle \zeta h h \rangle$

Small-momentum mode induces correlations



$$P_\gamma^{\rm mod}(\mathbf{x},\mathbf{k}) = P_\gamma(k)\left[1+\int \frac{d^3q_L}{(2\pi)^3}\, e^{i\mathbf{q}_L\cdot\mathbf{x}}\, F_{\rm NL}(\mathbf{q}_L,\mathbf{k})\,\zeta(\mathbf{q}_L)\right]$$

Applications of Anisotropies of the SGWB

Multi-Messenger Cosmology

[Adshead et al, Dimastrogiovanni et al]

`Recall: we can have` $\delta_{\mathrm{GW}} \propto \zeta$

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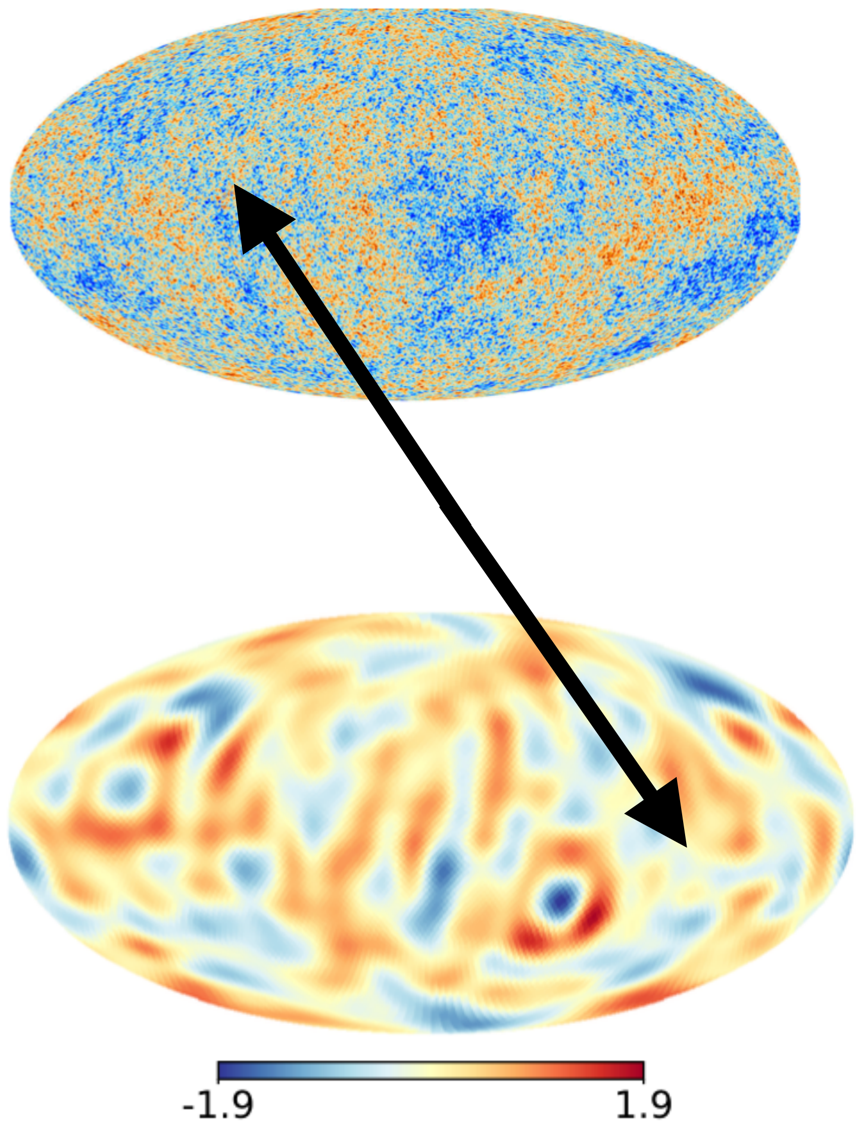

Then we can have $\langle \frac{\Delta T}{T} \, \delta_{\mathrm{GW}} \rangle \propto \mathcal{P}_\zeta$

Applications of Anisotropies of the SGWB

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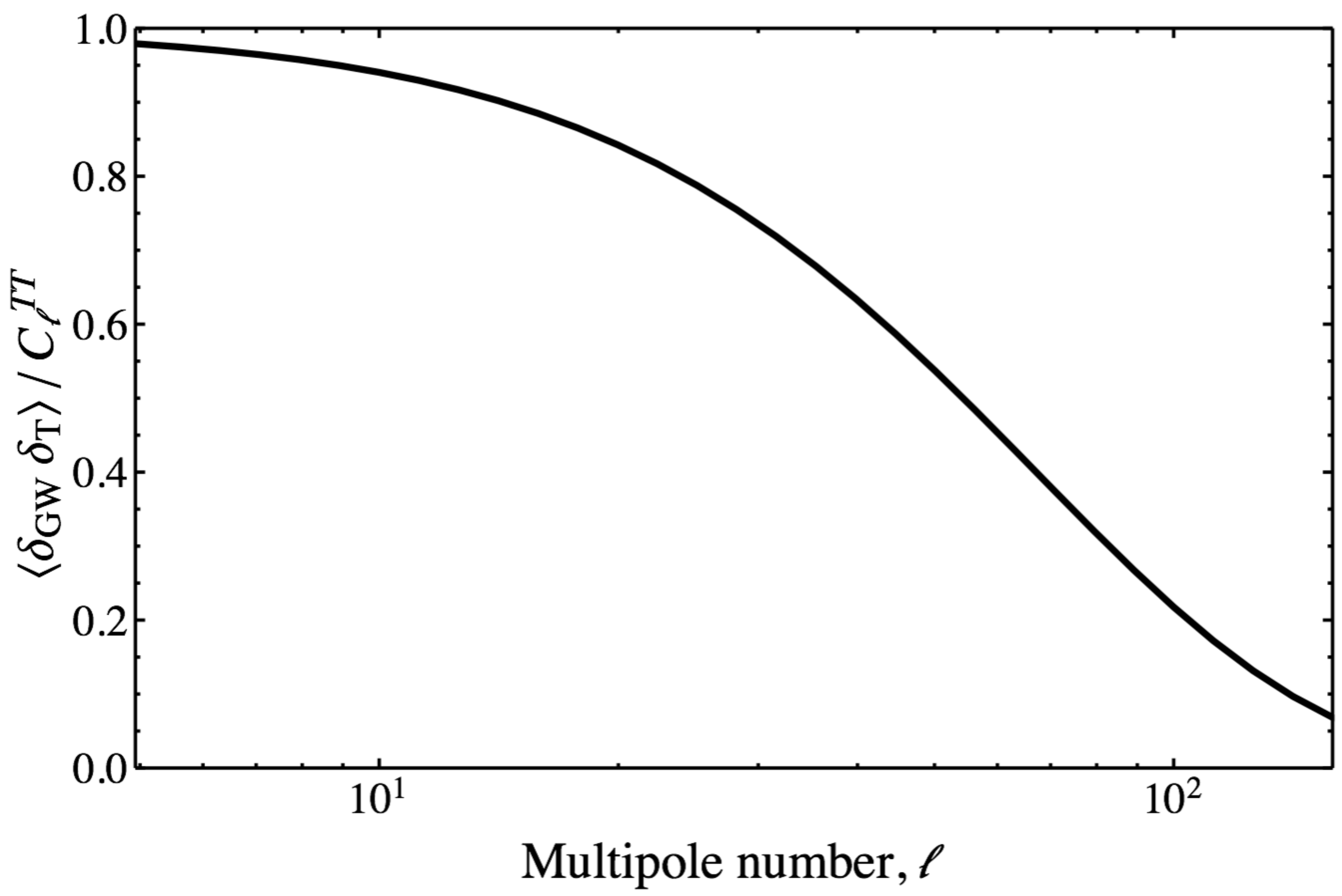

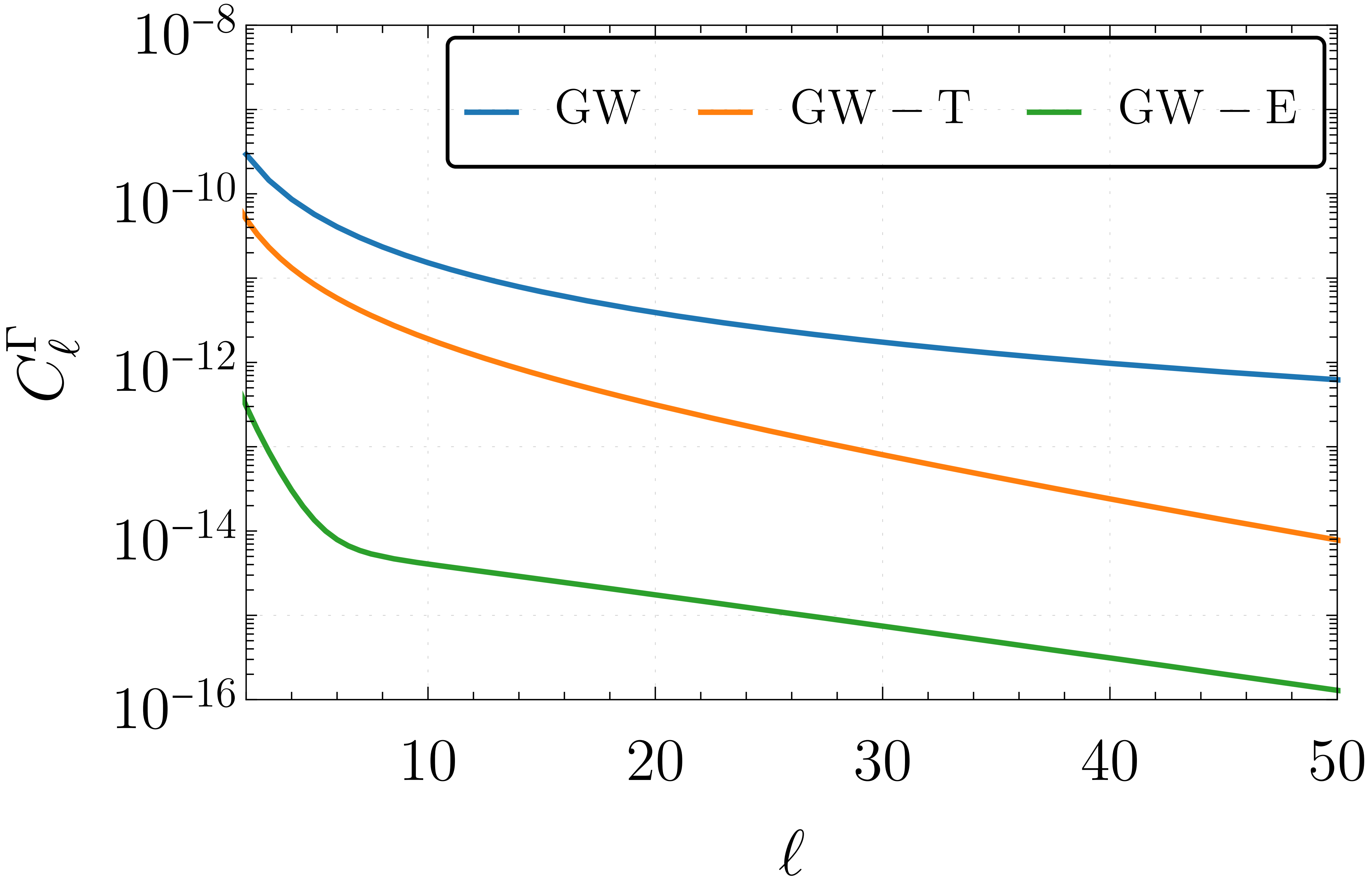

Circular polarisation

Circular polarisation

Intensity

$$\begin{pmatrix} \langle h_+^*(f,\hat{n})h_+(f',\hat{n}')\rangle & \langle h_+^*(f,\hat{n})h_\times(f',\hat{n}')\rangle \\ \langle h_\times^*(f,\hat{n})h_+(f',\hat{n}')\rangle & \langle h_\times^*(f,\hat{n})h_\times(f',\hat{n}')\rangle \end{pmatrix} = \frac{1}{2}\delta_D(f-f')\frac{\delta^{(2)}(\hat{n}-\hat{n}')}{4\pi}\begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix}$$

Circular polarization

Circular polarisation

Astro backgrounds characterized by circular polarization [Valbusa Dall'Armi et al]

- `Related with shot-noise` [Jenkins-Mairi, SatoPolito et al, Belgacem et al] `as well as propagation through cosmic structures` `[Cusin et al]`

Cosmological sources e.g. GW from axion-gauge fields [Unal et al. 2023 + more]

PTA blind to circular polarisation monopole — planar detector

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Circular polarisation

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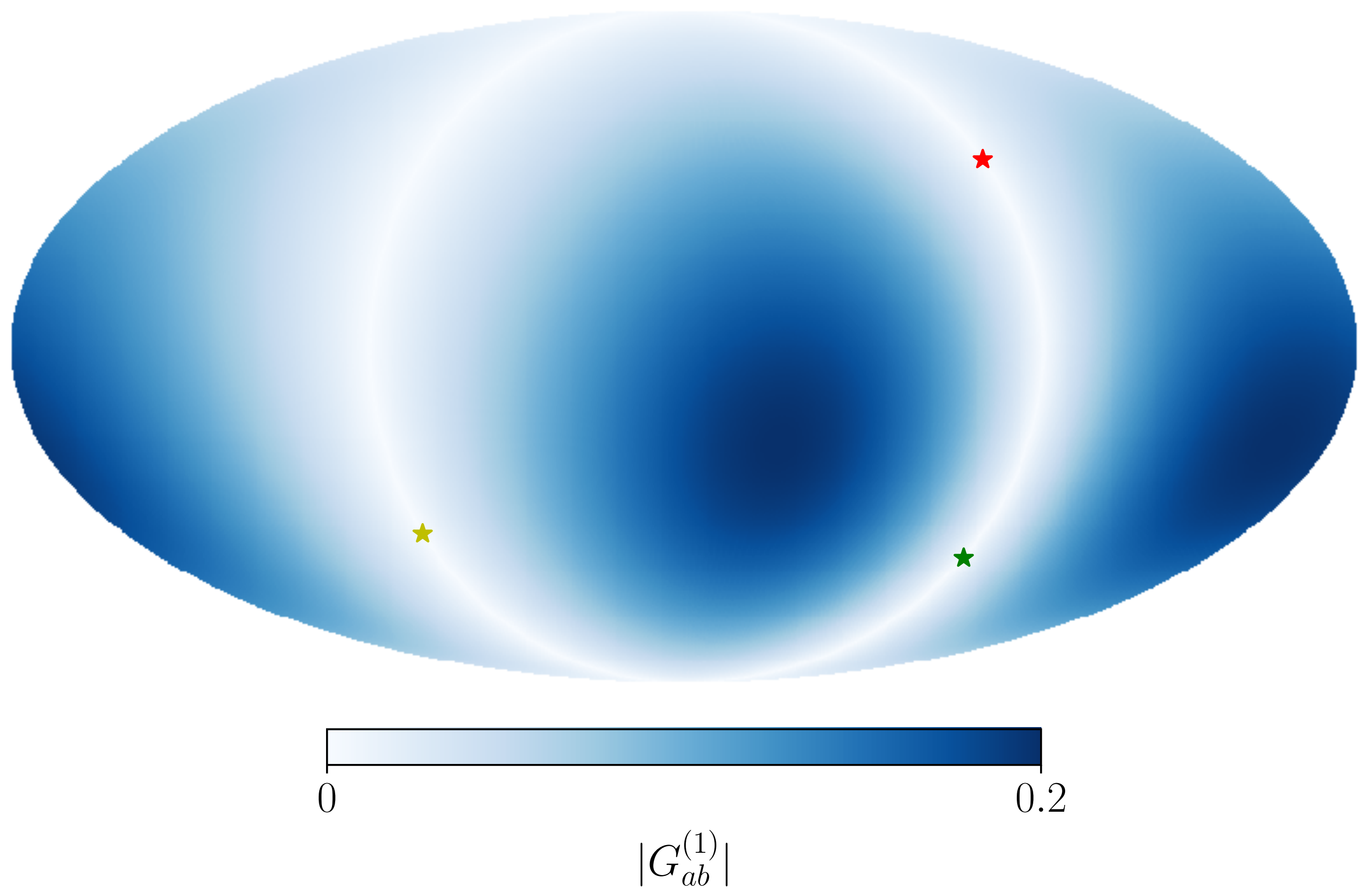
PTA blind to circular polarisation monopole — planar detector

PTA response begins at dipole

$$\Gamma^V_{ab} = \beta\ (n_V - 1)\ G^{(1)}_{ab}\ V$$

$$G^{(1)}_{ab} = -\left(\frac{1}{3} + \frac{y_{ab}\ln y_{ab}}{4(1-y_{ab})}\right)[\hat{v}\cdot(\hat{x}_a \times \hat{x}_b)]$$

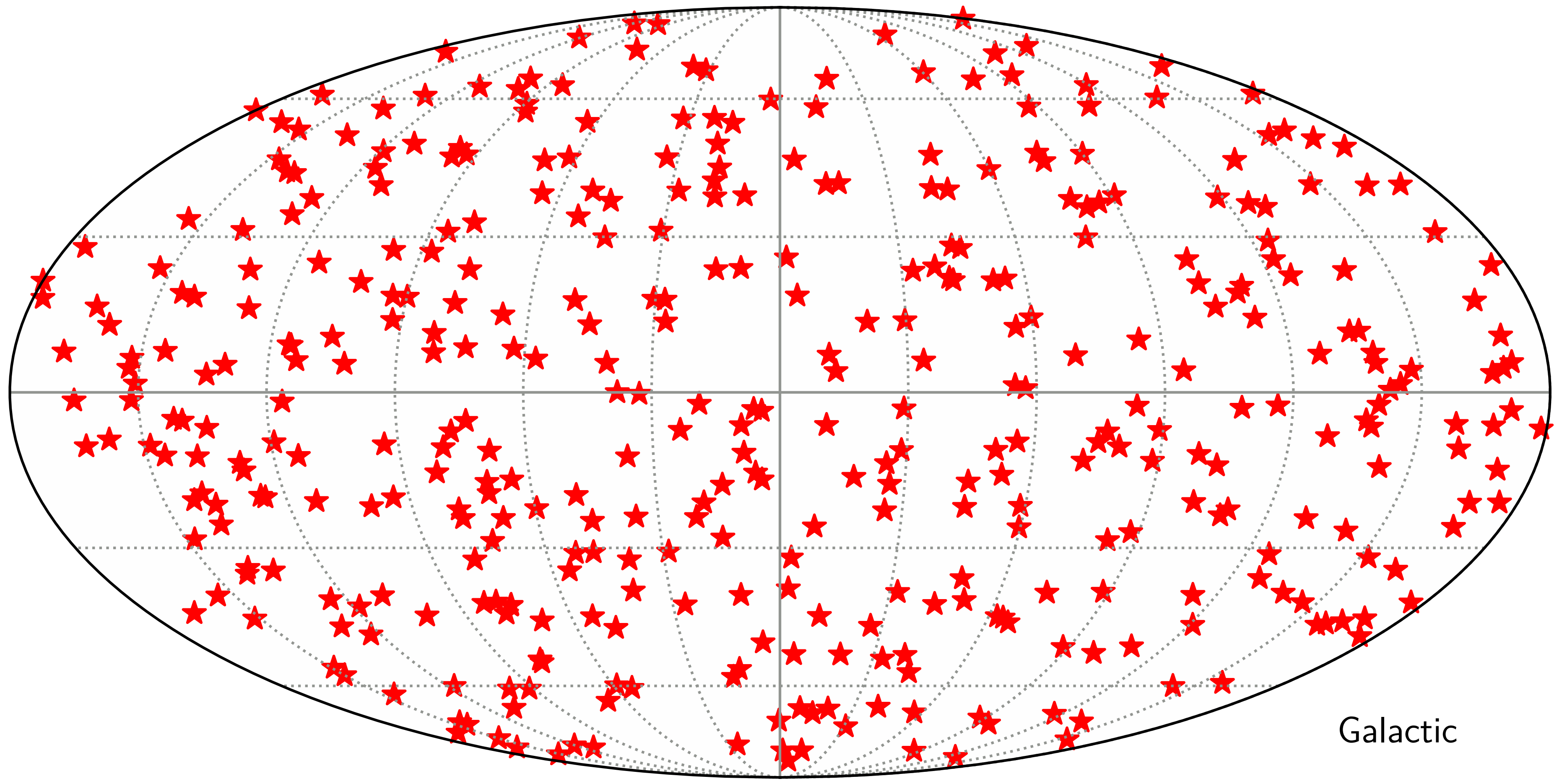


Enhanced when pulsars locations orthogonal $\vec{v}$

Forecasts: SKA era

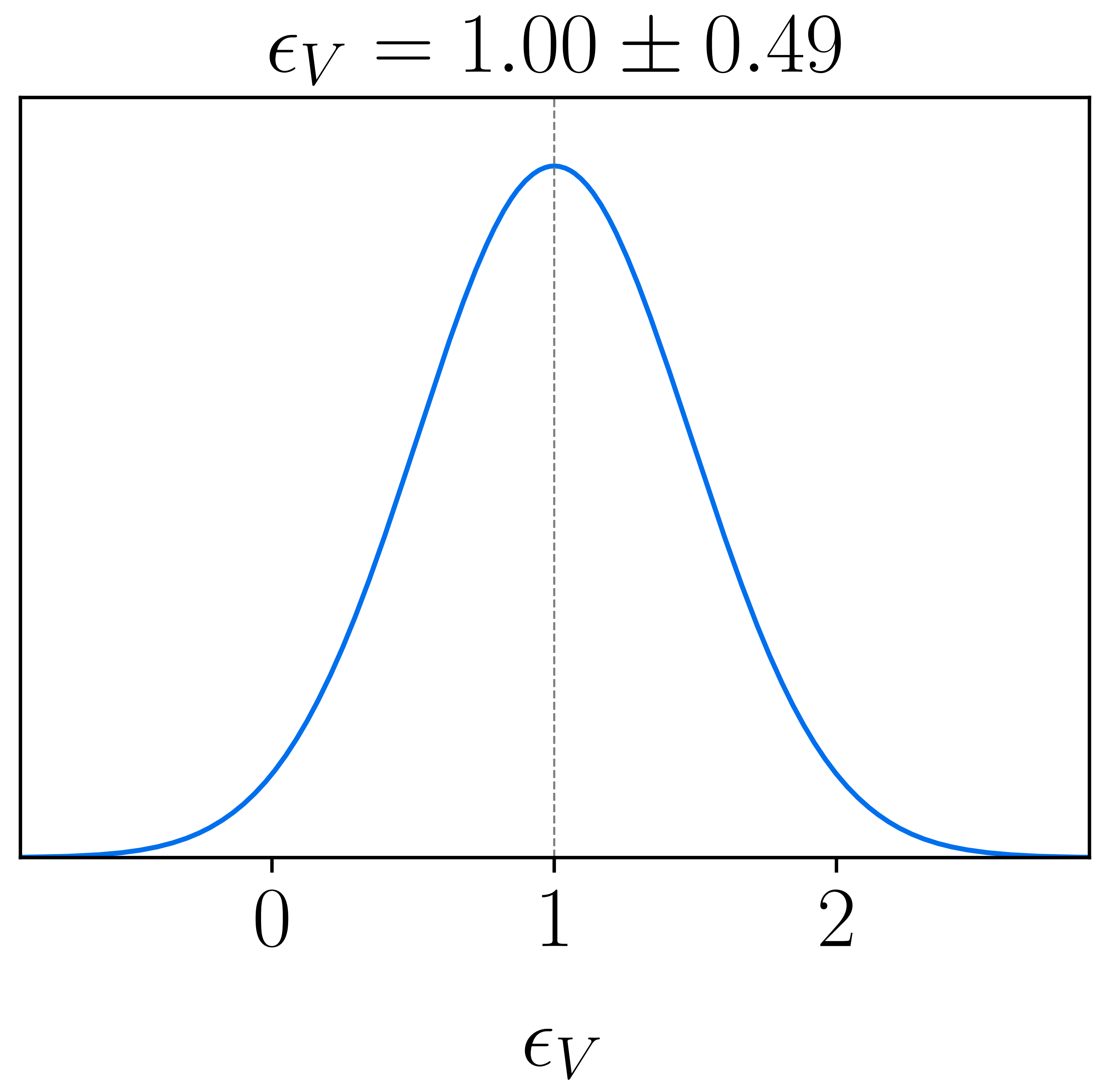
Idealised scenario with $N \gg 100$ identical pulsars distributed uniformly

We make several simplifying assumptions —> most optimistic estimate



[Keane et al. (2015), Janssen et al. (2015)]

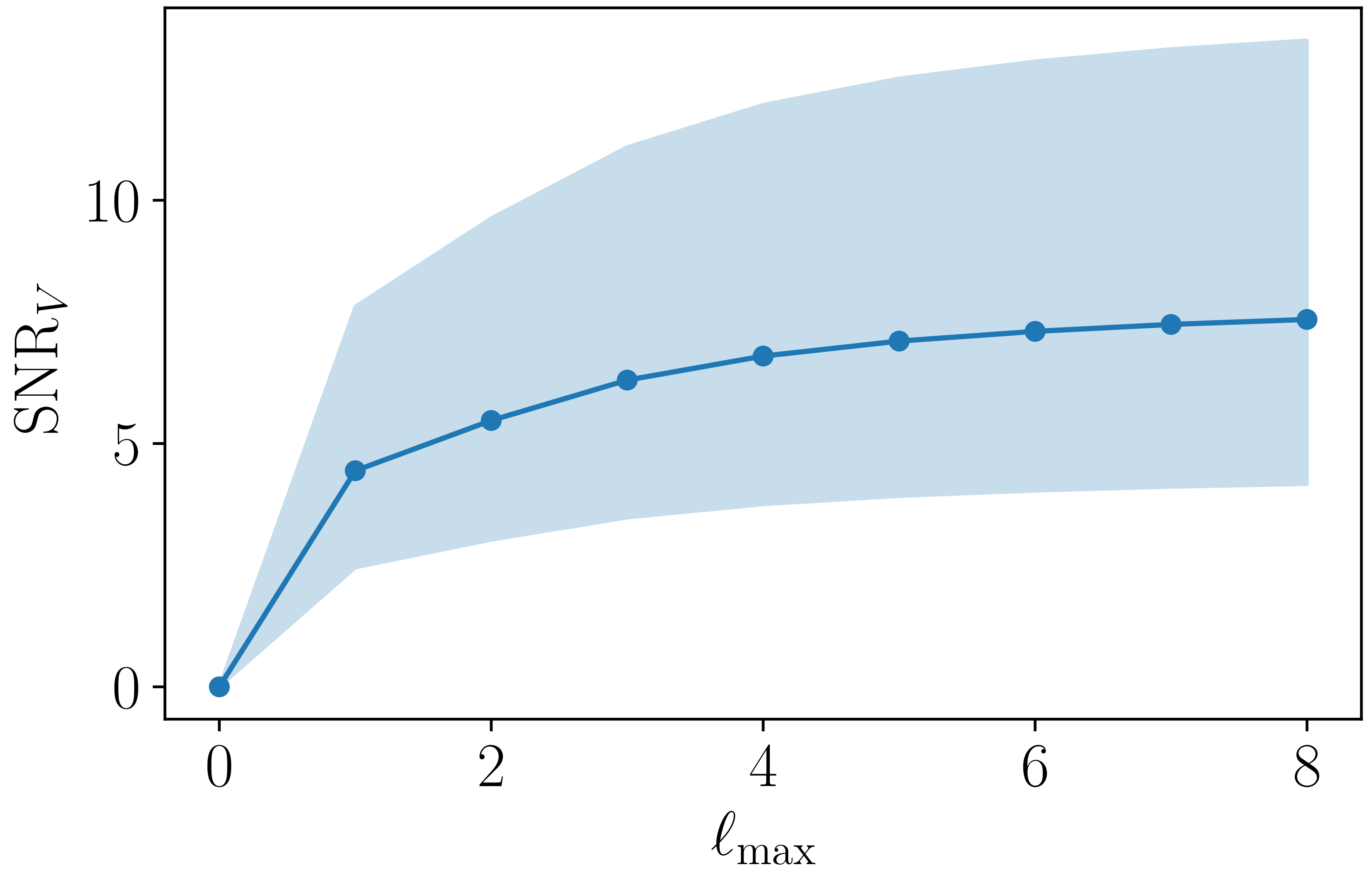
Circular polarisation (for general anisotropies)



Degree of circular polarisation

$$\epsilon_V = \frac{V}{I}$$

Unconstrained by current data



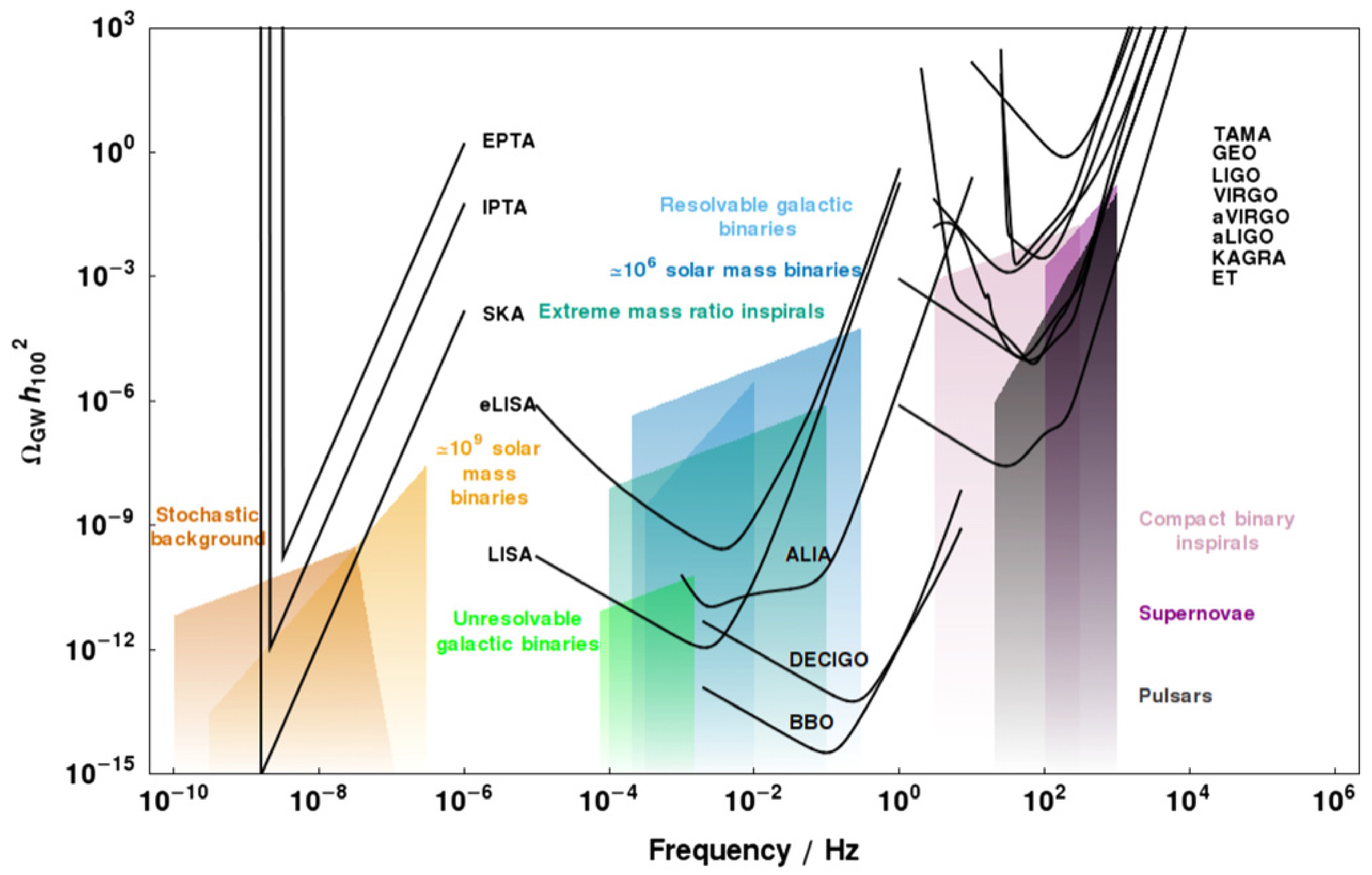
Near maximal polarisation may be detected with SKA ($N_{\rm psr} \gtrsim 10^3$)

`...or with the help of astrometry [in prep]`

Conclusions

I discussed three topics towards characterisation SGWB:

- **Frequency dependence**
- **Anisotropies**
- **Circular polarization**



Experimentally challenging to measure, but if successful will be very useful for deciding if Signal is primordial or not