Seeded phase transitions

Simone Blasi
DESY Hamburg

SB, Mariotti [2203.16450] PRL

SB, Jinno, Konstandin, Rubira, Stomberg [2302.06952] JCAP

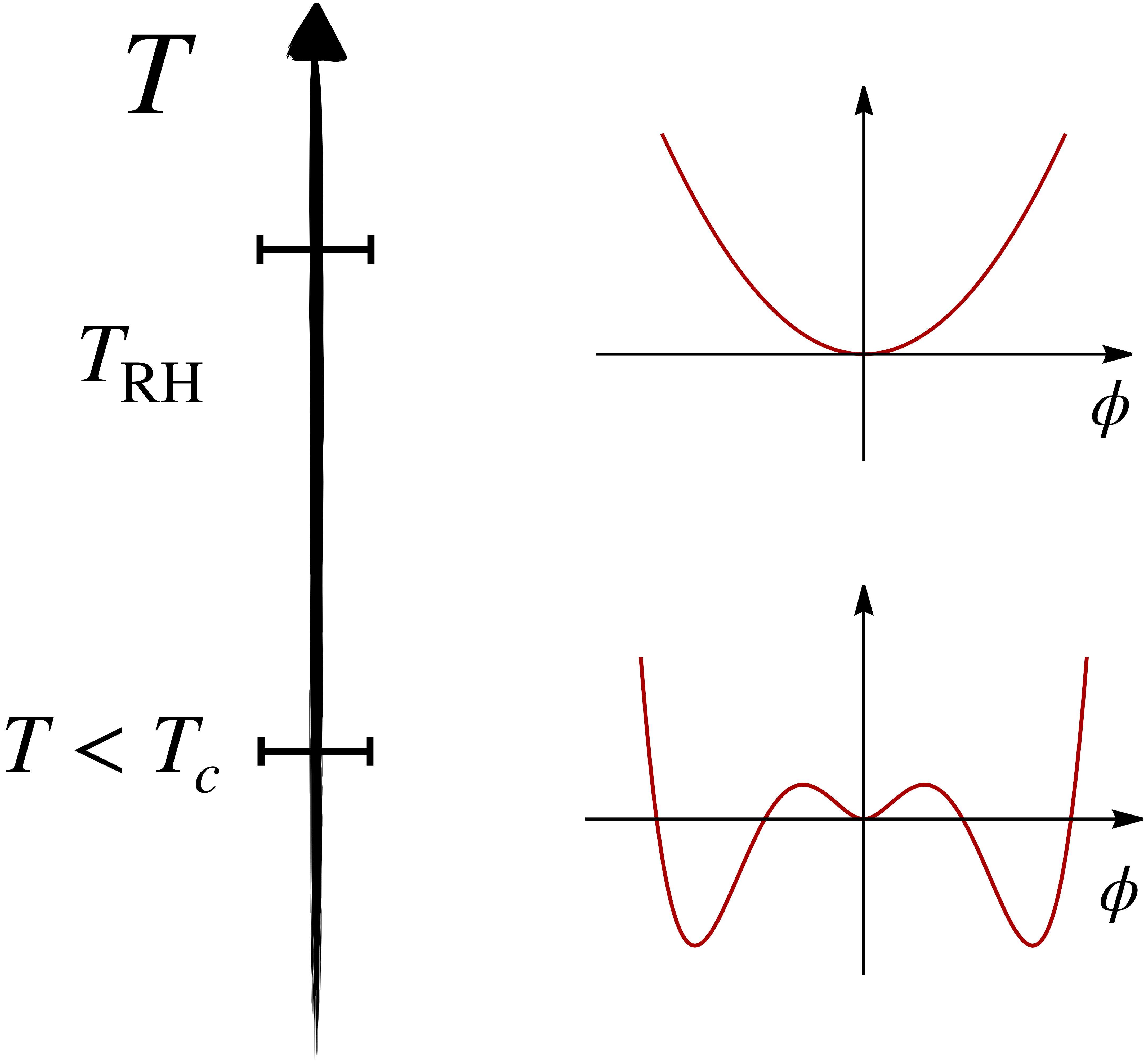
Agrawal, SB, Mariotti, Nee [2312.06749] JHEP

SB, Mariotti [2405.08060] SciPost Phys.

+ ongoing

GW BSM 4, Warsaw, 26.06.2025

Introduction



Symmetries are **restored** at high temperatures/early times

$$\langle \phi \rangle : G \to H$$

Spontaneous breaking while the Universe expands and cools down

⇒ Cosmological phase transitions

Key to address open questions: **baryogenesis**

Aftermath directly observable in **GWs**

Evidence for **new** fundamental **physics**

Phase transitions source GWs

Phase transition at T_c

$\langle \phi \rangle : G \to H$

Topology of the vacuum:

Formation of defects and annihilation

Strength:

Bubble collision, hydrodynamics

Phase transitions source GWs

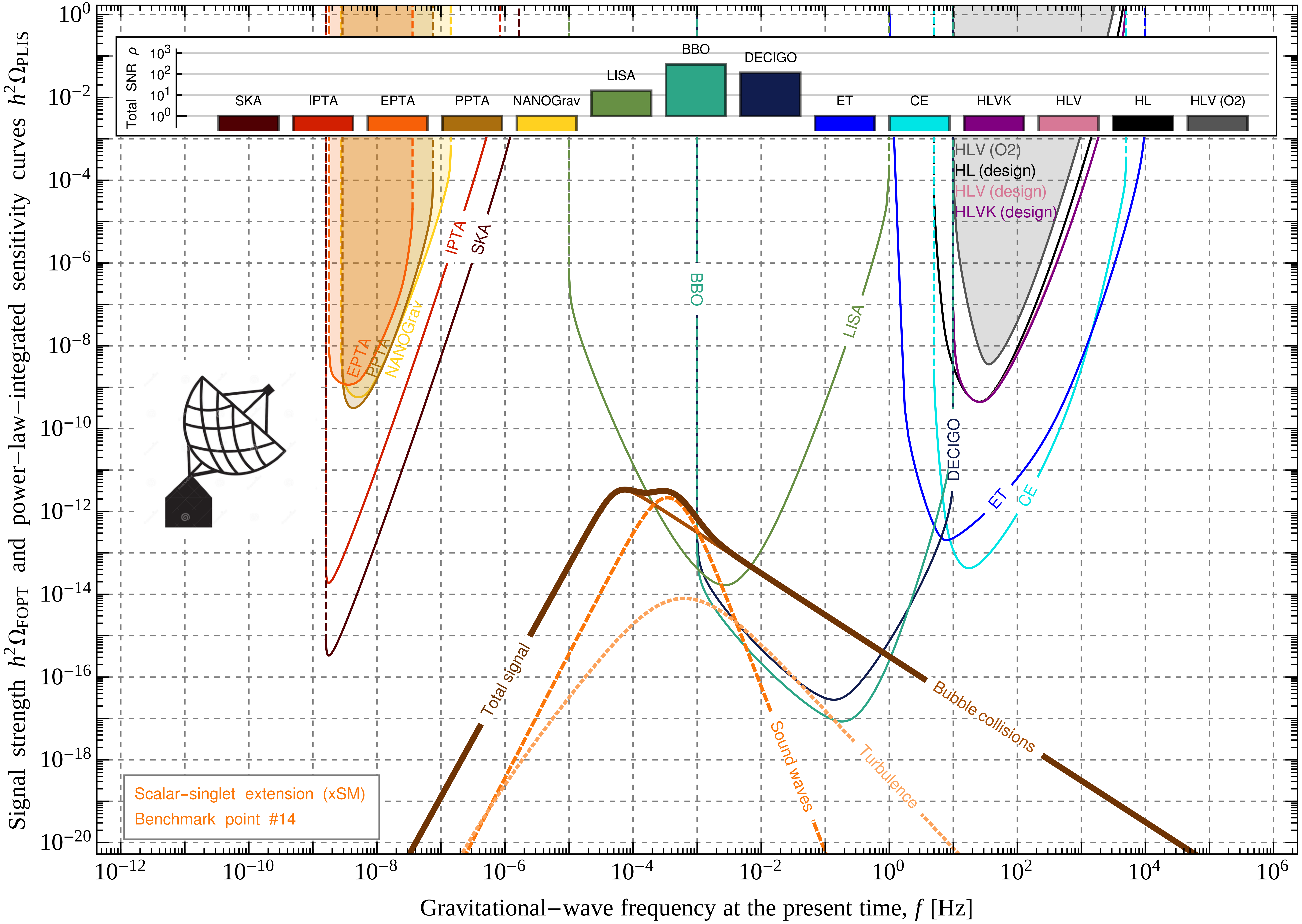


Fig. from Schmitz [2002.04615] JHEP

Nucleation theory

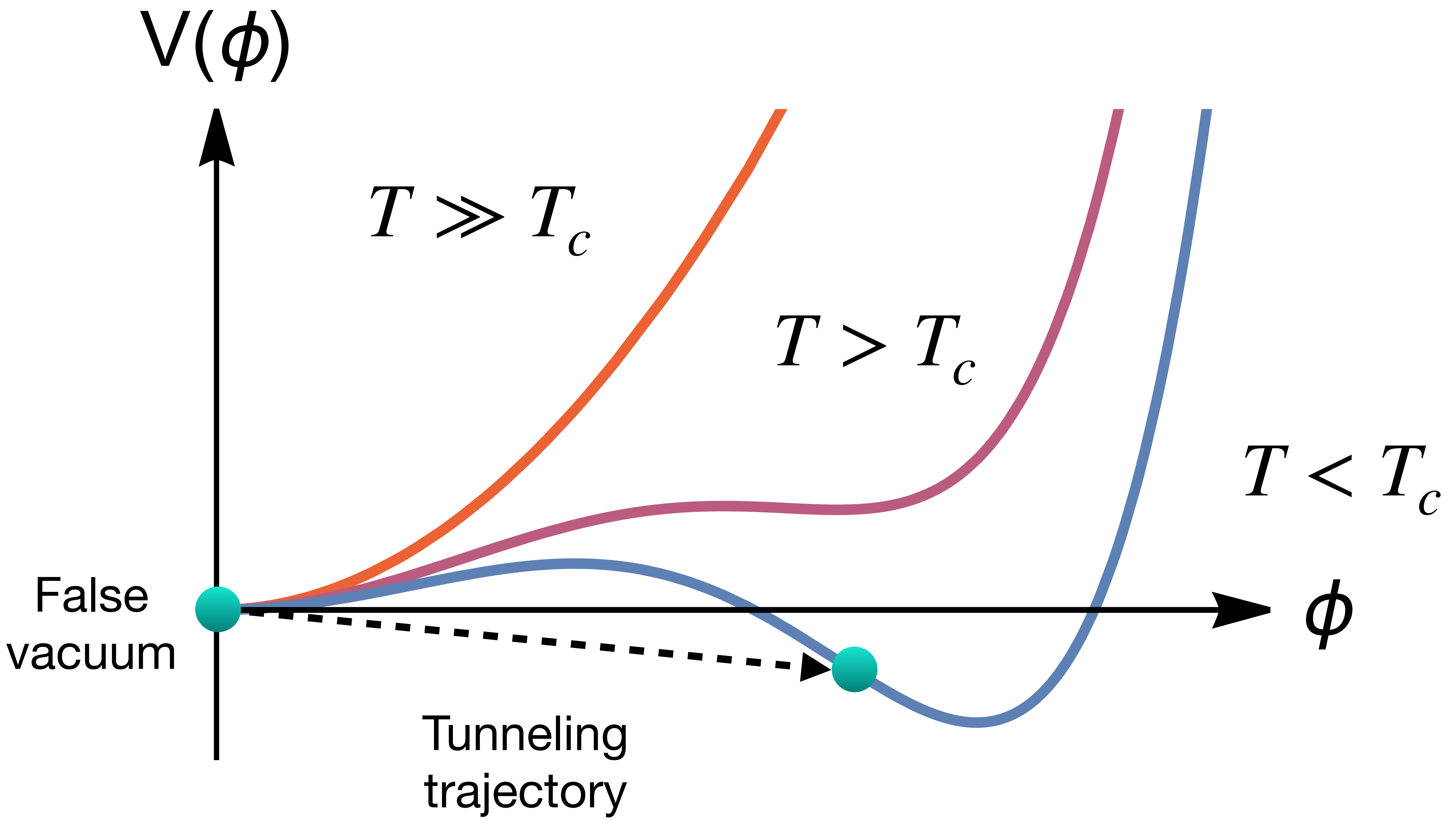
Coleman 1977 (PRD)
Callan, Coleman 1977 (PRD)
Linde 1983 (NPB)

- Assume thermal fluctuations in homogeneous spacetime:

$$\phi(\mathbf{x}, \tau) = \phi(r), \quad r = |\mathbf{x}|$$

- Tunneling rate per unit volume given by O(3) action S_3/T

$$\gamma_V \sim T^4 \exp(-S_3/T)$$



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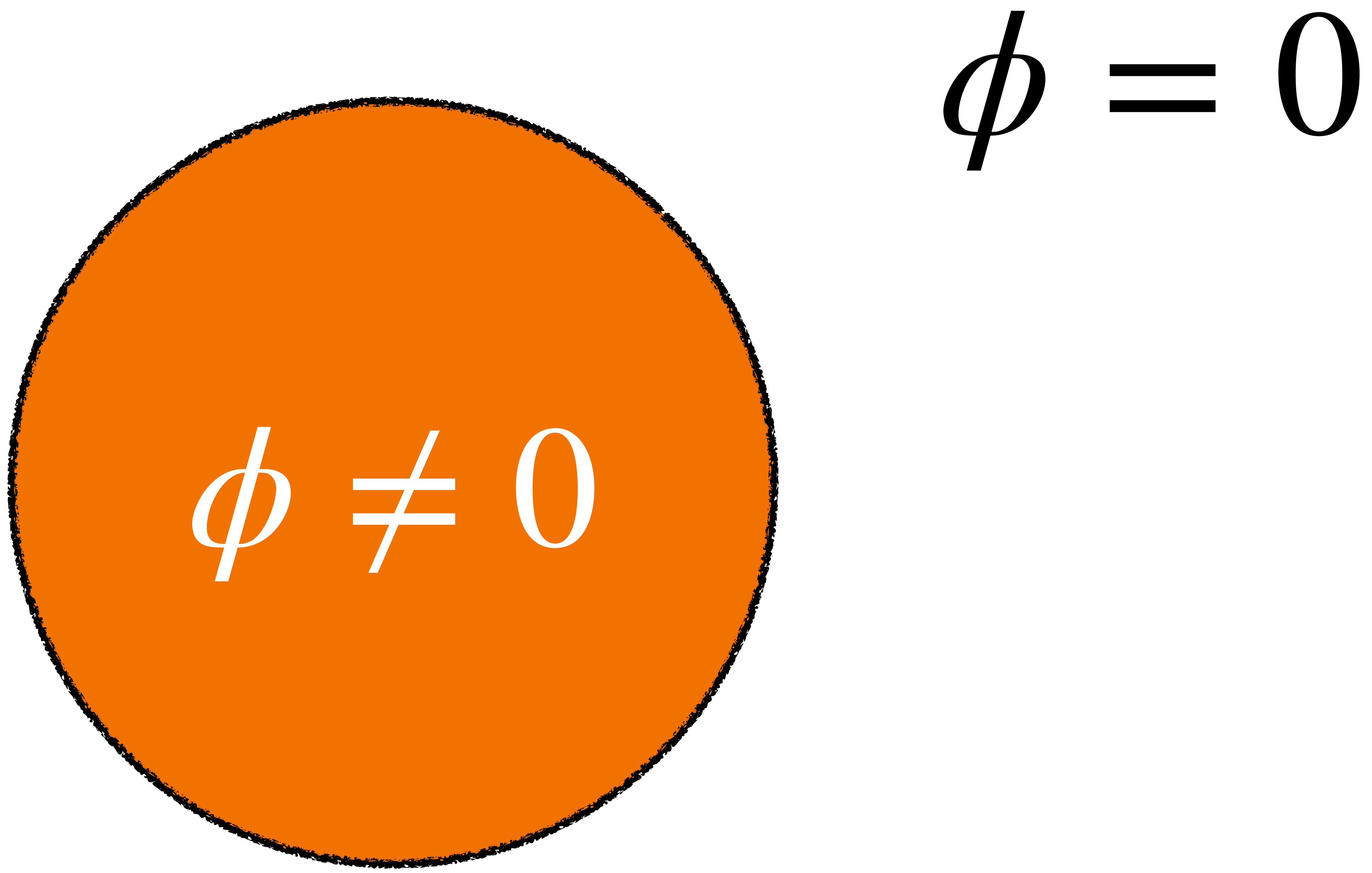
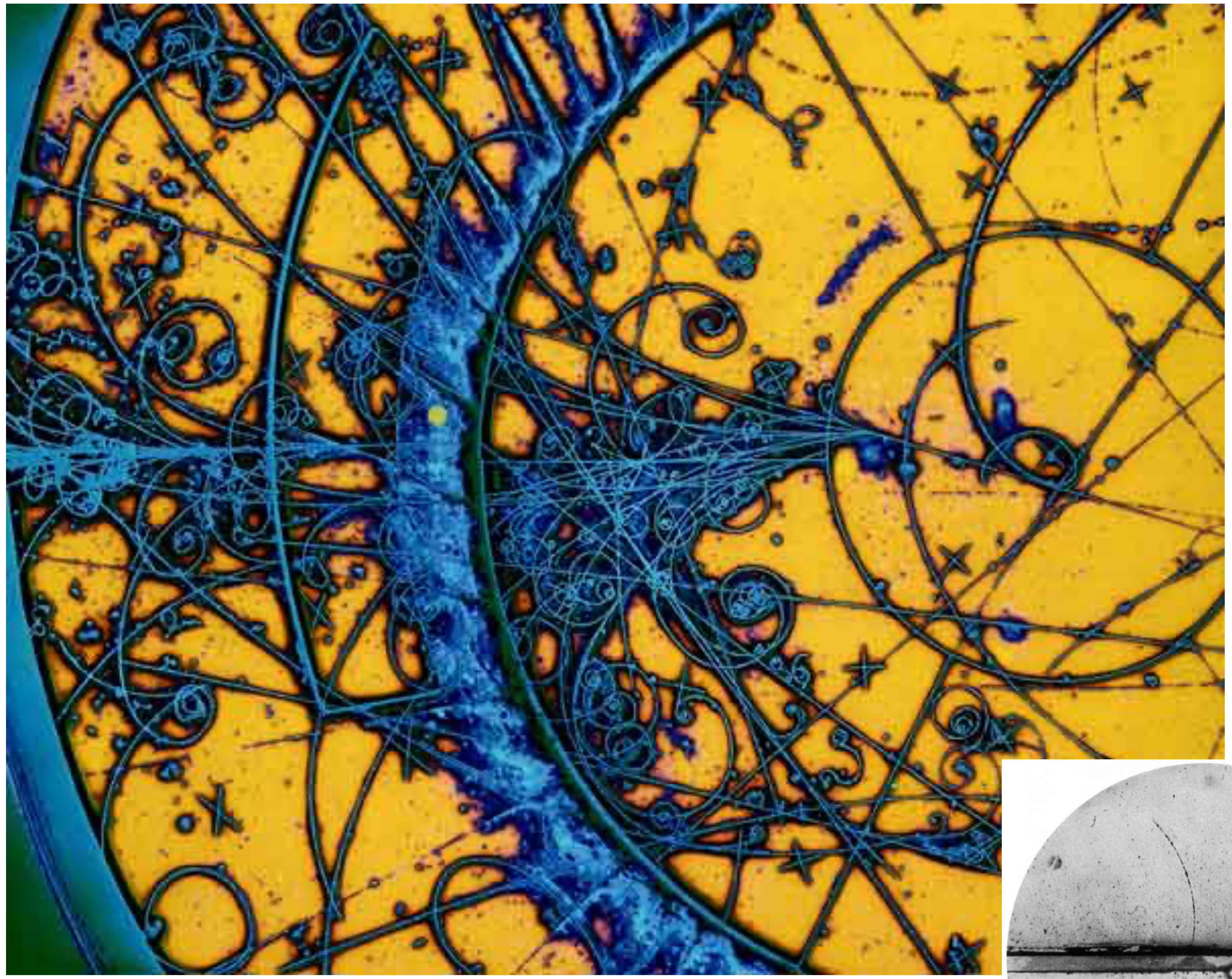


What about impurities?



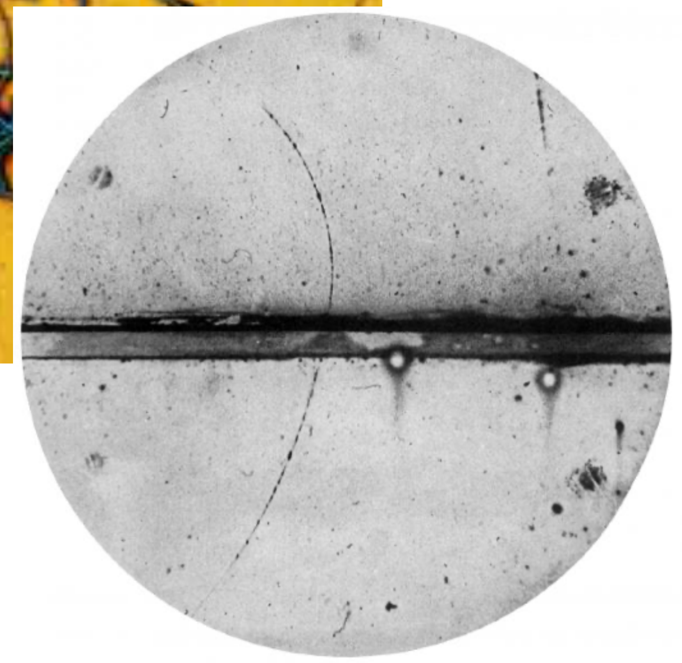


Figure: Bubble chamber

MONOPOLE AND VORTEX DISSOCIATION AND DECAY OF THE FALSE VACUUM

Paul Joseph STEINHARDT

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Received 17 February 1981

"**If monopole** (or vortex) **solutions exist** for a metastable or false vacuum, **a finite density of monopoles** (or vortices) **can act as impurity sites that trigger inhomogeneous nucleation** and decay of the false vacuum."

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Impurities in the early universe

Yutaka Hosotani

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104

(Received 1 November 1982)

“Now one has to ask the following question: **Is the early universe really sufficiently pure in order for supercooling to take place**? The aim of this paper is to show that in most cases the early universe is very pure. […] In this paper we consider **ordinary particles as impurities**.”

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Cosmic separation of phases

Edward Witten*

Institute for Advanced Study, Princeton, New Jersey 08540

(Received 9 April 1984)

"In particle physics it is often assumed that phase transitions are nucleated by thermal fluctuations. In practice, […] except in very pure, homogeneous samples, **phase transitions are often nucleated by various forms of impurities and inhomogeneities of nonthermal origin**."

"What if the transition was nucleated by impurities? In this case **the mean spacing between bubbles has nothing to do with free energies** of nucleation and is simply the spacing between the relevant impurities. "

The nature of impurities

- Compact objects and gravitational effects

(Coleman-de Luccia, PRD, 1980)

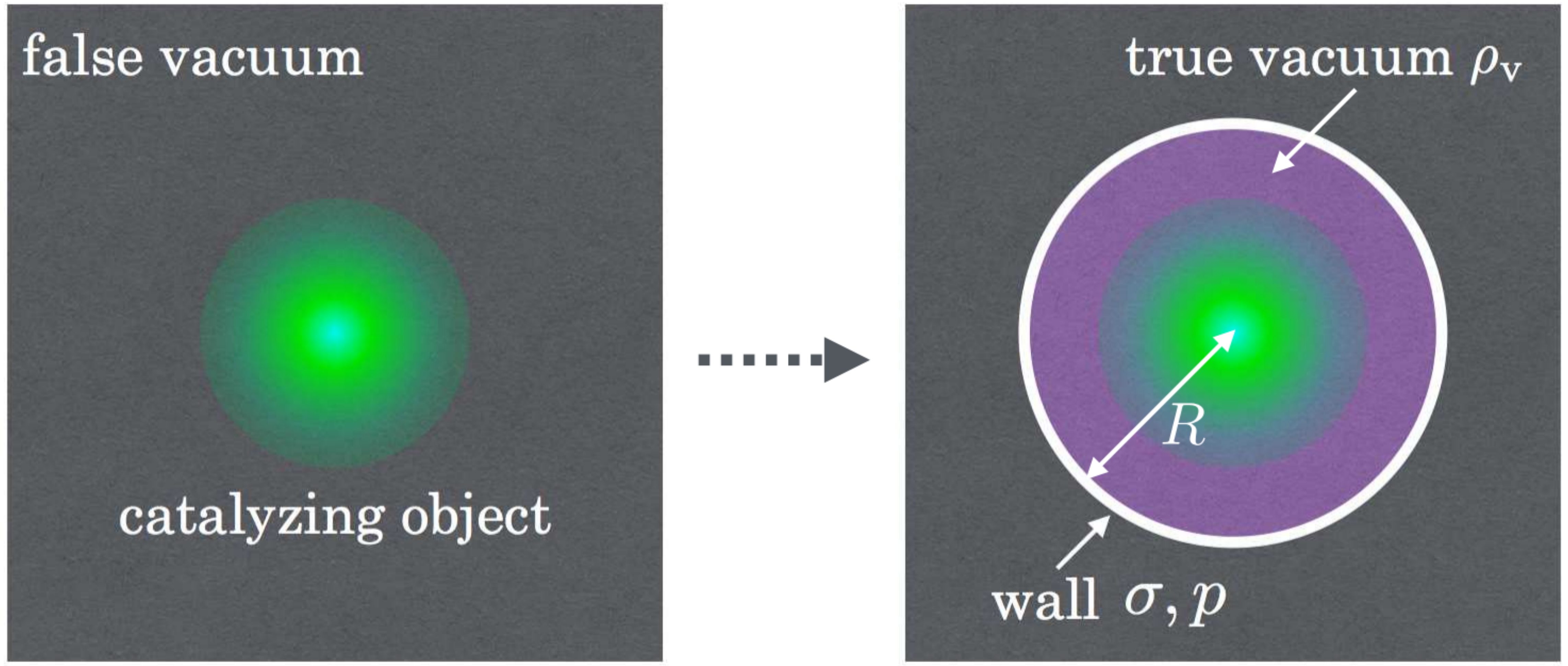

Fig. from Oshita, Yamada, Yamaguchi [1808.01382], PLB

Jinno, Kume, Yamada [2310.06901], PLB

Hiscock, PRD, 1987; Burda, Gregory, Moss [1501.04937], PRL

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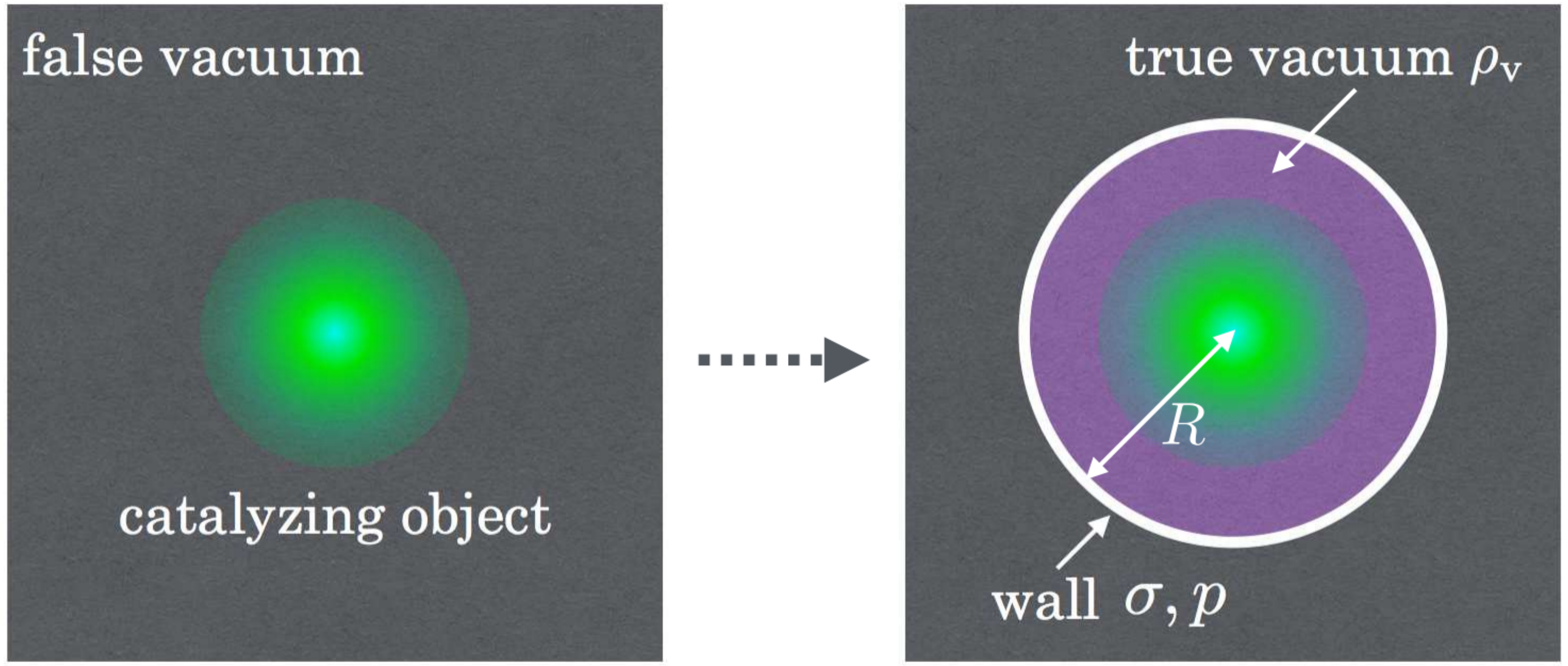

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- Primordial density fluctuations

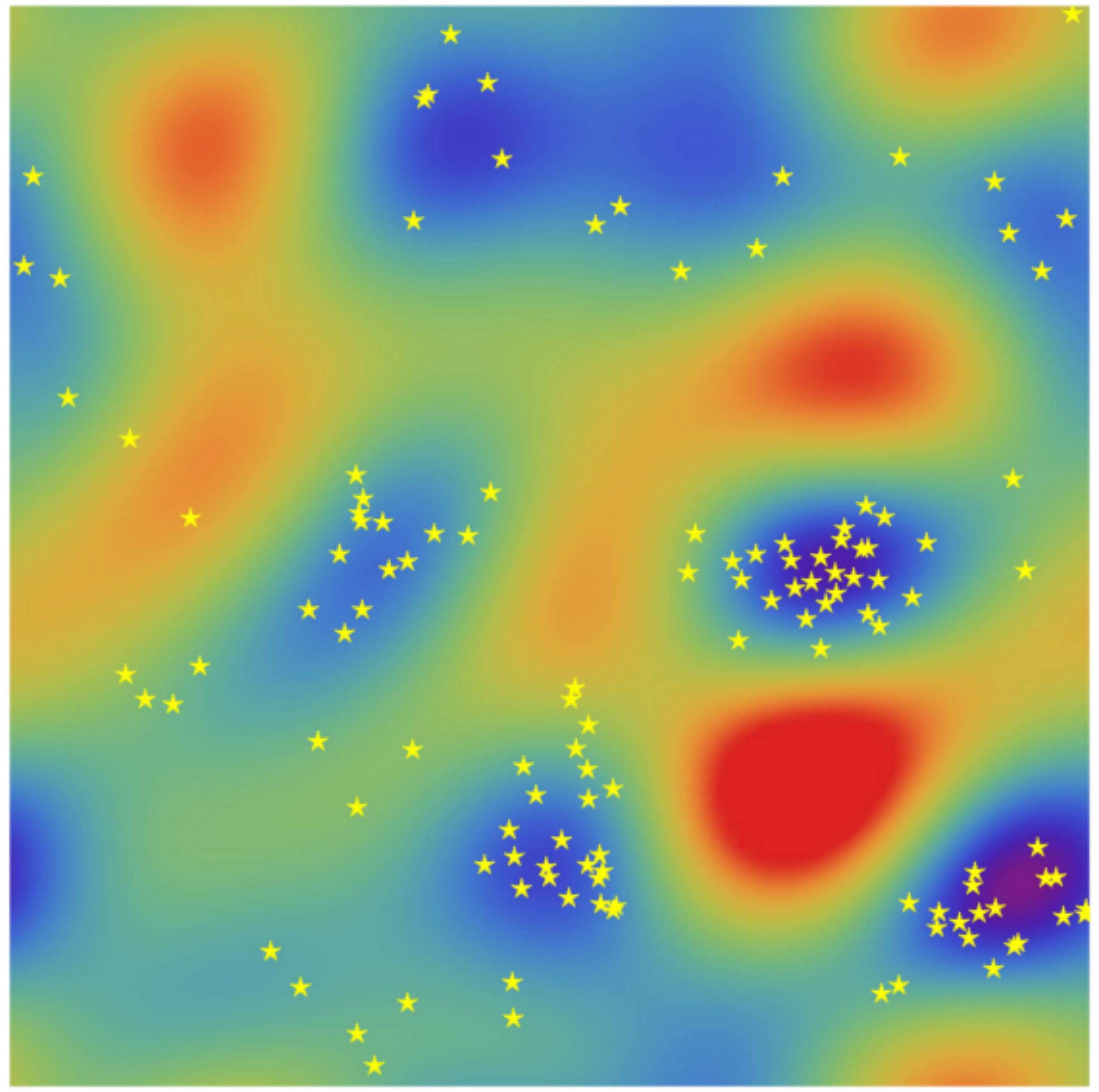
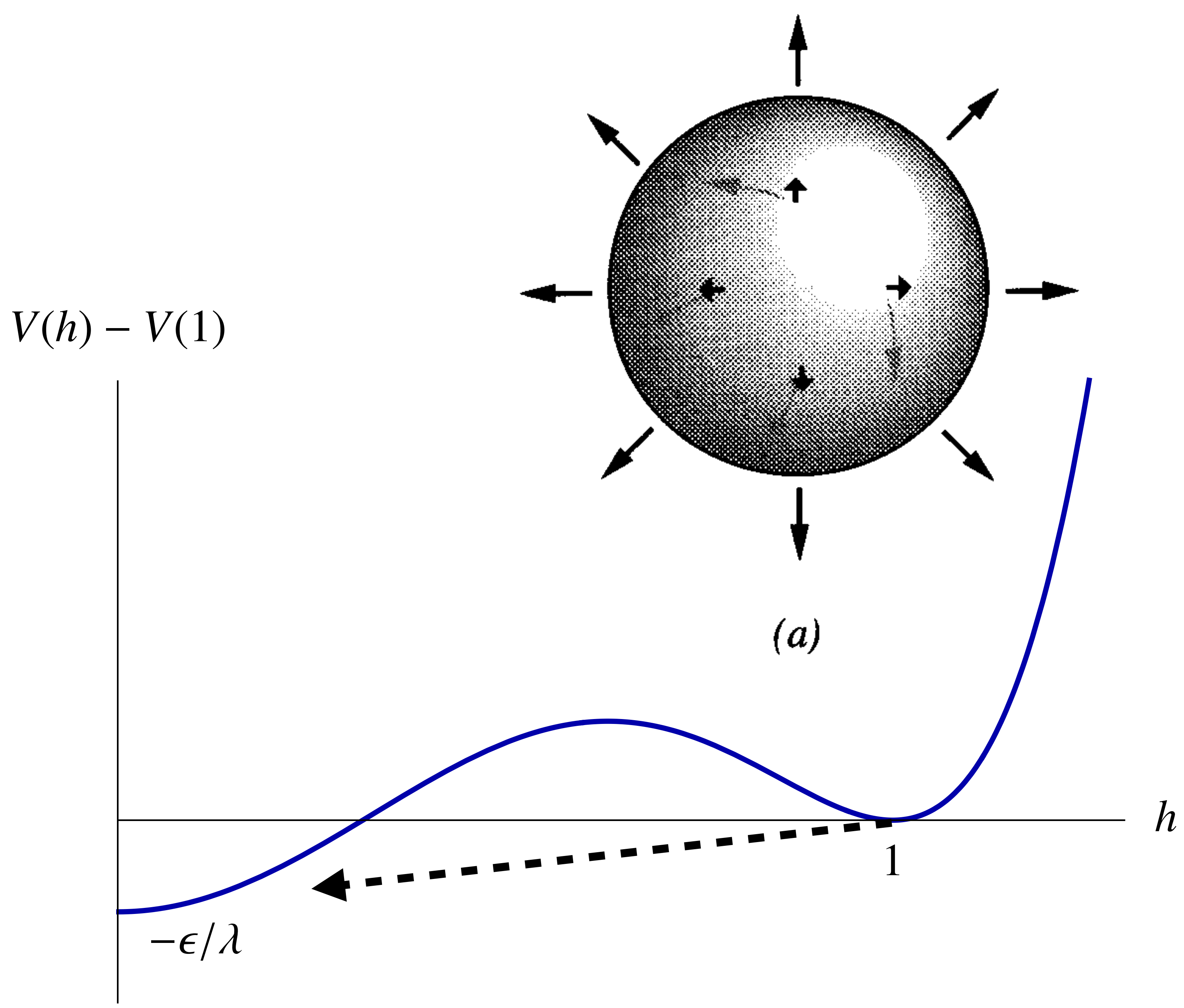


Fig. from Jinno, Konstandin, Rubira, van de Vis, [2108.11947], JCAP

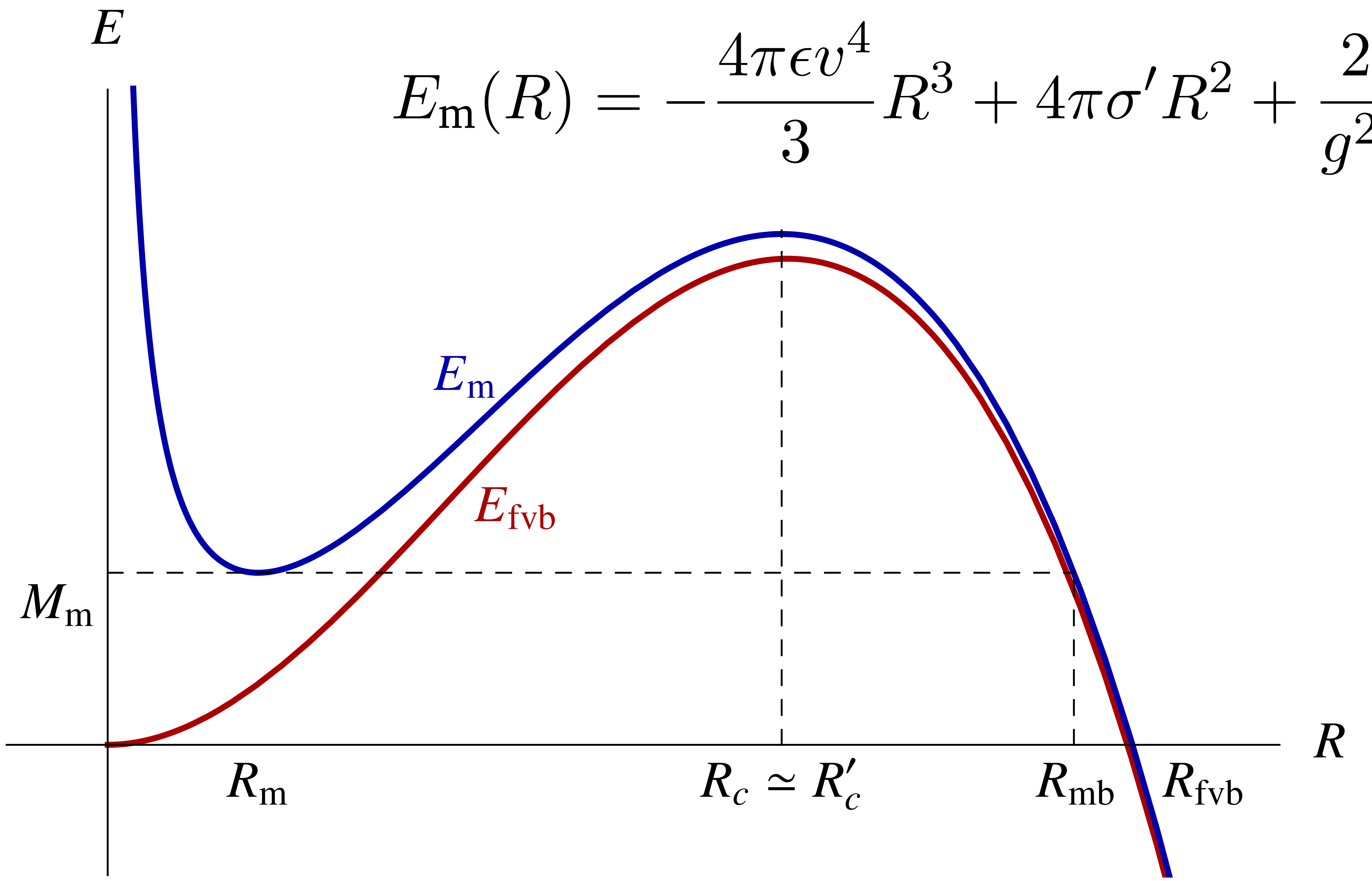
The nature of impurities

- Topological defects

Monopoles



$$E_{\mathrm{m}}(R) = -\frac{4\pi\epsilon v^4}{3}R^3 + 4\pi\sigma' R^2 + \frac{2\pi}{g^2 R}$$



Figs. from Agrawal, Nee [2202.11102] SciPost Phys.

The nature of impurities

- Topological defects

Domain walls

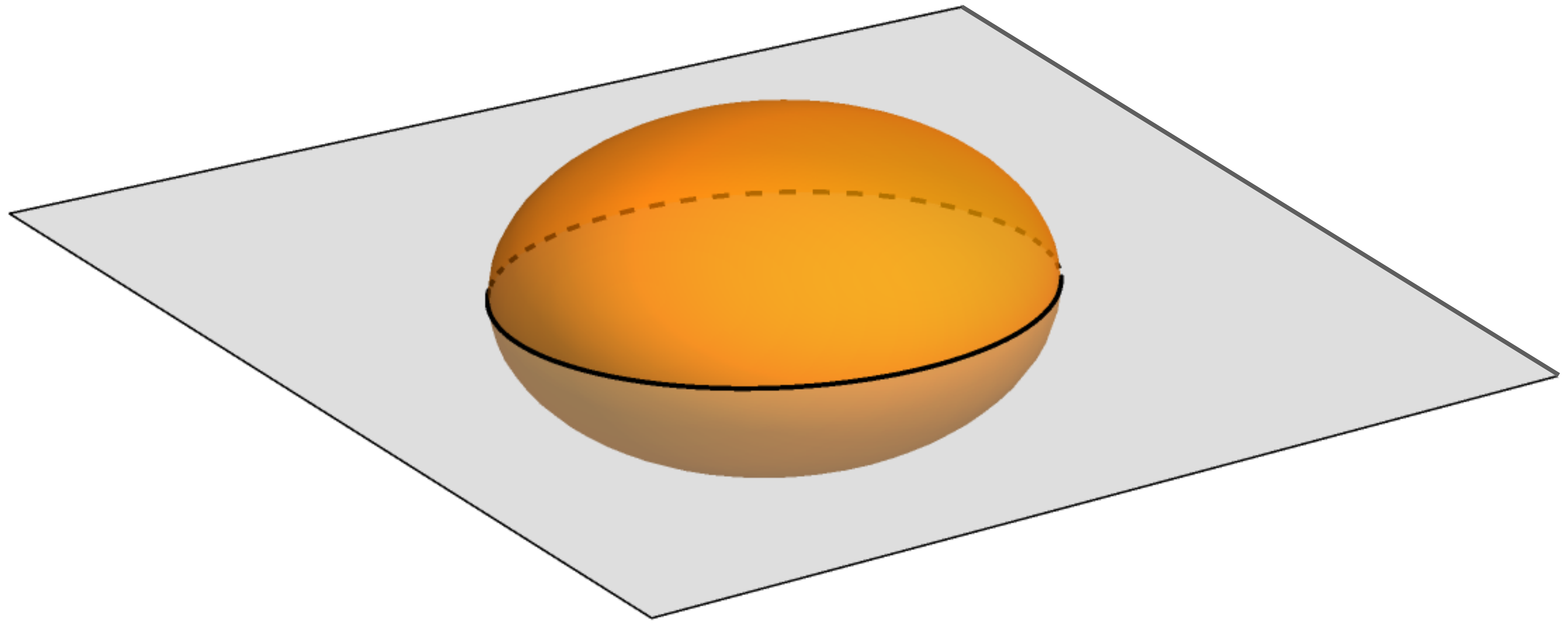


Fig. from Agrawal, SB, Mariotti, Nee [2312.06749] JHEP

Cosmic strings

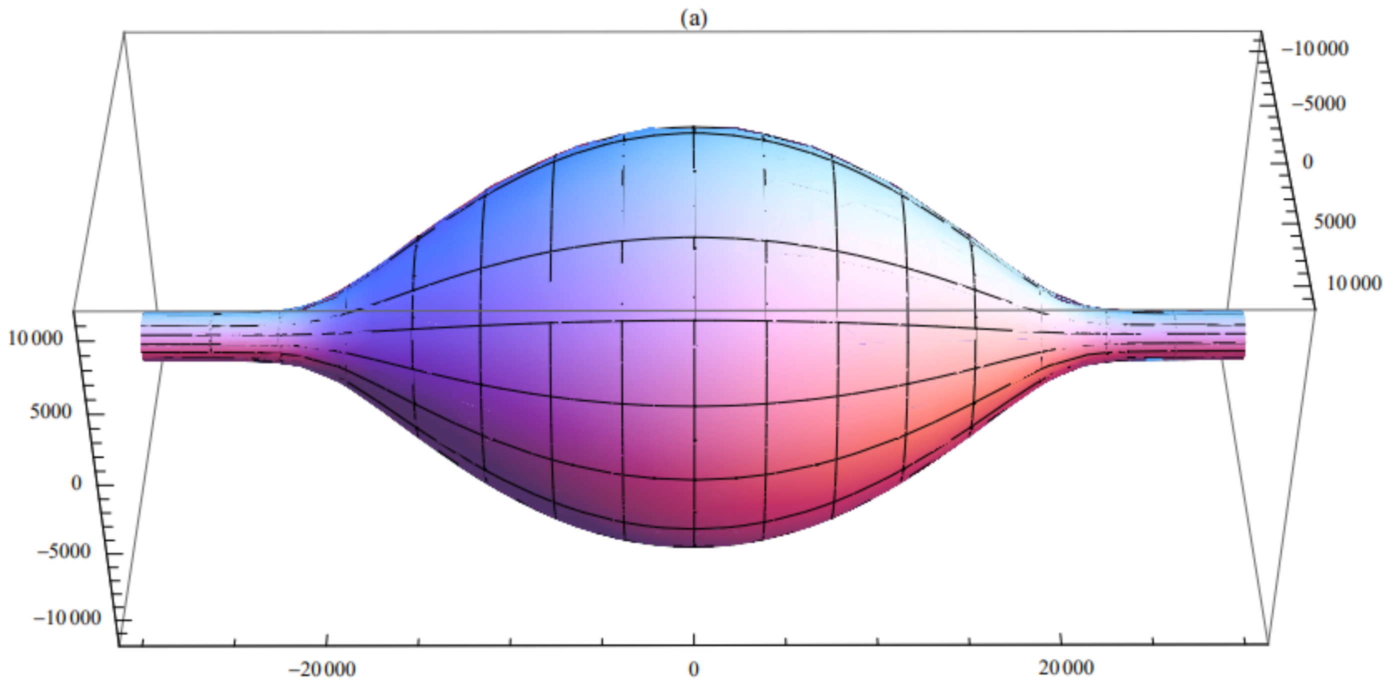
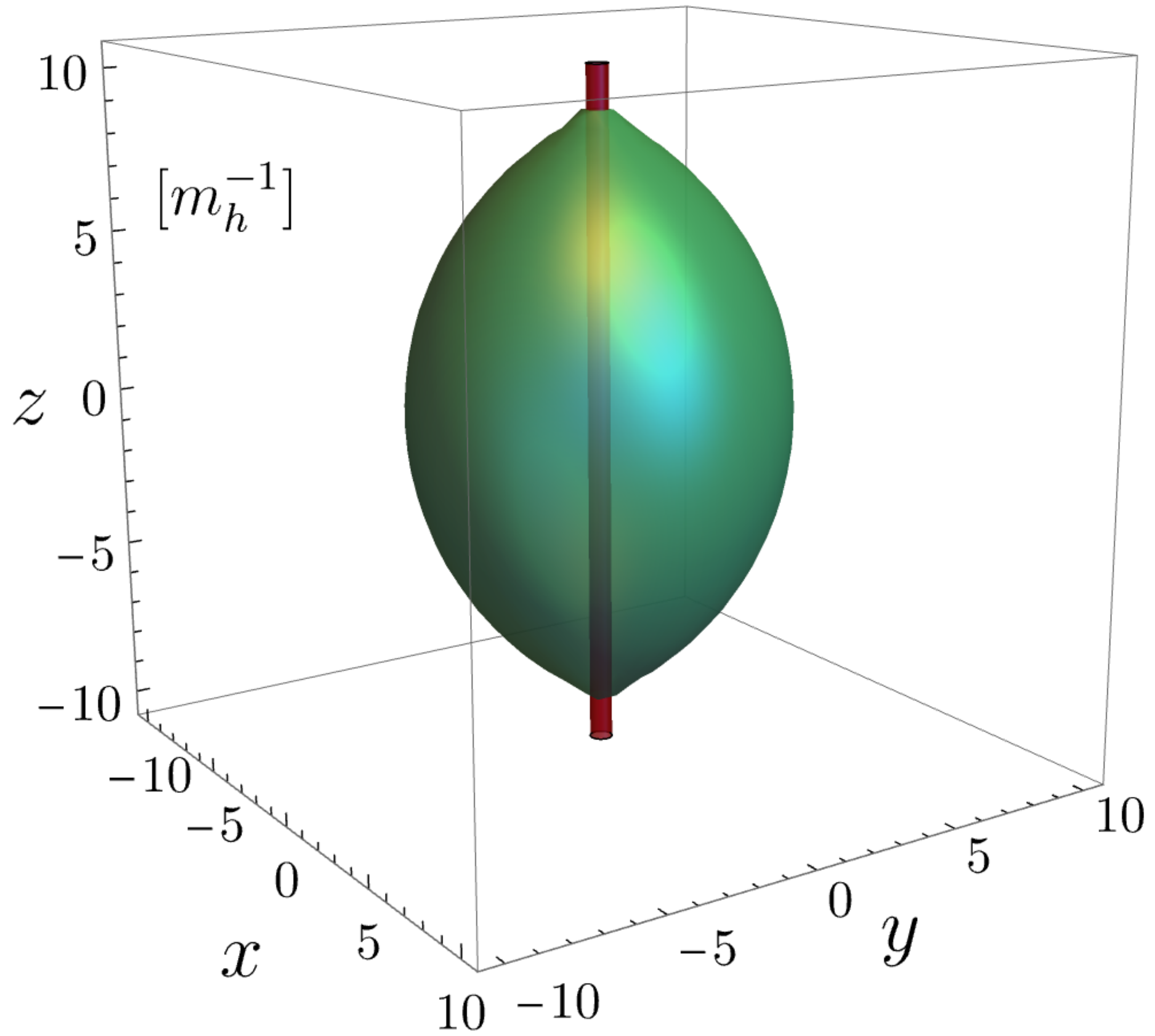


Fig. from Lee et al., [1310.3005], PRD

Yajnik, PRD, 1986

Fig. from SB, Mariotti, [2405.08060] SciPost Phys.



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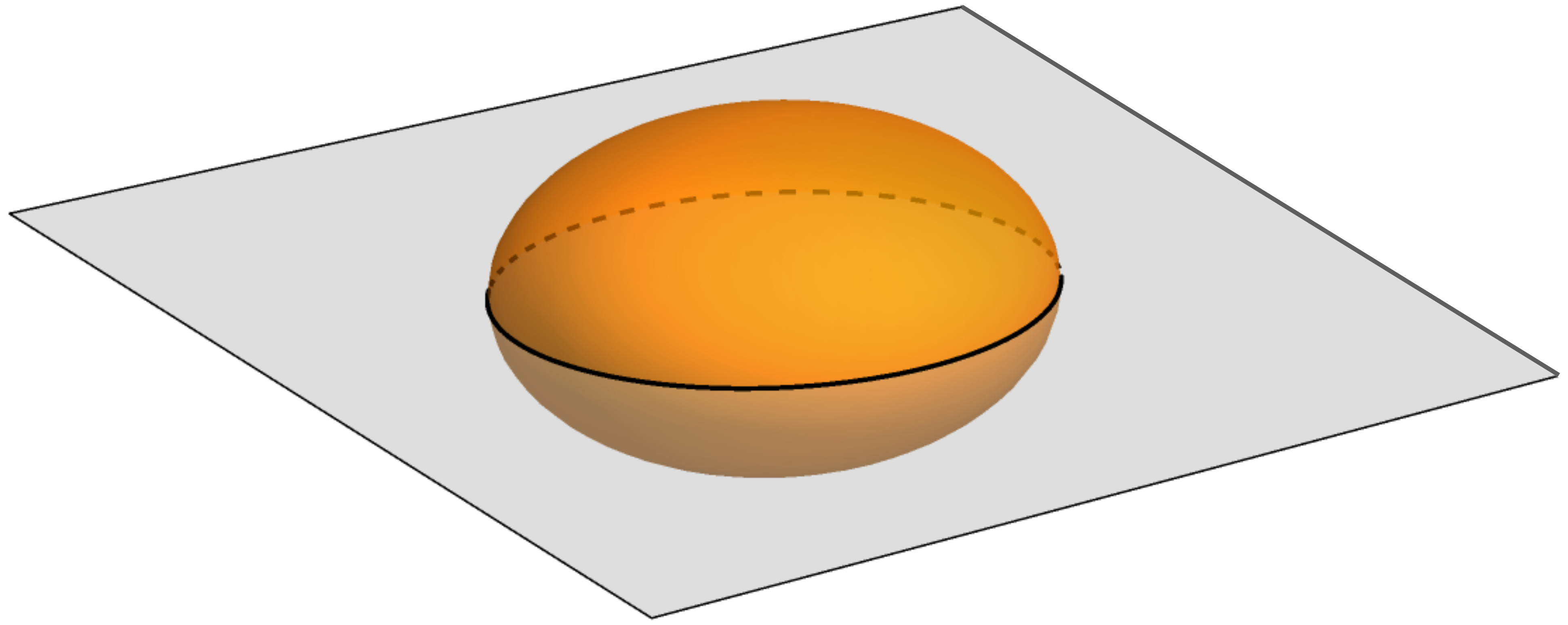

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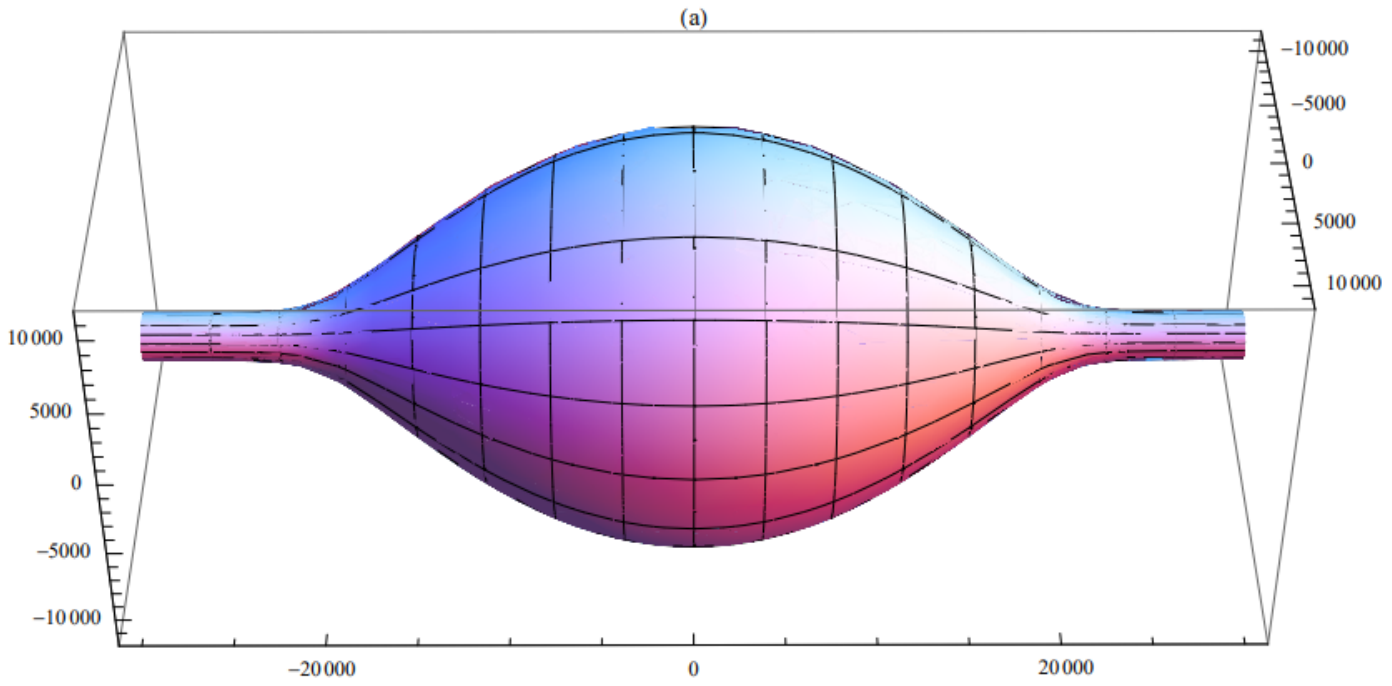

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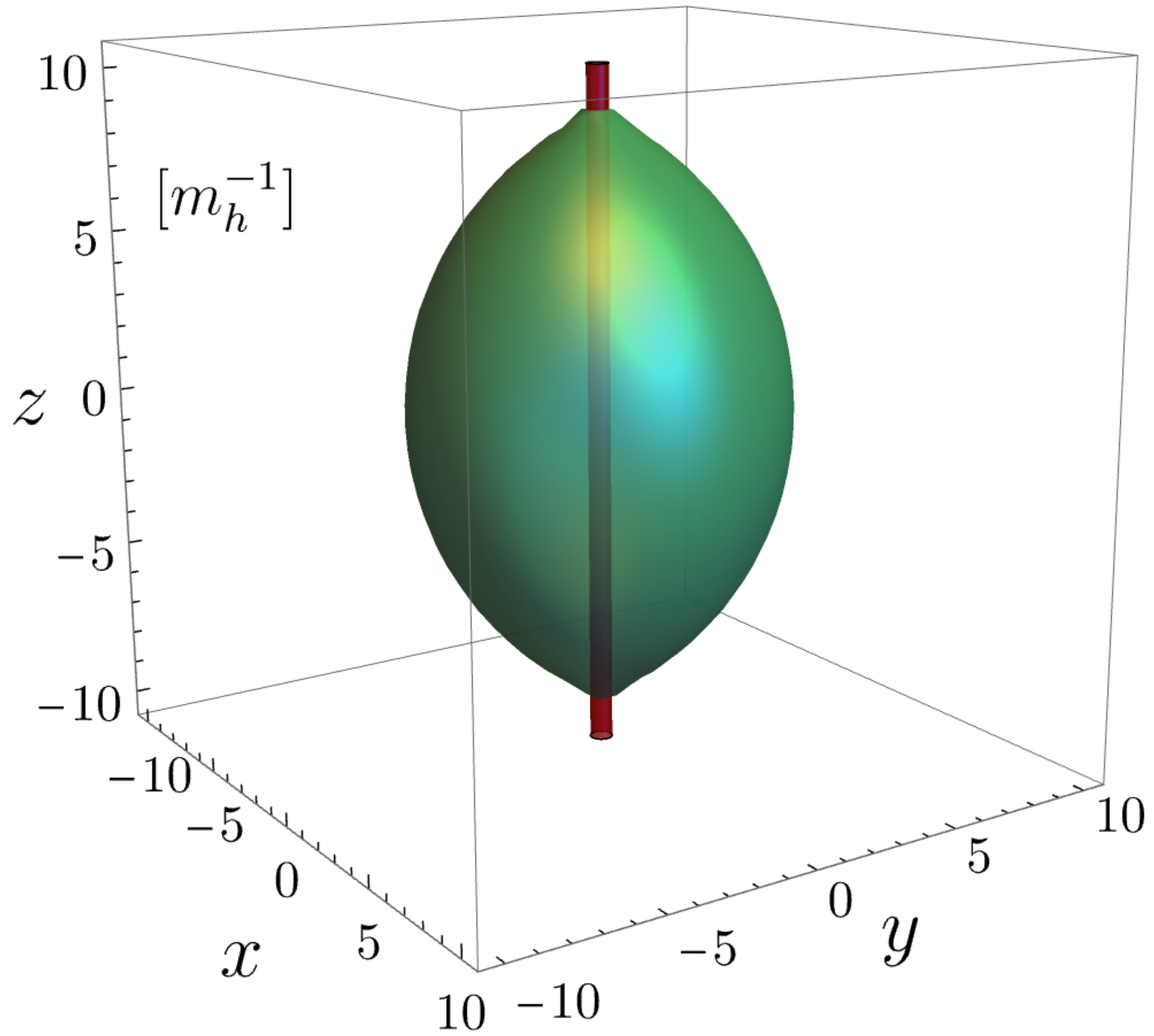
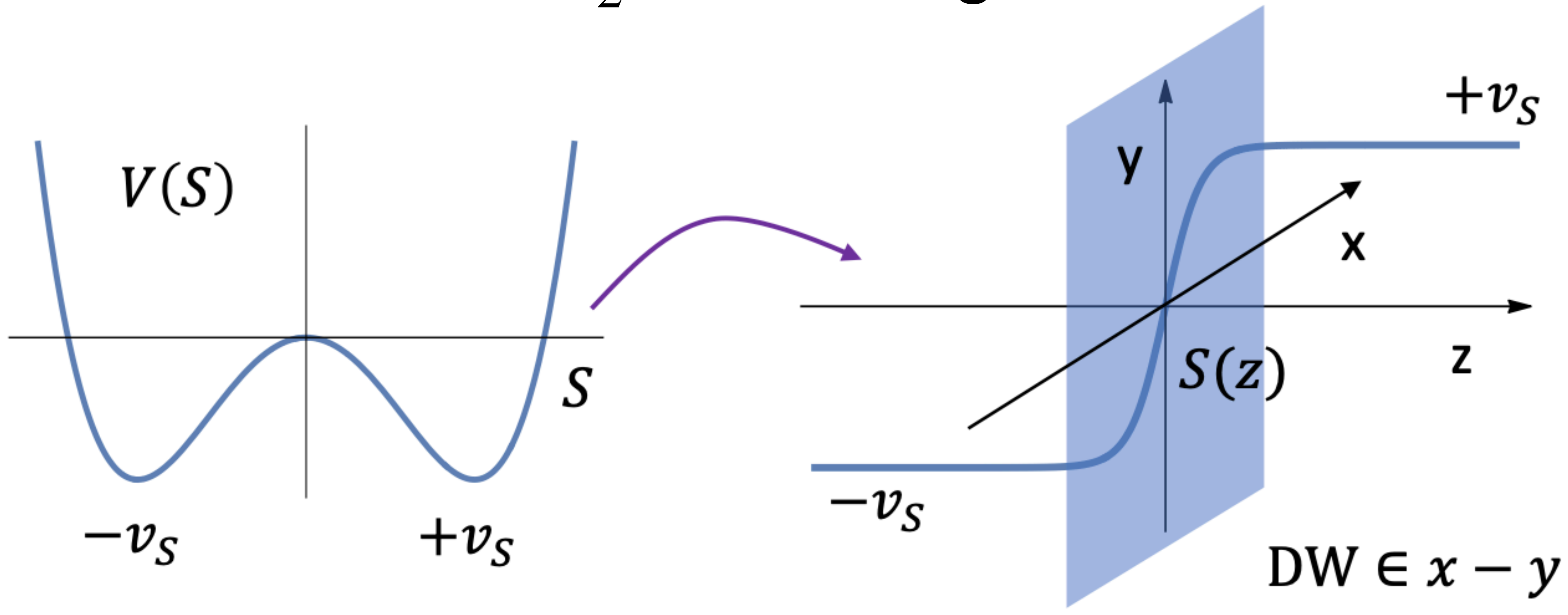


Can be realized in superfluid He!

Topological classification

Defect	Dimension	Homotopy	Mass
Domain walls	2	$\pi_0(\mathcal{M})$	σL^2
Cosmic strings	1	$\pi_1(\mathcal{M})$	μL

$\mathbb{Z}_2 \to$ nothing



$\mathrm{U}(1) \to$ nothing

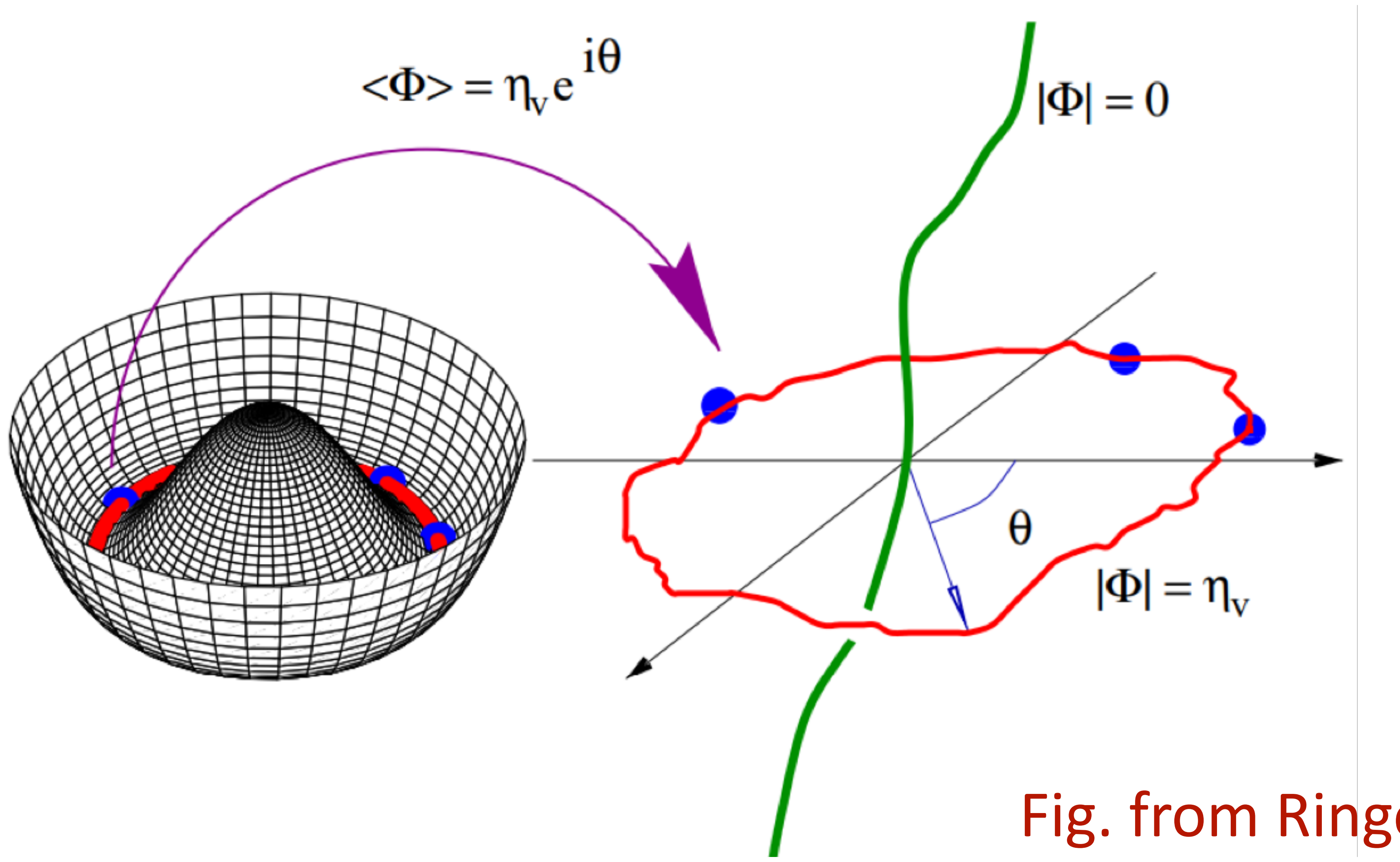


Fig. from Ringeval 2010

EWPT with a singlet

- SM + scalar singlet with $\mathbb{Z}_2 : S \to -S$

$$
\begin{aligned}
V = & -\frac{1}{2}(\mu^2 - c_h T^2)h^2 + \frac{1}{4}\lambda\, h^4 \\
& -\frac{1}{2}(m^2 - c_s T^2)S^2 + \frac{1}{4}\eta\, S^4 \\
& +\frac{1}{2}\kappa\, h^2 S^2
\end{aligned}
$$

*$\mathbb{Z}_2$ breaking terms destabilize the wall network and are set to zero in the following

See e.g. Espinosa, Gripaios, Konstandin, Riva [1110.2876] JCAP

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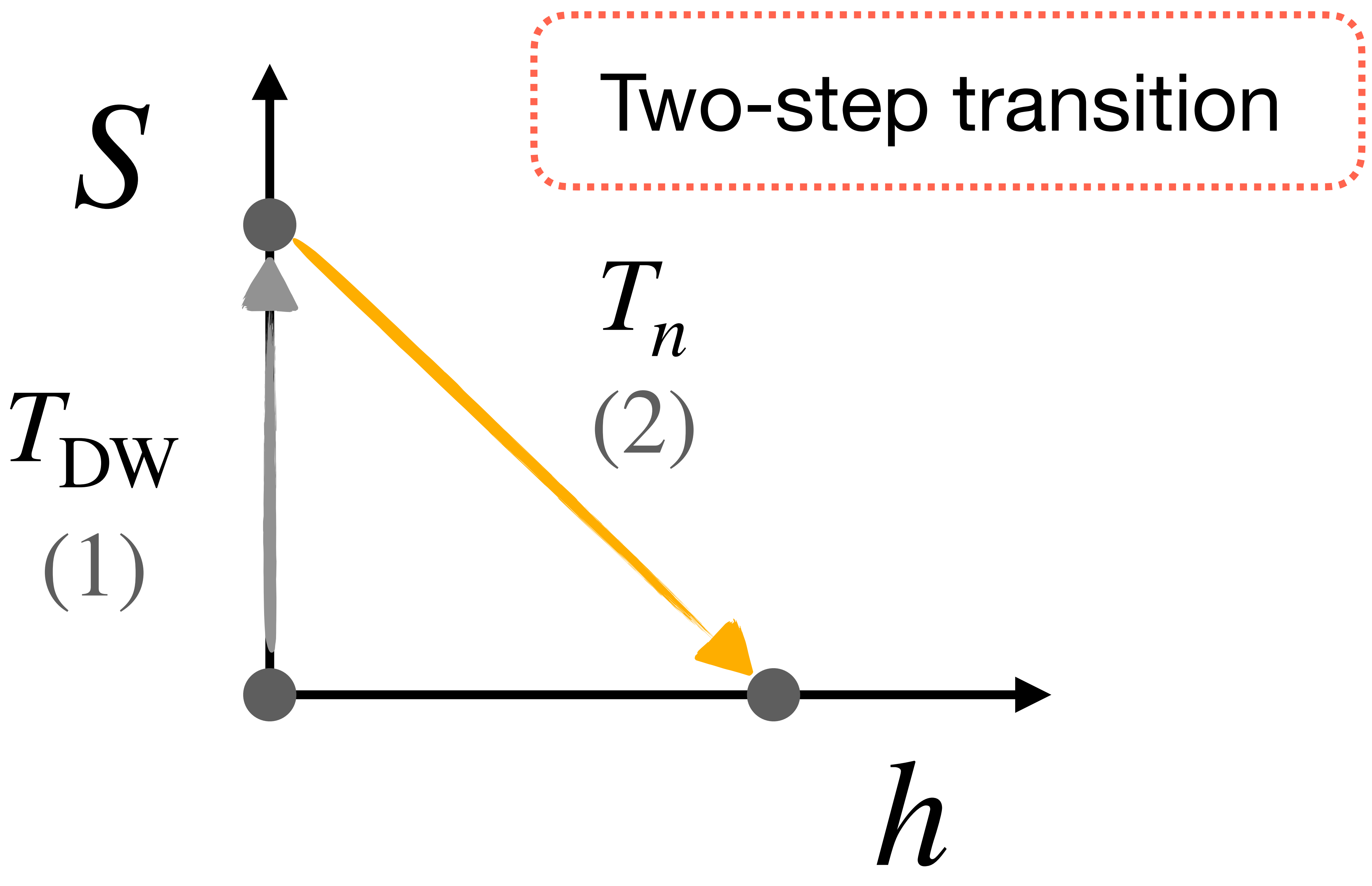

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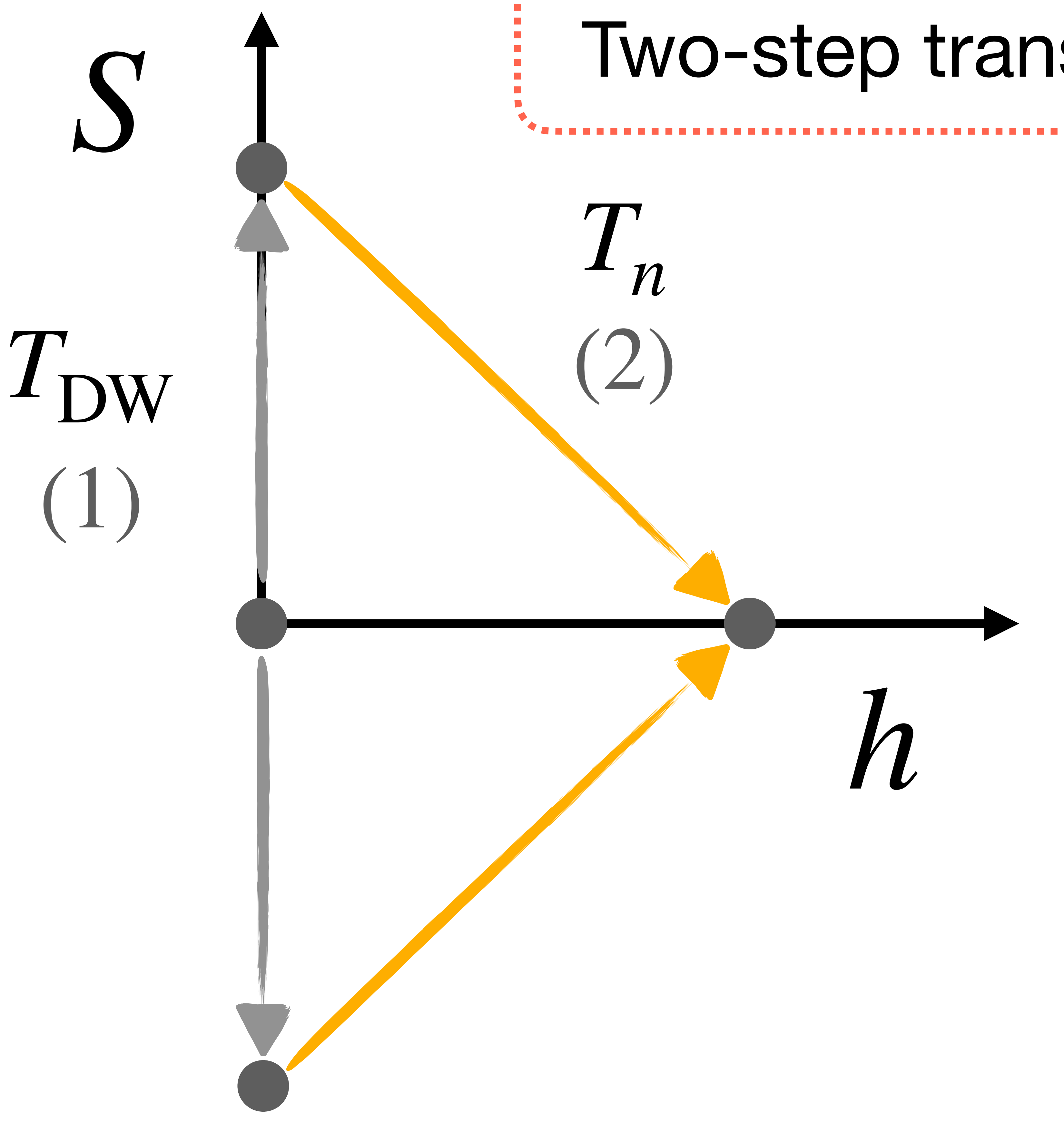
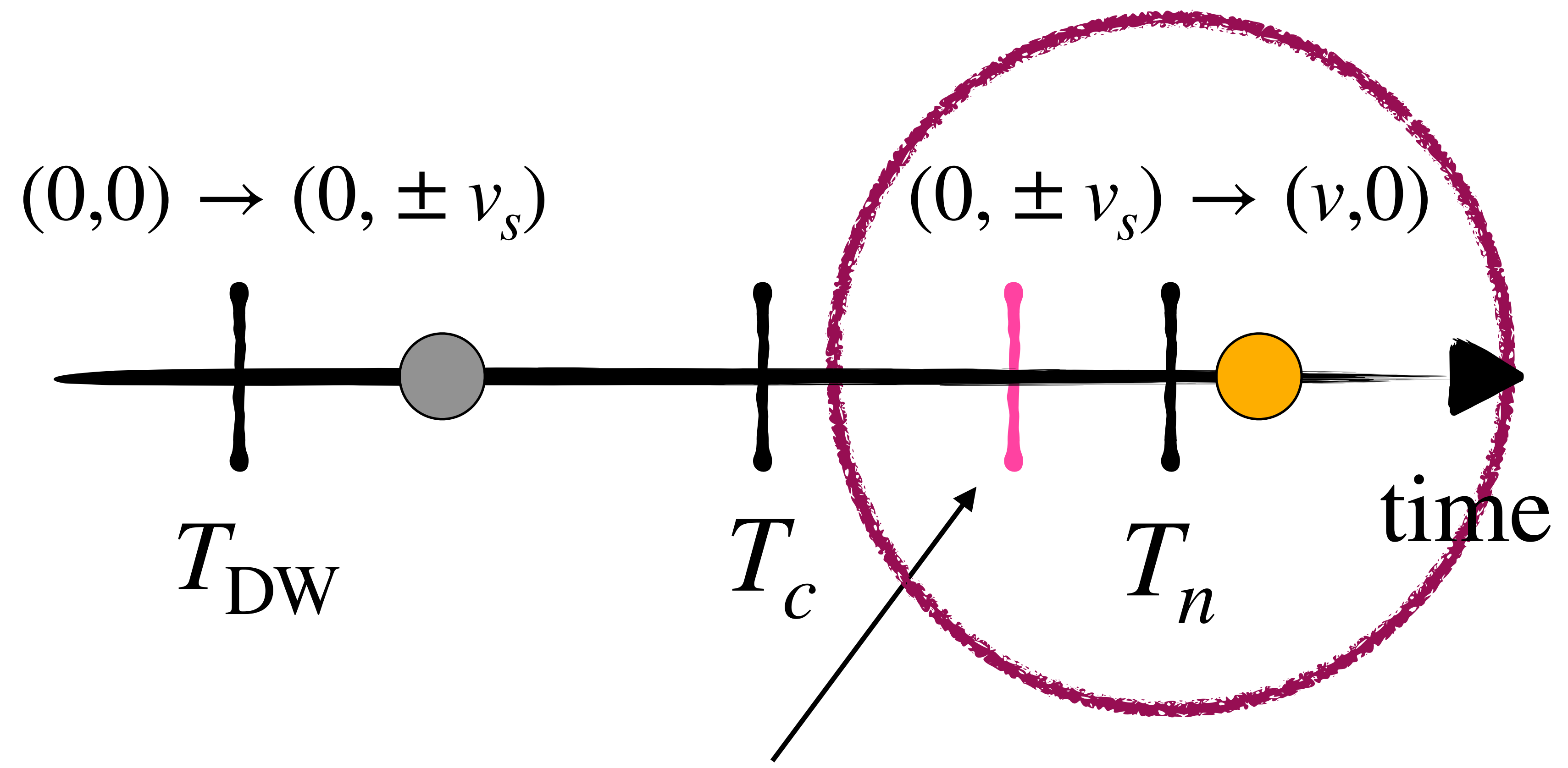
Two-step transition

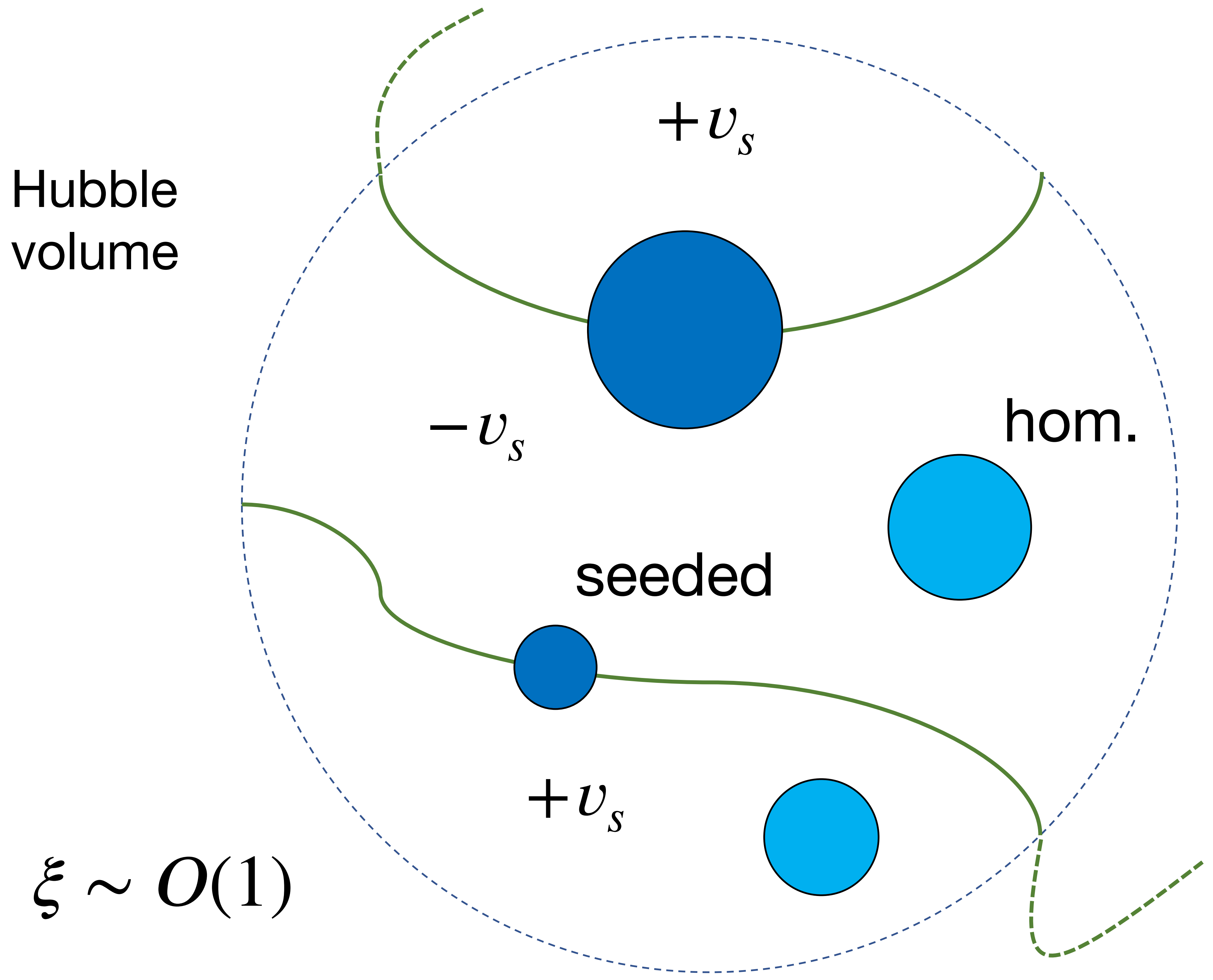


EWPT with a singlet

- Competition between **homogenous** and **seeded nucleation** for 2nd step:



At T_n^{seed} bubbles may nucleate *on* the walls



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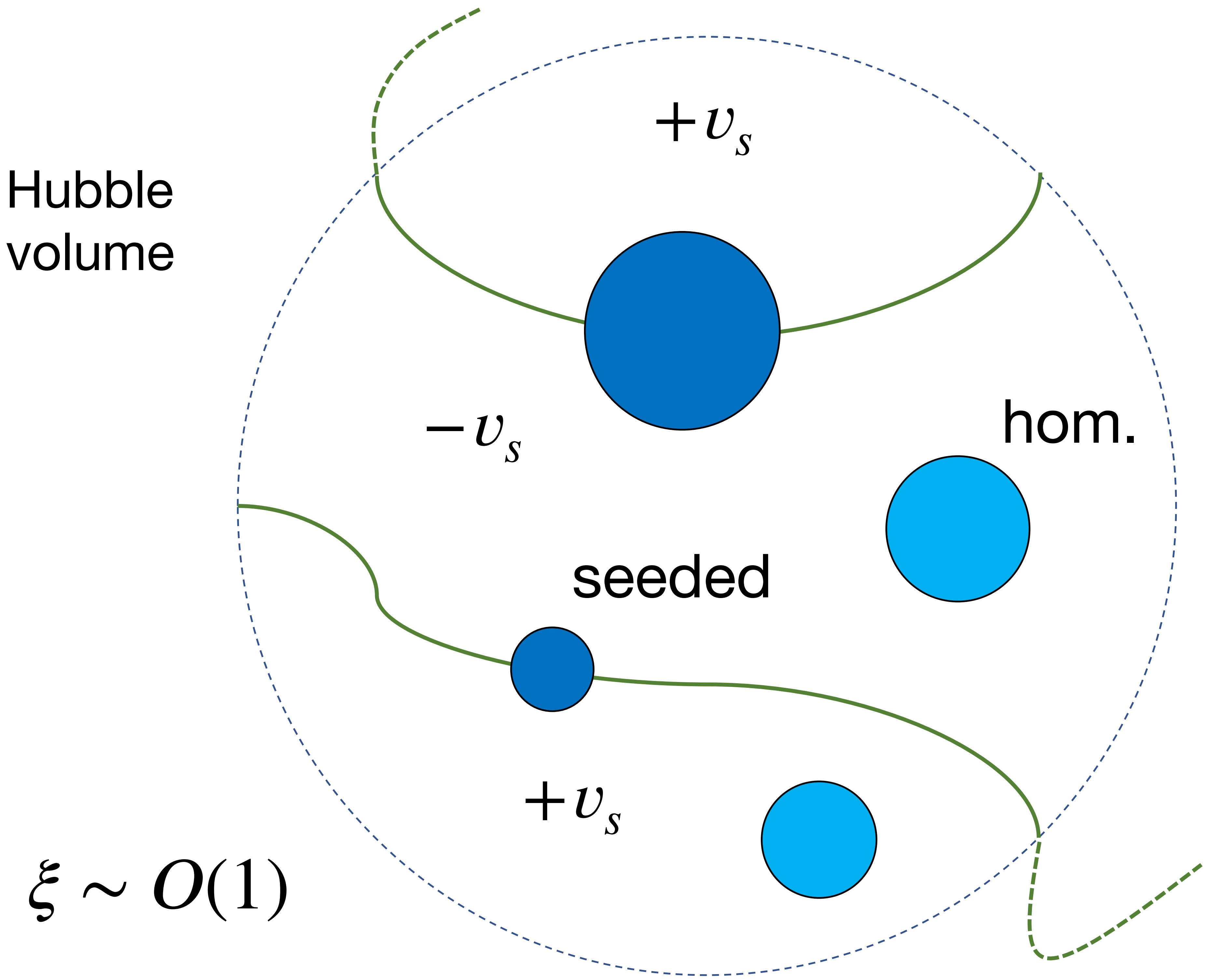
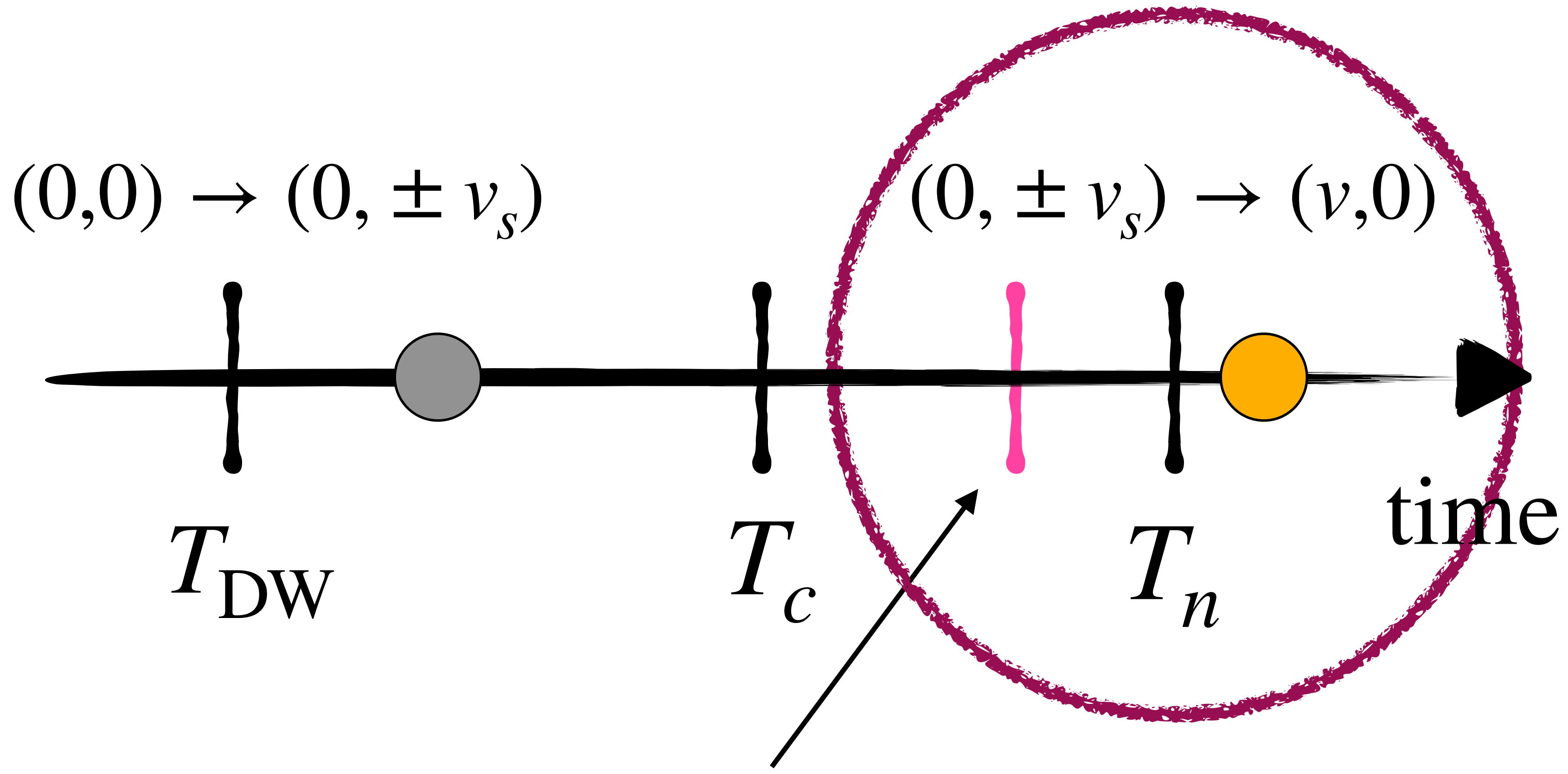


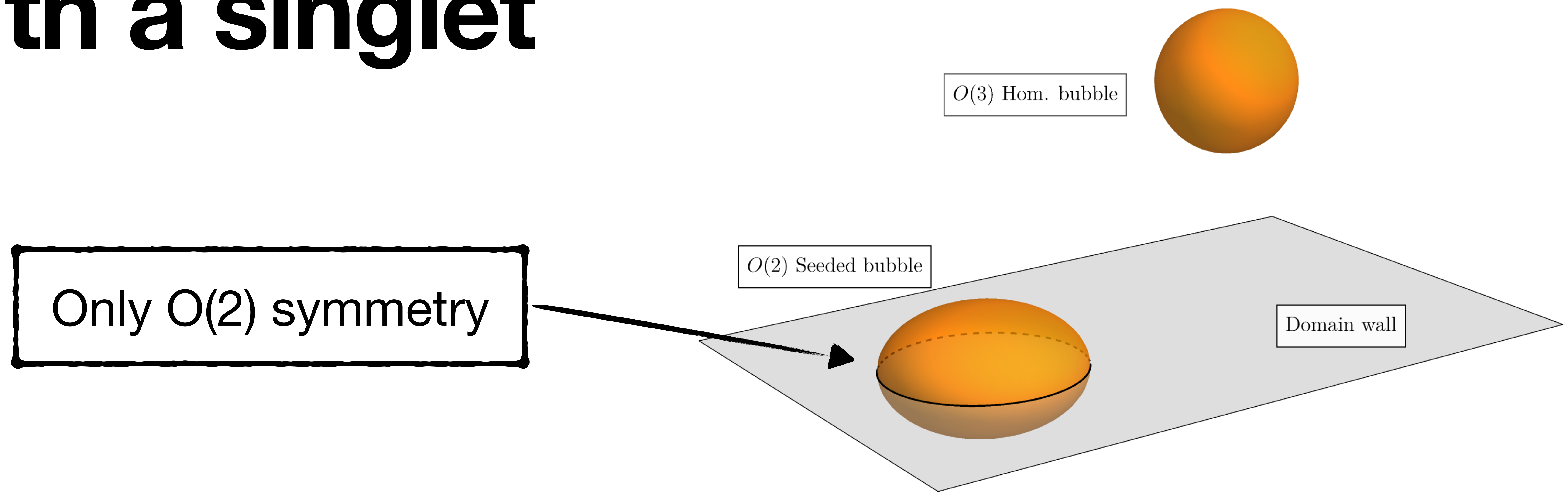


At T_n^{seed} bubbles may nucleate *on* the walls

- Define nucleation rate per unit surface
- Stricter nucleation condition (only on sub manifold)

Lazarides, Shafi, Kibble 1982, PRD
Perkins, Vilenkin 1992, PRD

EWPT with a singlet



1. Solving coupled **system of PDEs**
   - "Exact"
   - Physical picture?
   - Which initial conditions for the algorithm?

2. **Thin wall** approximation
   - Limited validity
   - Intuitive picture
   - Simple calculation

3. **Kaluza-Klein** decomposition
   - Quantitative results
   - Still intuitive
   - Initial conditions for num. algorithms and cross-checks

Agrawal, SB, Mariotti, Nee [2312.06749] JHEP

SB, Mariotti [2203.16450] PRL

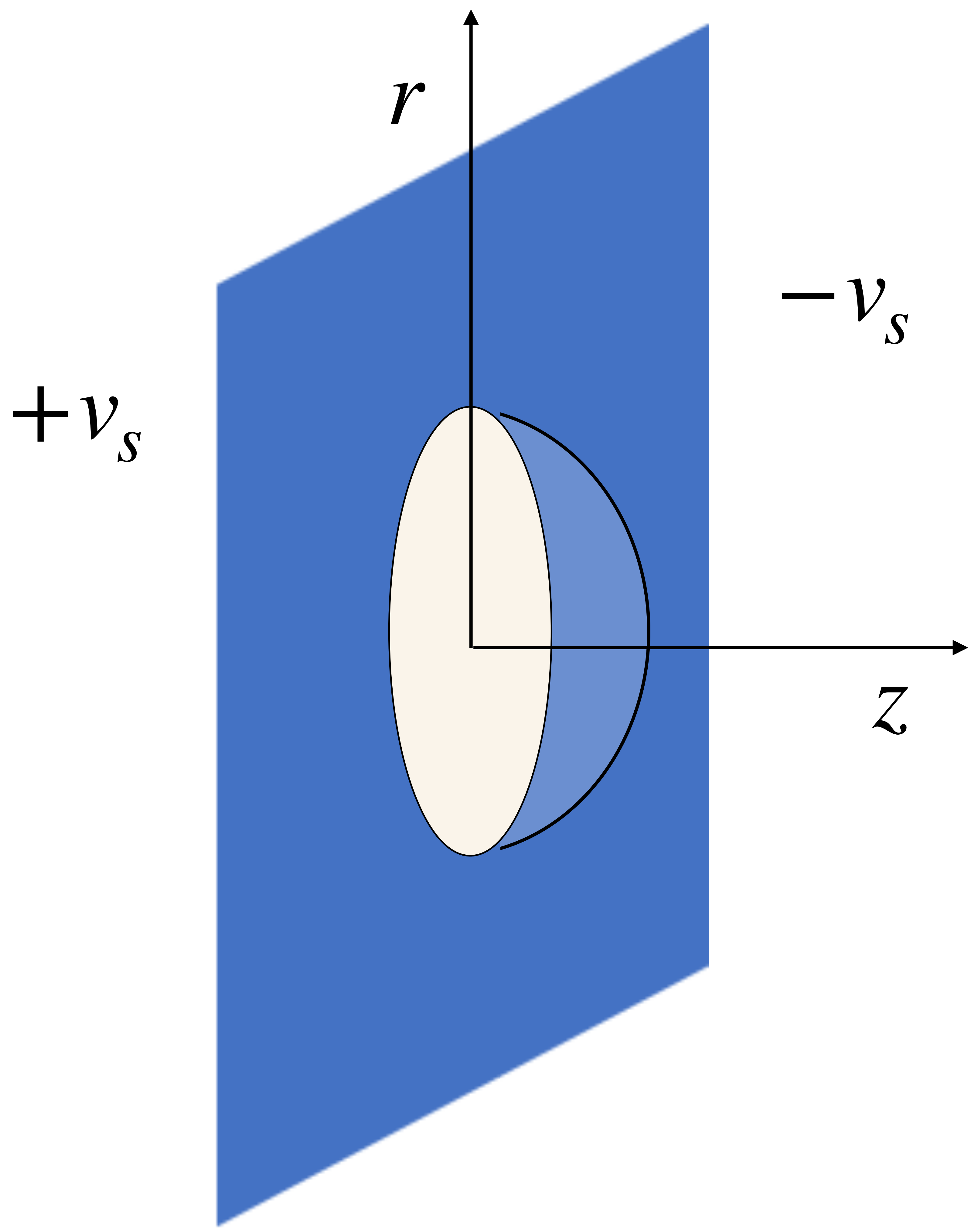
1. Equations of motion

False vacuum is non-trivial as it depends on z

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = \frac{\partial V}{\partial \phi}, \qquad \phi = h, S$$

$$S(\infty, z) = S_{DW}(z), \qquad S(r, \pm\infty) = \pm v_s,$$

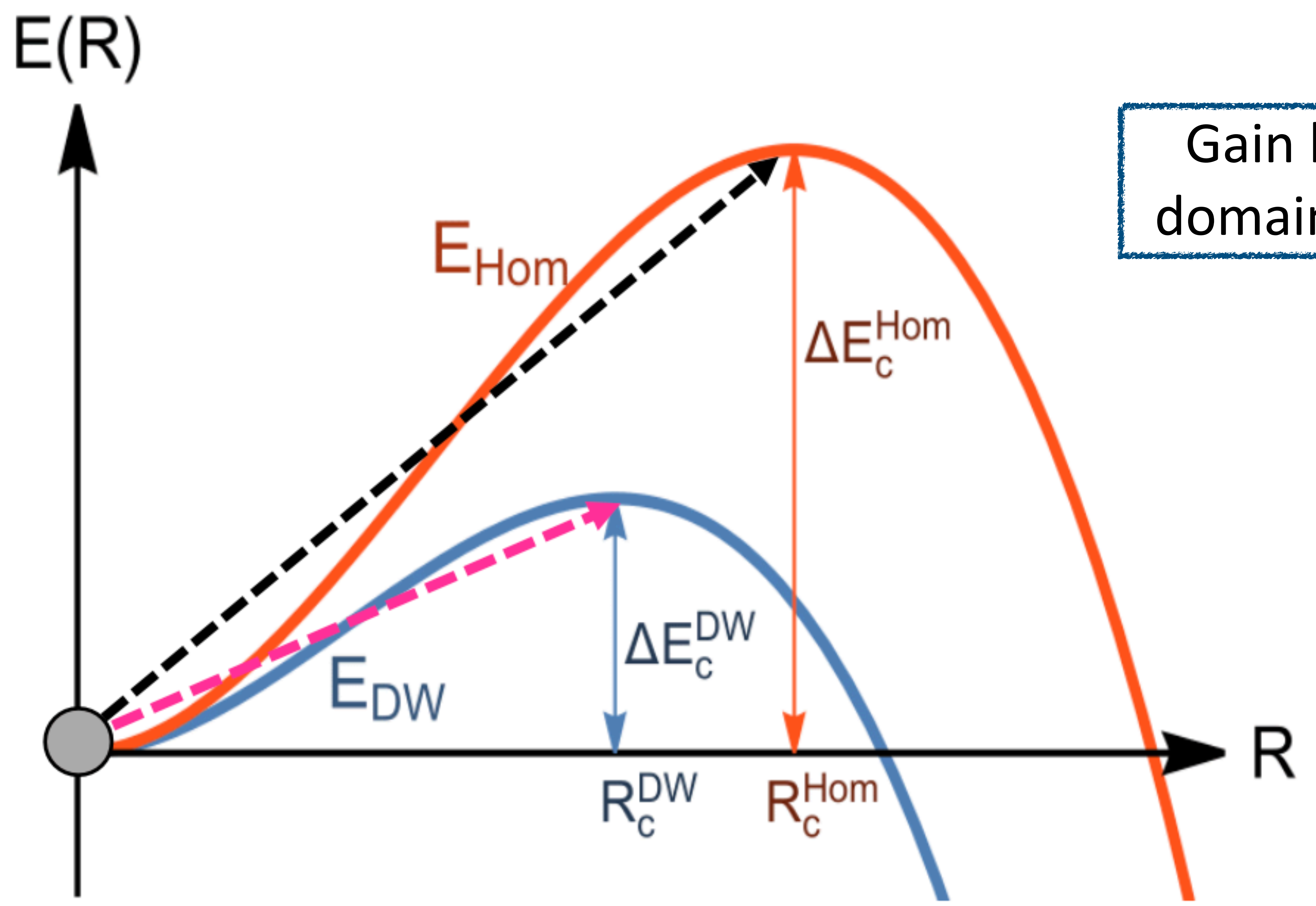
$$h(\infty, z) = h(\rho, \pm\infty) = 0$$

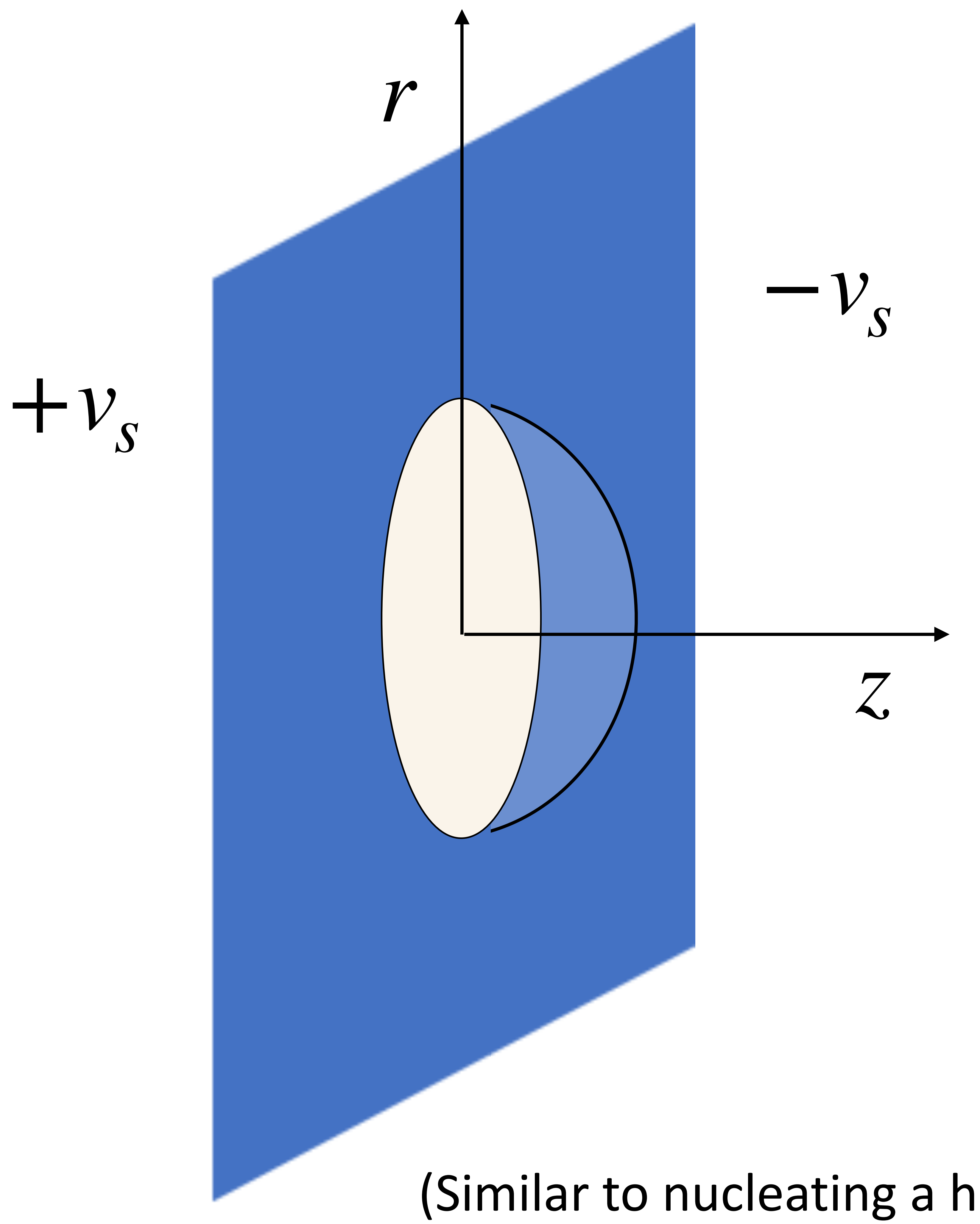


2. Thin wall approximation

$$E(R) = -\frac{4\pi}{3}\epsilon R^3 + 4\pi\left(\sigma_B - \frac{1}{4}\sigma_{DW}\right)R^2$$

Gain by eating up domain wall surface





(Similar to nucleating a hole in string-wall networks)

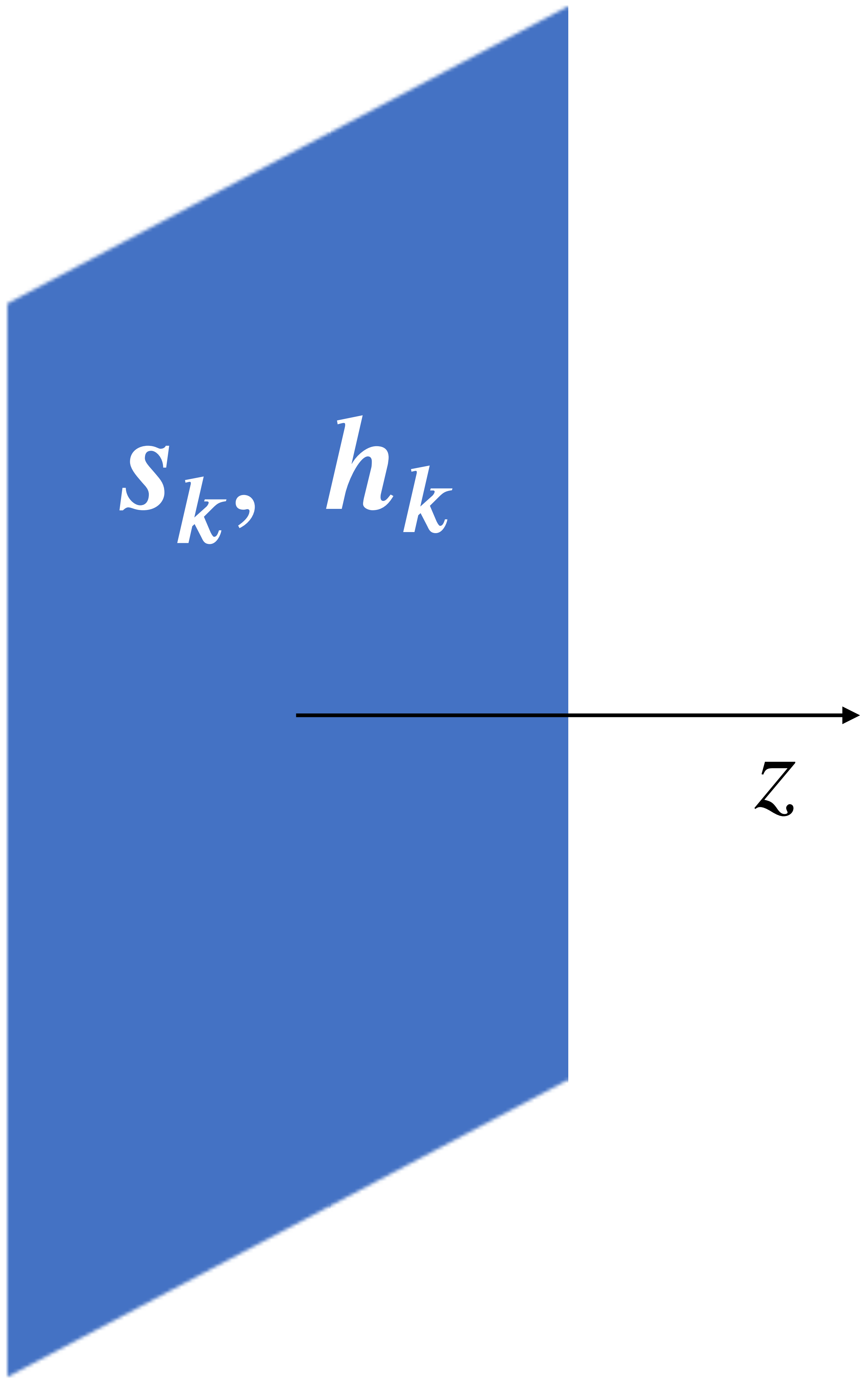
3. Kaluza-Klein decomposition

- Expand the fields around the domain wall background:

$$S = S_{\mathrm{DW}}(z) + \sum_k s_k\left(x_\mu\right)\sigma_k(z)$$

$$h = \sum_k h_k\left(x_\mu\right)\phi_k(z)$$

$$x_\mu = t, x, y$$



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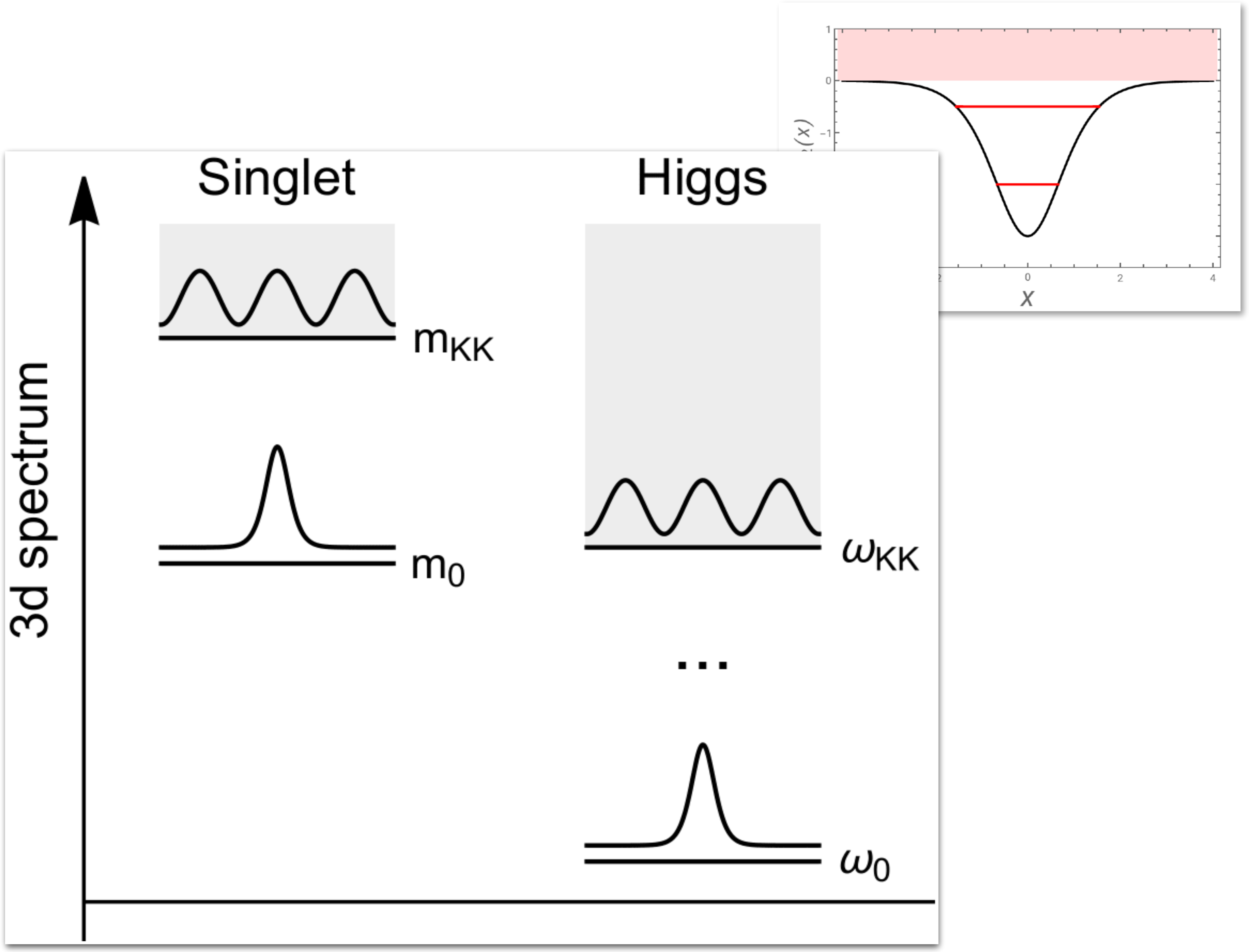
- Eigenspectrum of excitations:



See also “Solitons and instantons”, Rajaraman 1982

3. Kaluza-Klein decomposition

- Integrate along z to obtain 3d action and integrate out continuum excitations:

$$S_{3d} = \int d^3x \, \frac{1}{2}(\partial_\mu h_0)^2 + \frac{1}{2}(\partial_\mu s_0)^2 - V_{3d}^{\text{eff}}(h_0, s_0)$$

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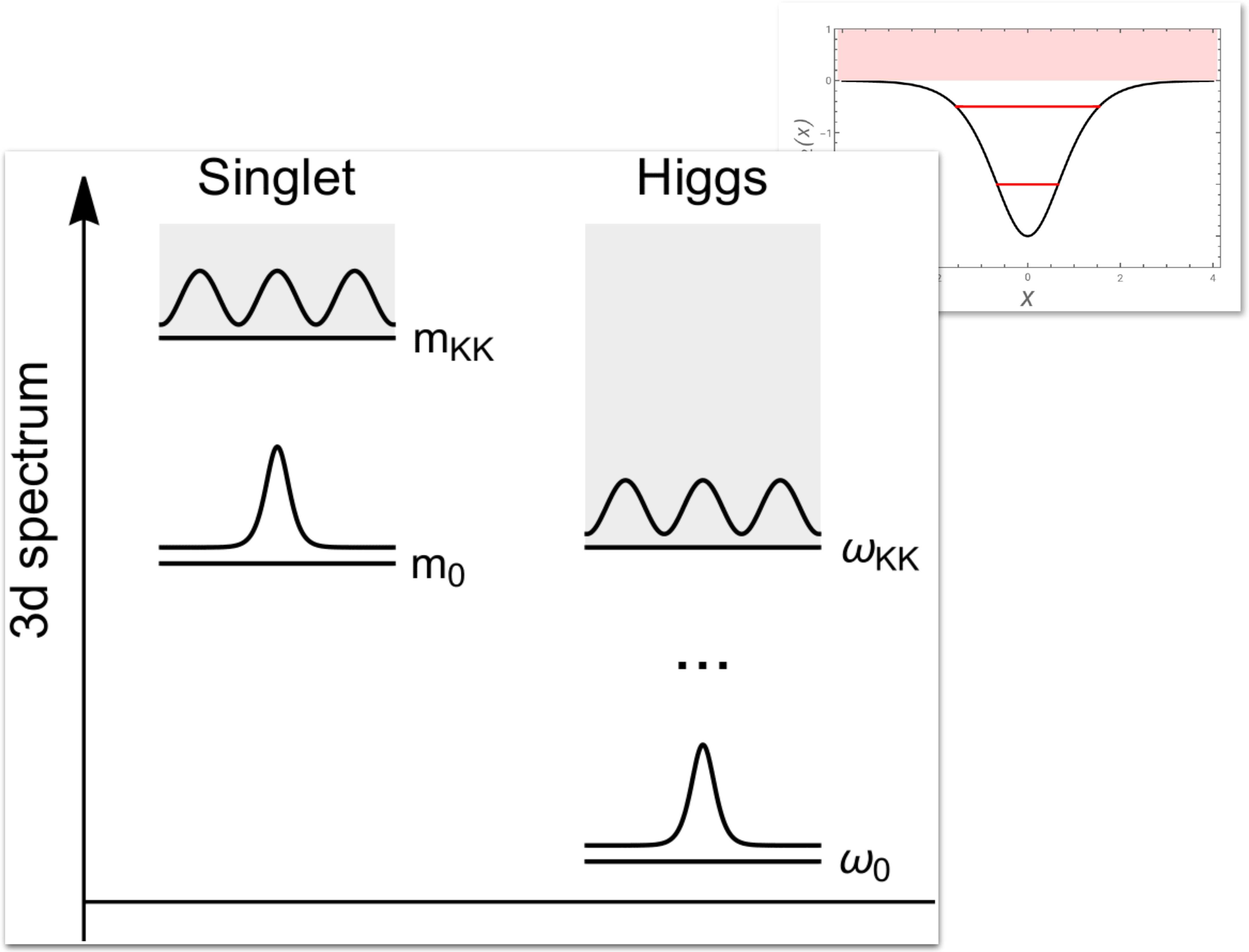

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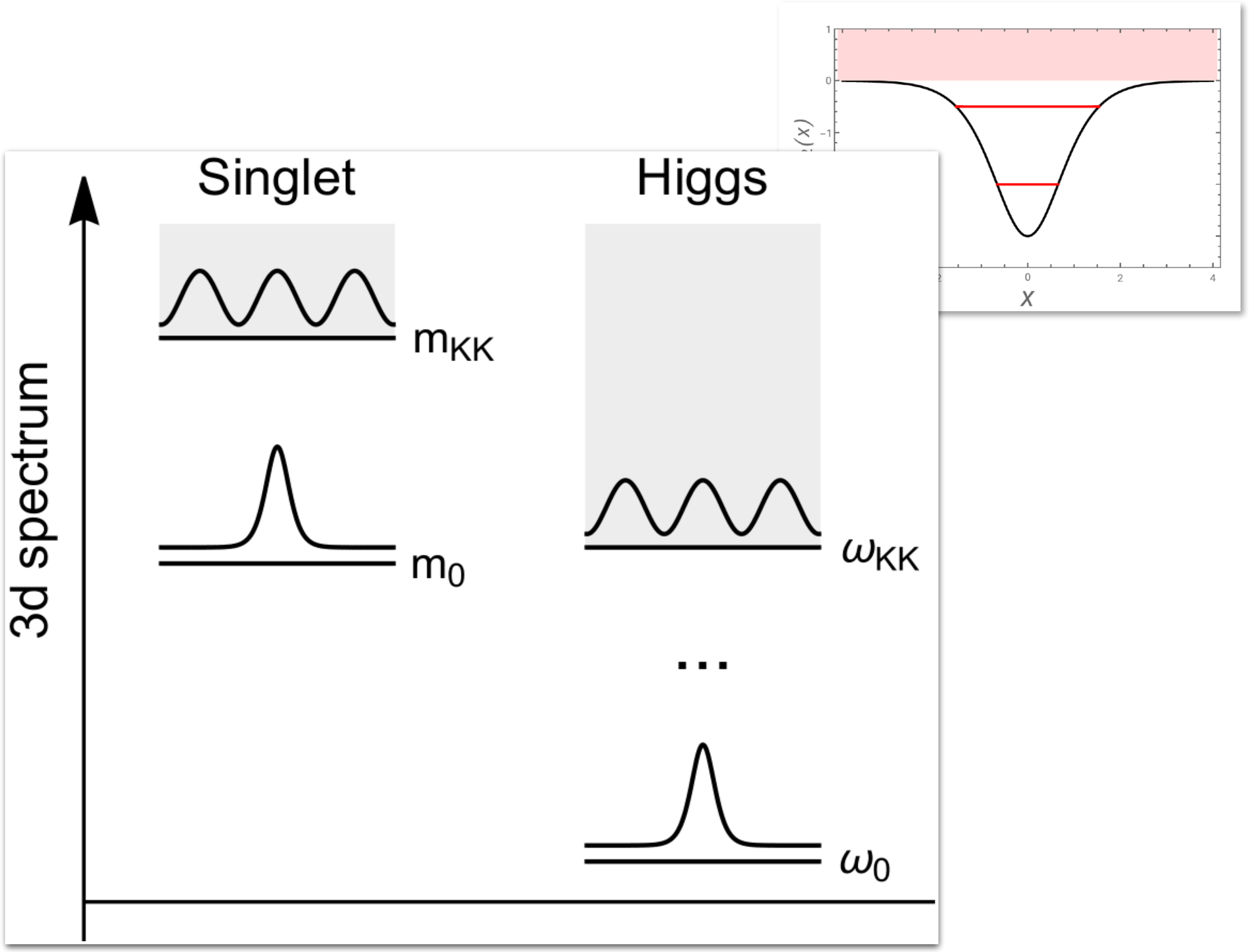
Seeded tunneling as homogeneous problem in lower dimension!

↓

Tunneling trajectory can be obtained with standard tools `CosmoTransitions`

- Eigenspectrum of excitations:



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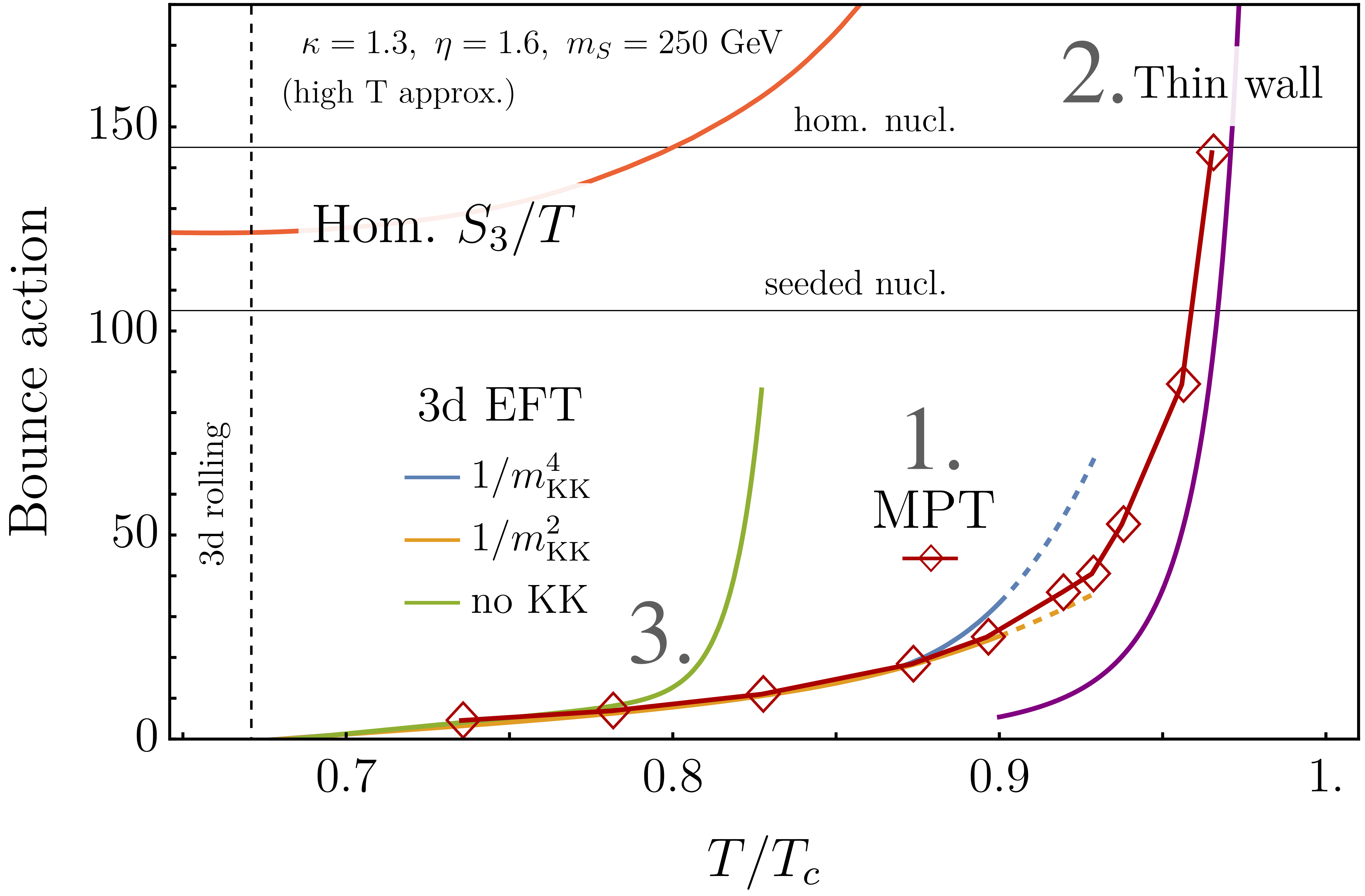
EWPT with a singlet

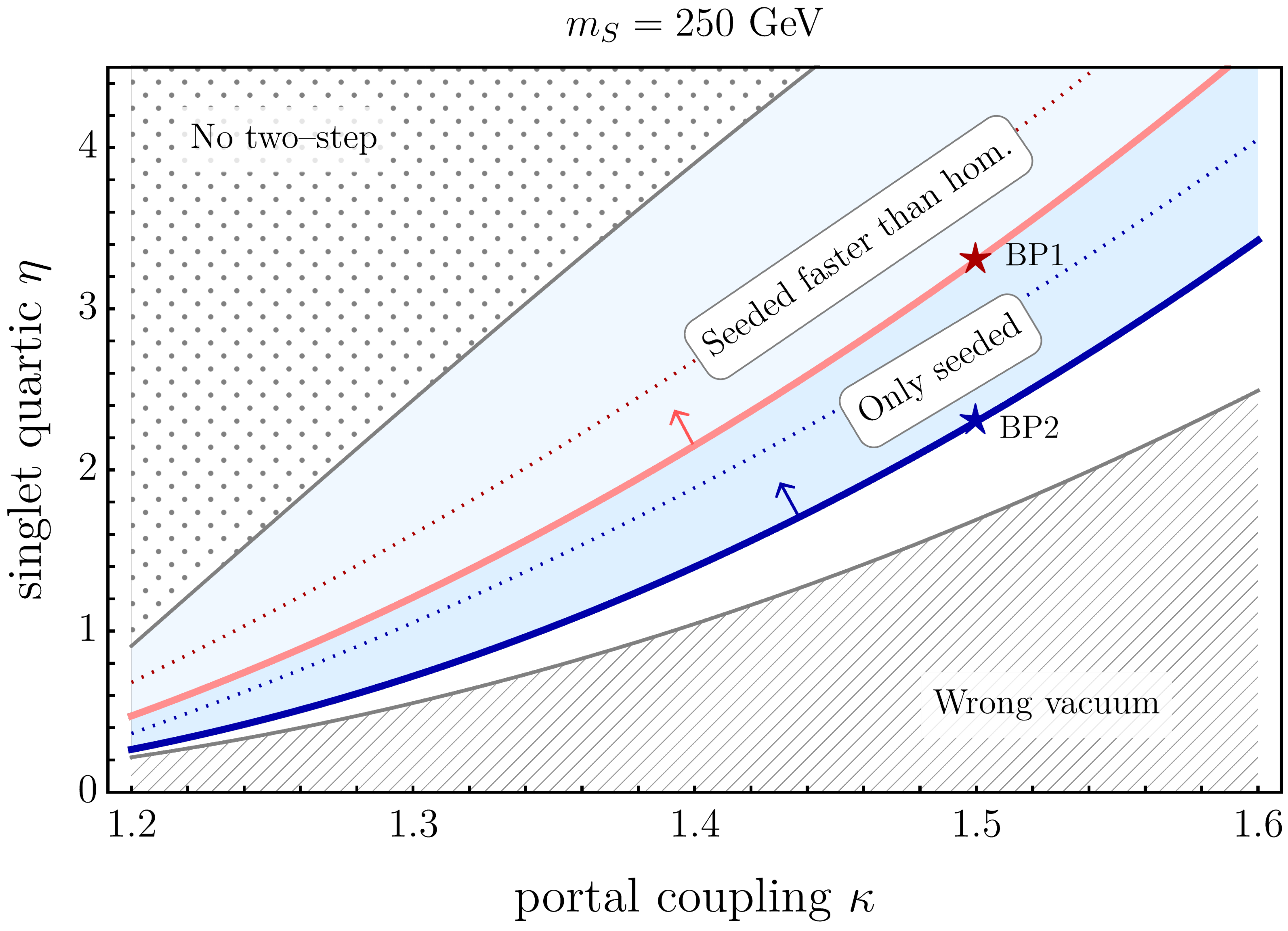
- Comparison of the various methods:



Agrawal, SB, Mariotti, Nee
[2312.06749] JHEP

Pheno implications

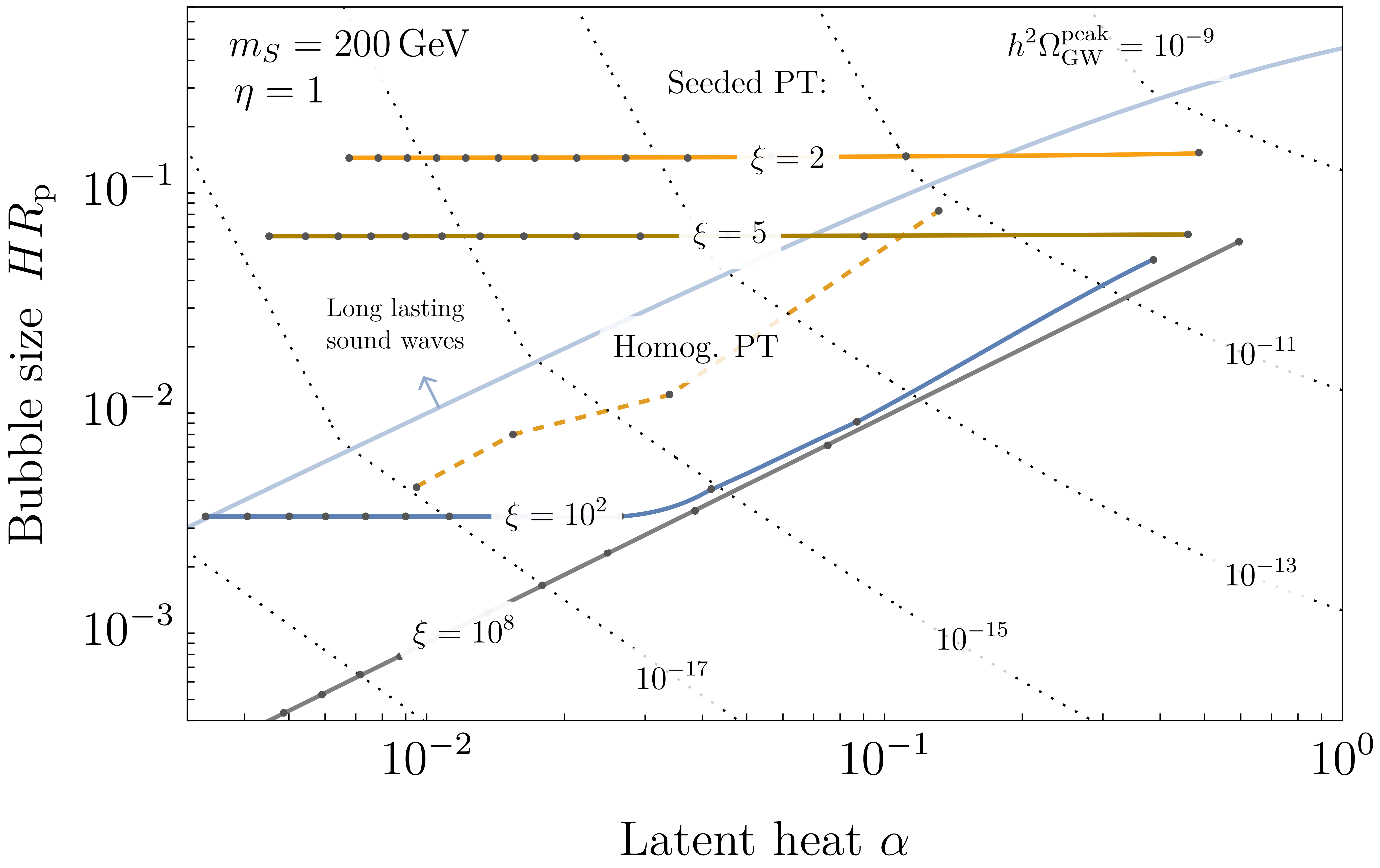
- Seeded transition is faster in all the two—step parameter space
- New parameter space becomes viable thanks to seeded tunneling



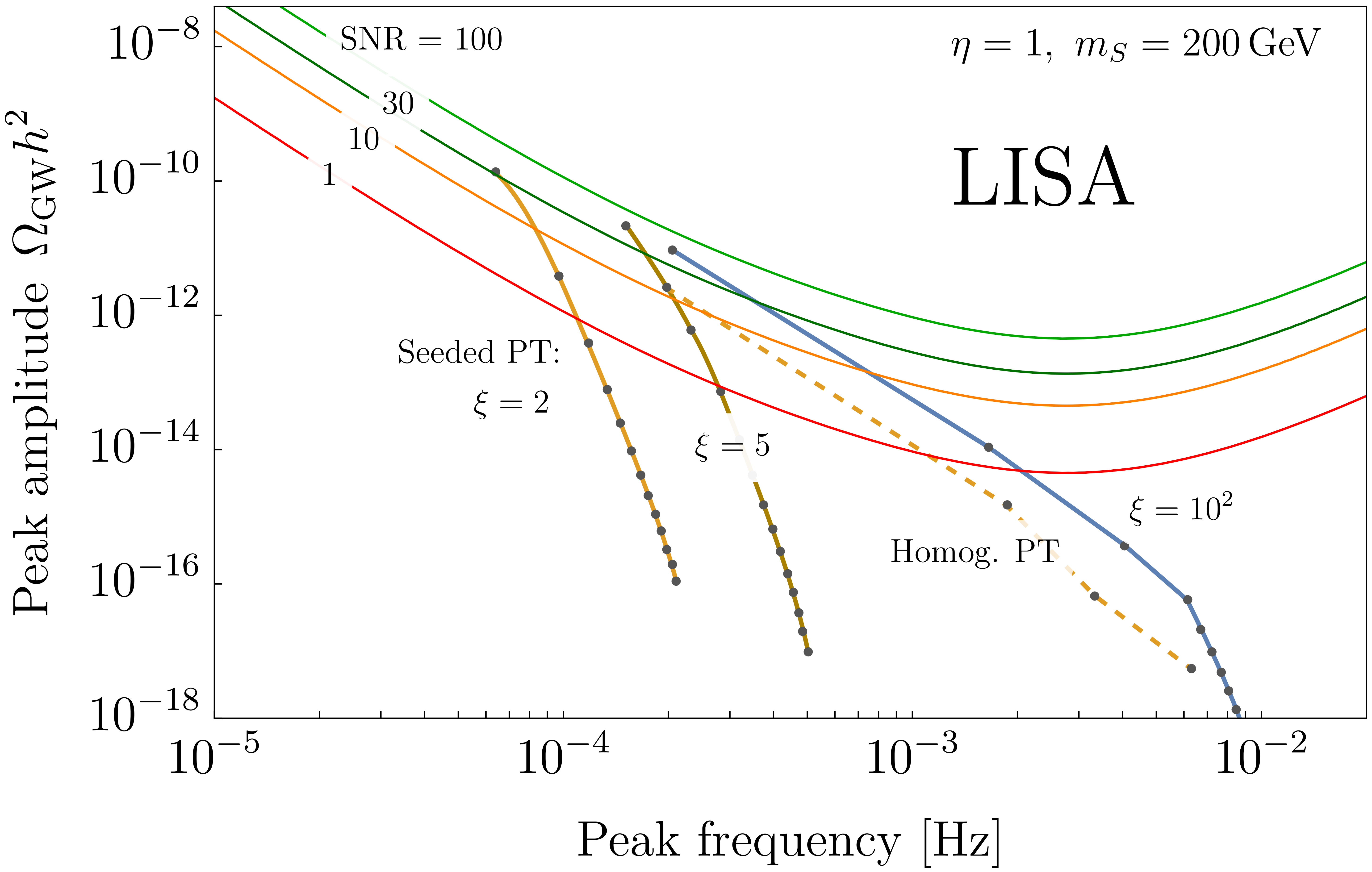
Pheno implications

- ***Homogenous PT***: bubble size and strength are strongly correlated
- ***Seeded PT***: pheno controlled by the # of defects per Hubble volume ξ

1. Sparse networks $\xi \sim 1$: size determined by average distance between defects
2. Dense networks $\xi \gg 1$: bubble size determined by nucleation rate



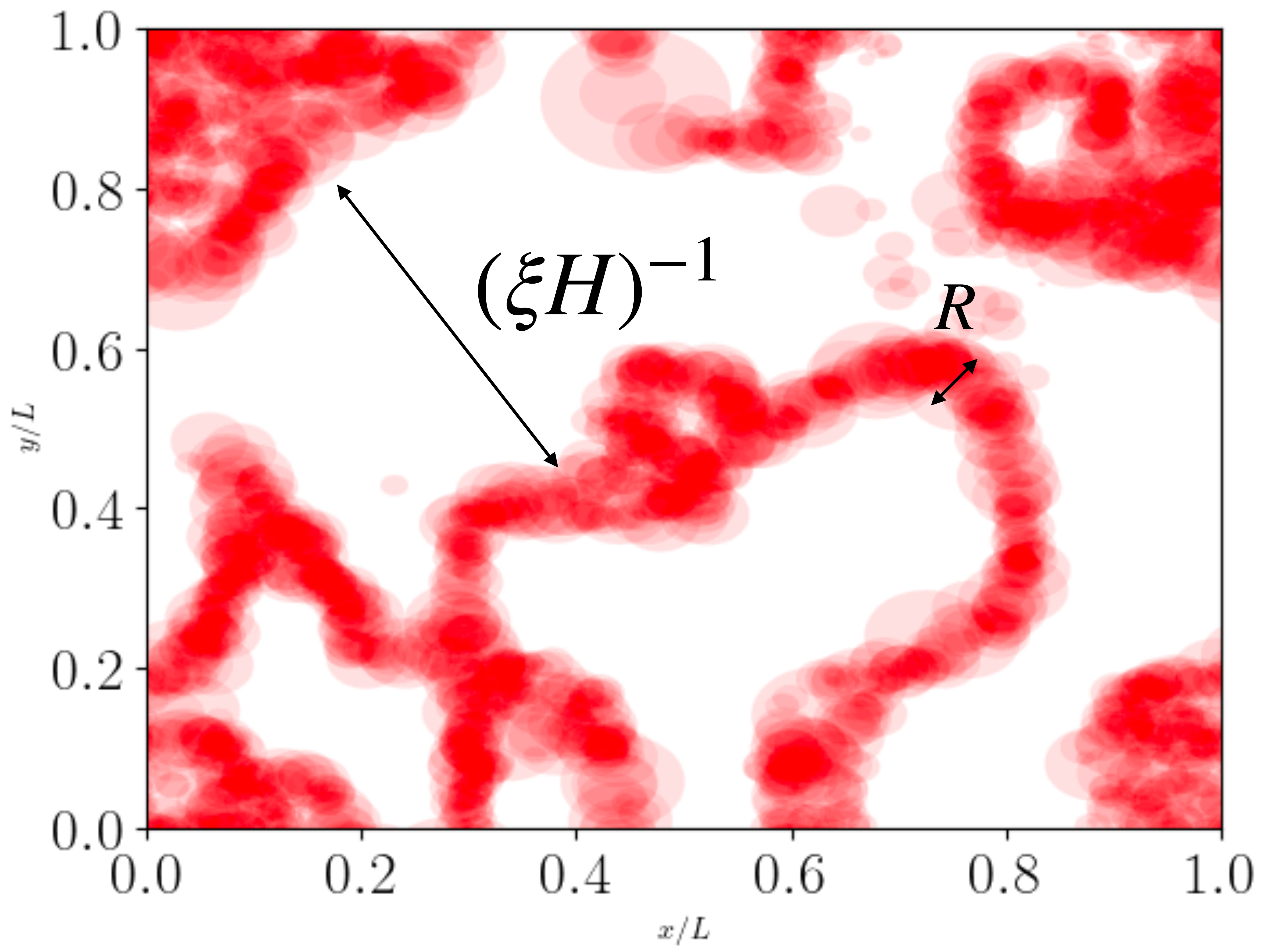
EWPT with a singlet



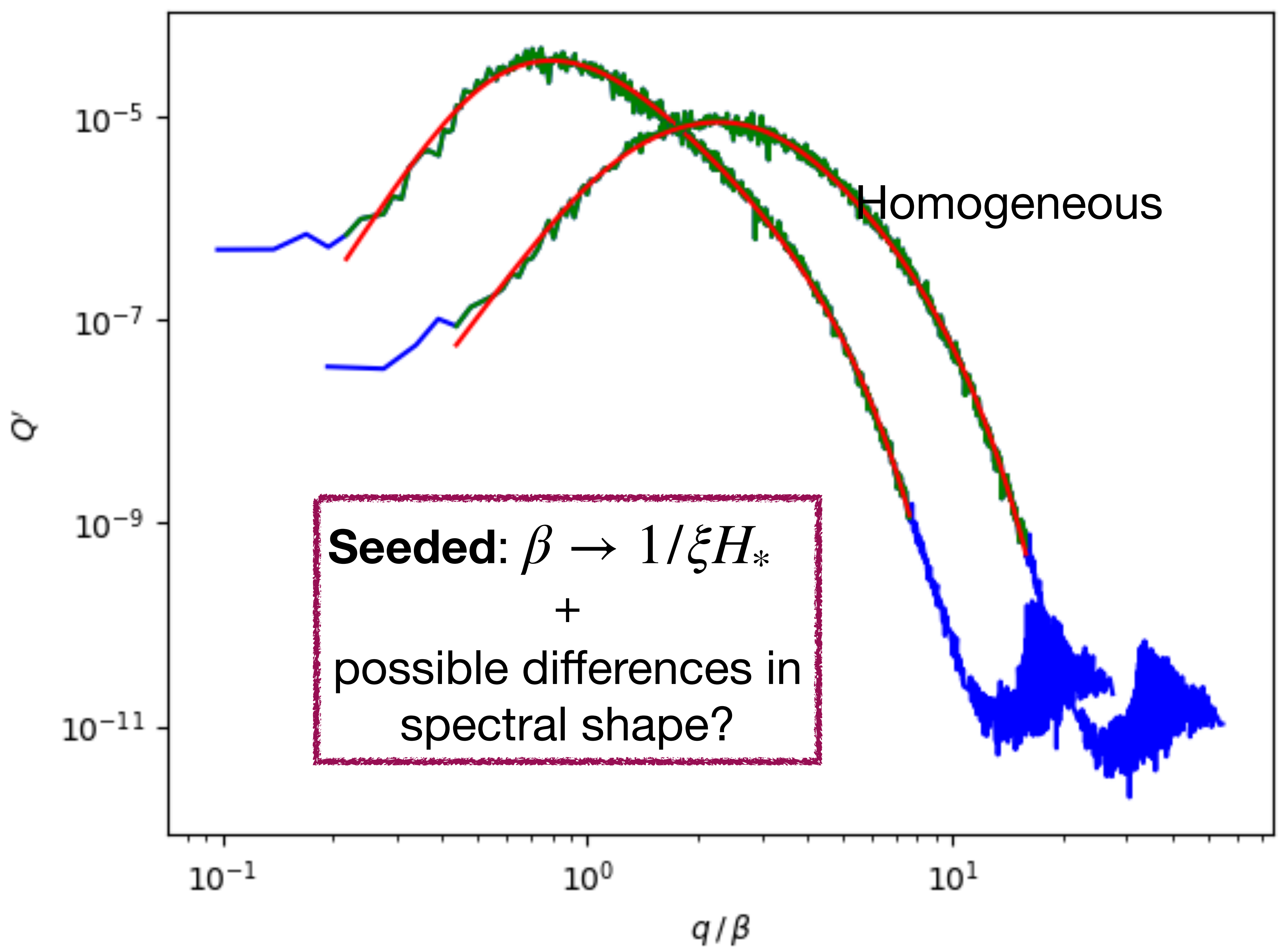
Higgsless-hydro simulation

Fig. from SB, Jinno, Konstandin, Rubira, Stomberg [2302.06952] JCAP

- Domain wall network mimicked by Ising model



- Spectrum shifted to IR with enhanced amplitude



Seeded: $\beta \to 1/\xi H_*$
+
possible differences in spectral shape?

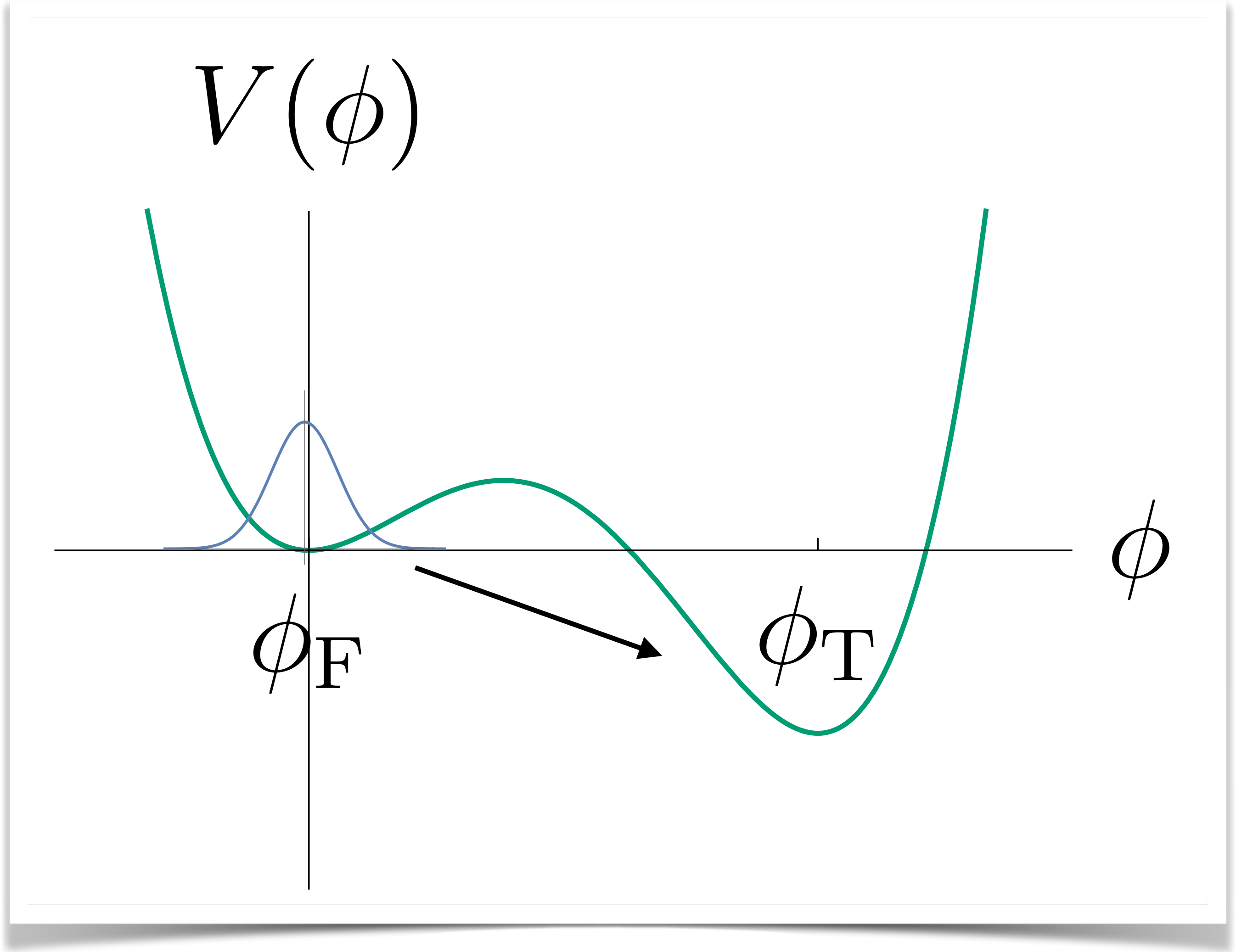
Tunneling from real-time simulations

- Langevin approach to mimic the thermal bath via fluctuation-dissipation:

$$(\partial_t^2 - \nabla^2)\phi(\vec{x},t) + \frac{\partial V(\phi)}{\partial \phi} + \eta\dot{\phi}(\vec{x},t) = \xi(\vec{x},t)$$

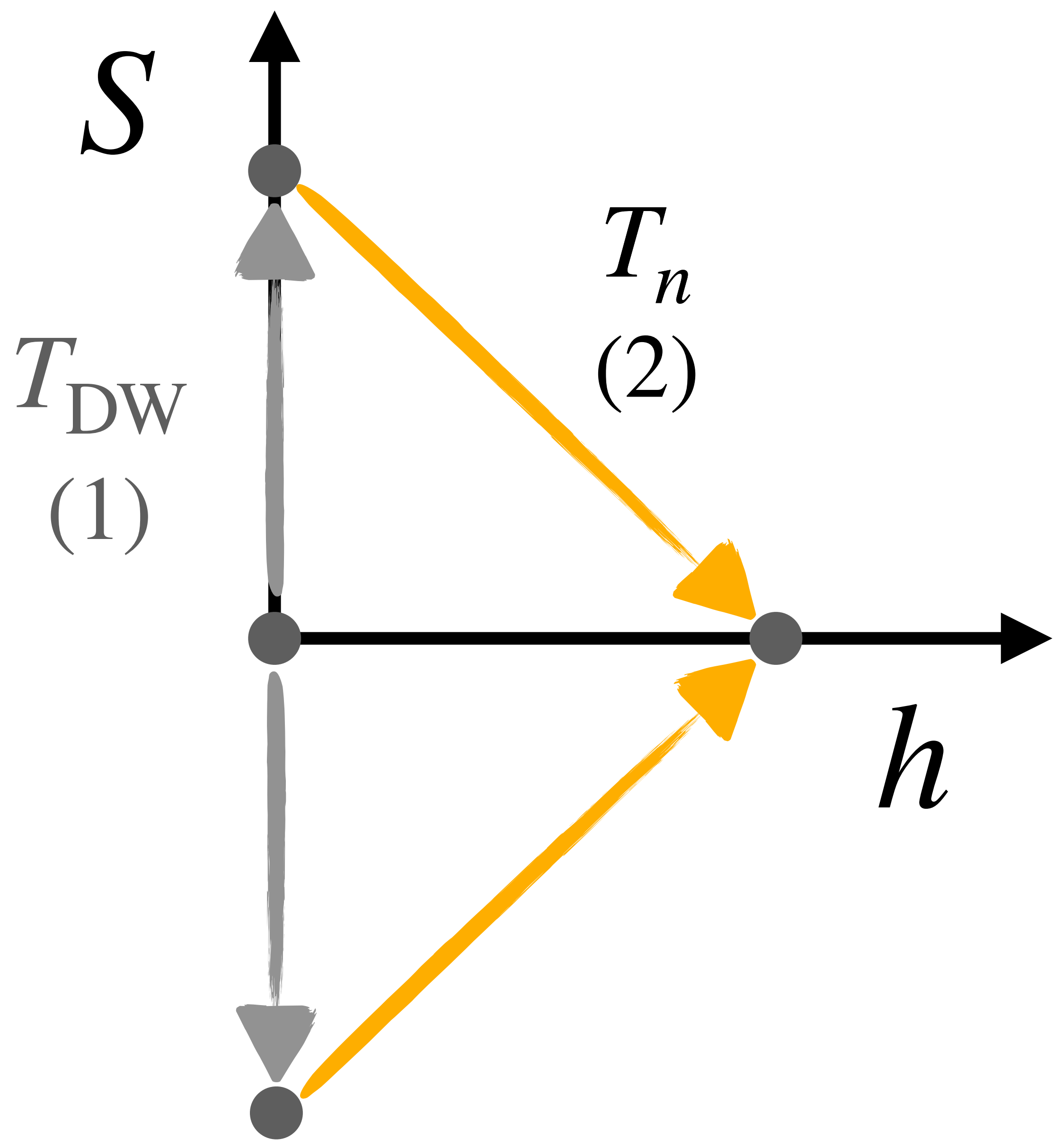
$$\langle \xi(\vec{x},t)\xi(\vec{x}',t')\rangle = \Omega\delta^3(\vec{x}-\vec{x}')\delta(t-t') \qquad \Omega = 2\eta k_B T$$

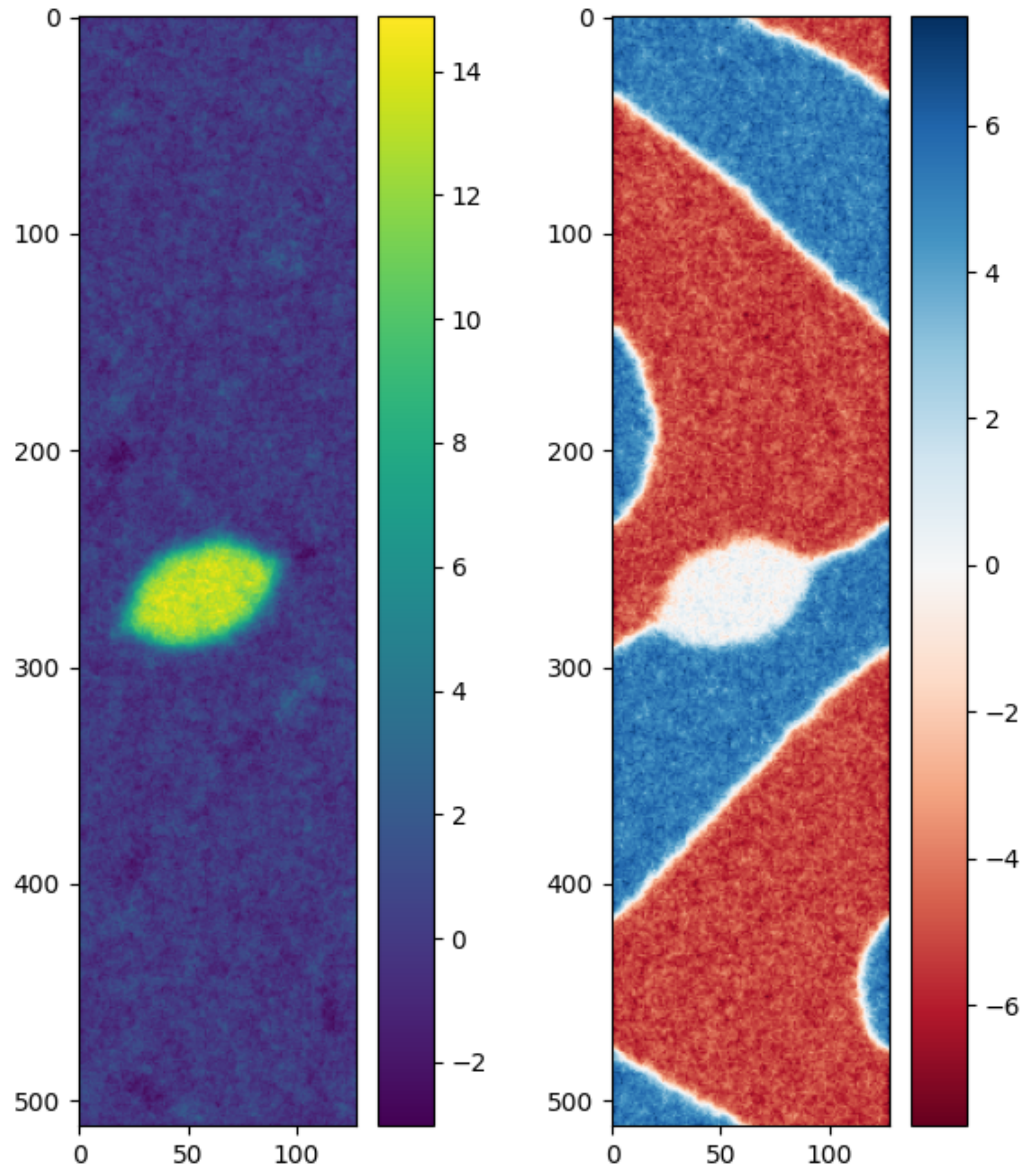
Initialize the system in the false vacuum, evolve in time, and wait for nucleation



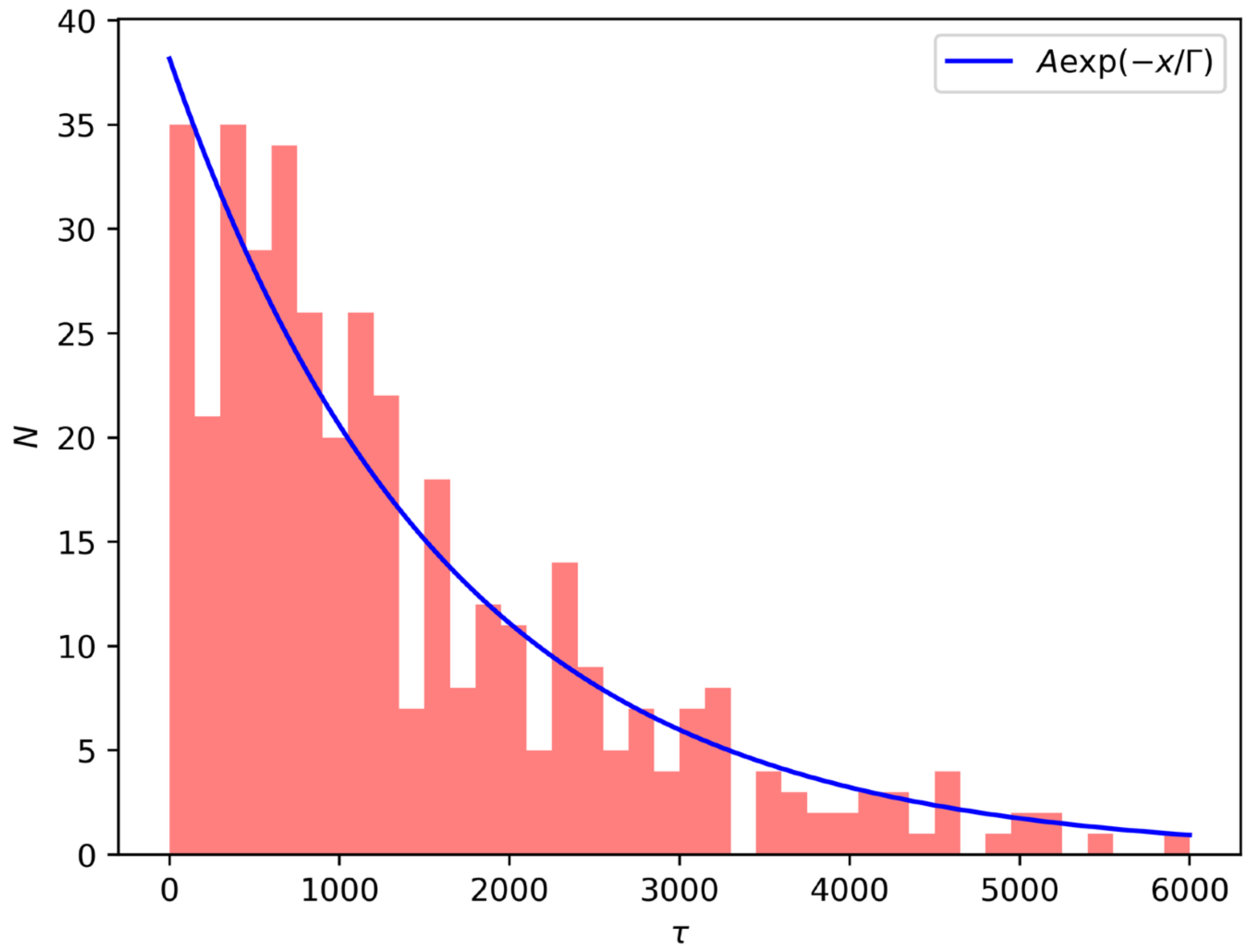
Tunneling from real-time simulations

- Applying this method for the first time to seeded nucleation in $d = 2 + 1$:



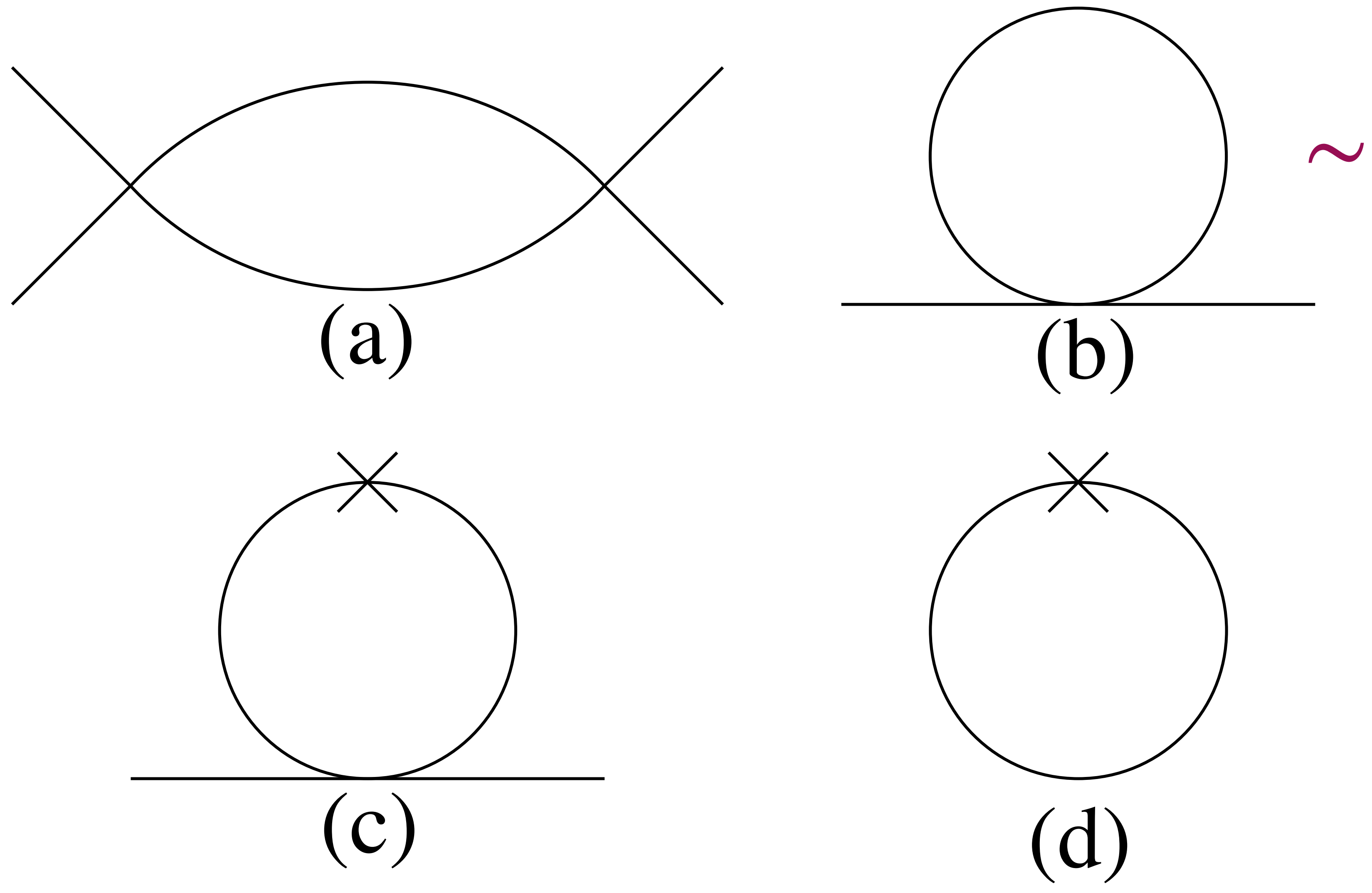


SB, Ekstedt, Hällfors, Rummukainen, in prep.



Tunneling from real-time simulations

- Extracting the continuum limit with lattice counter-terms (1-loop):



$\sim \log a/\mu$

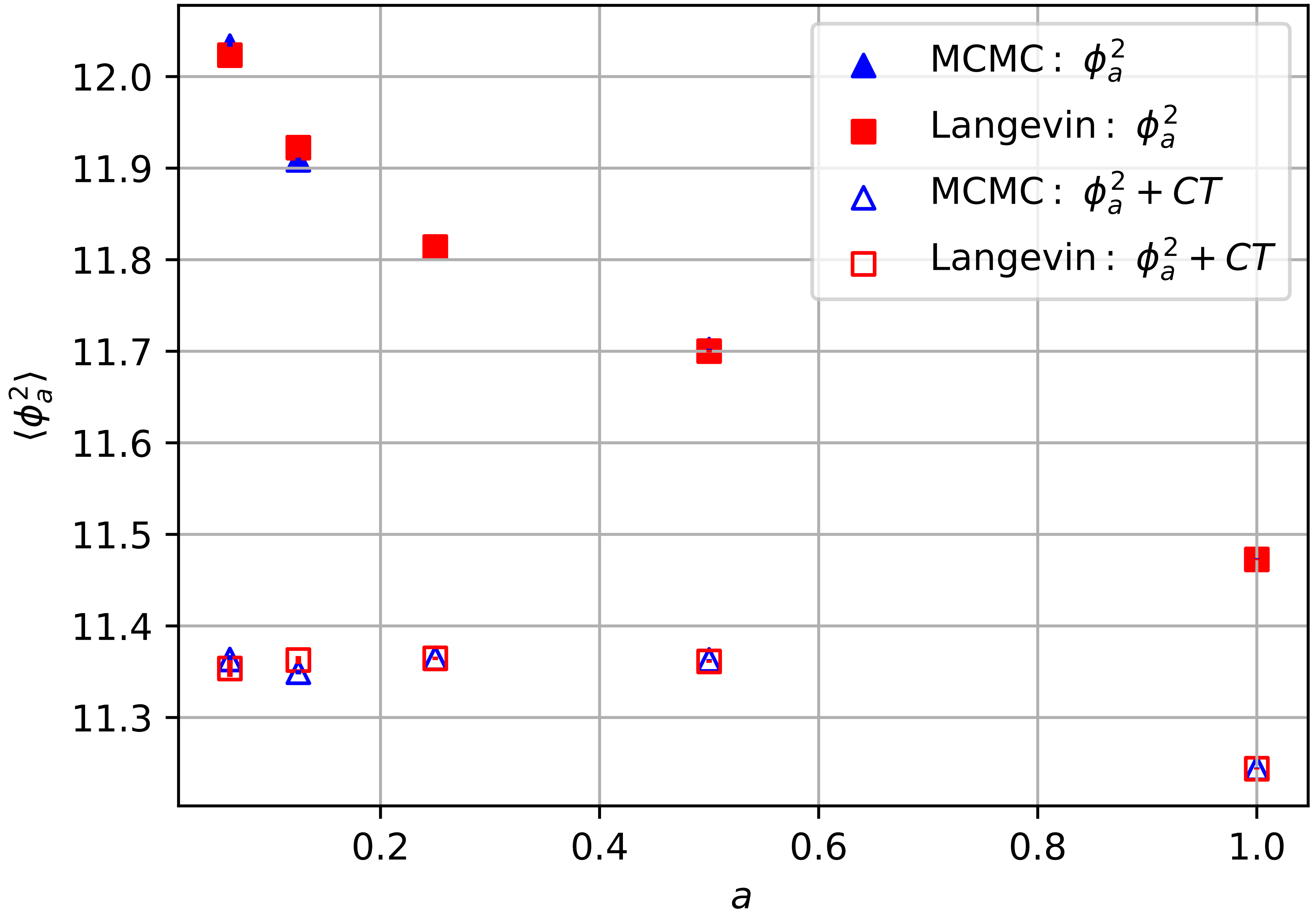


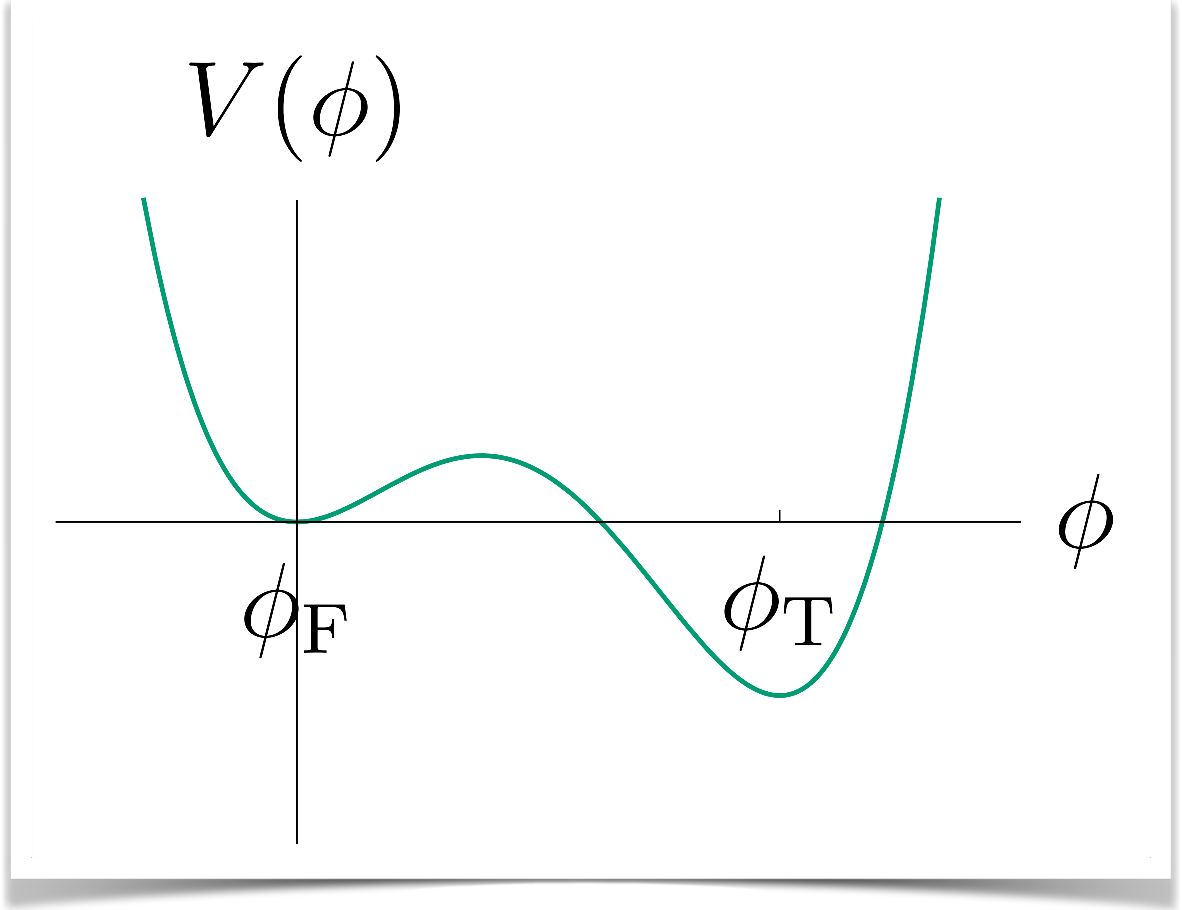
Fig. from Moore, Rummukainen, Tranberg [hep-lat/0103036], JHEP

Theory prediction: homogenous

- Thermal nucleation rate per unit volume in $d+1$:

$$A_{\mathrm{stat}} = \left(\frac{S[\phi_{\mathrm{b}}]}{2\pi}\right)^{d/2} \left|\frac{\det'\left(-\nabla^2 + V''(\phi_{\mathrm{b}})\right)}{\det\left(-\nabla^2 + V''(\phi_{\mathrm{F}})\right)}\right|^{-1/2} e^{-(S[\phi_{\mathrm{b}}]-S[\phi_{\mathrm{F}}])}$$

$$A_{\mathrm{dyn}} = \frac{1}{2\pi}\left(\sqrt{|\lambda_-| + \frac{\eta^2}{4}} - \frac{\eta}{2}\right)$$



$$\boxed{\Gamma_T = A_{\mathrm{dyn}} \times A_{\mathrm{stat}}}$$

See e.g. Ekstedt, Gould, Hirvonen [2308.15652], JHEP

Theory prediction: homogenous

- In general the determinant involves a product of spherical harmonics:

$$\frac{\det\left(-\nabla^2 + W(r)\right)}{\det\left(-\nabla^2 + W(\infty)\right)} = \Pi_{l=0}^{\infty}\left[\frac{\det\left(-\nabla_l^2 + W(r)\right)}{\det\left(-\nabla_l^2 + W(\infty)\right)}\right]^{\deg(d;l)}$$

$$\nabla_l^2 = \partial^2 + \frac{d-1}{r}\partial - \frac{l(l+d-2)}{r^2}$$

- Need to account for zero modes and renormalization

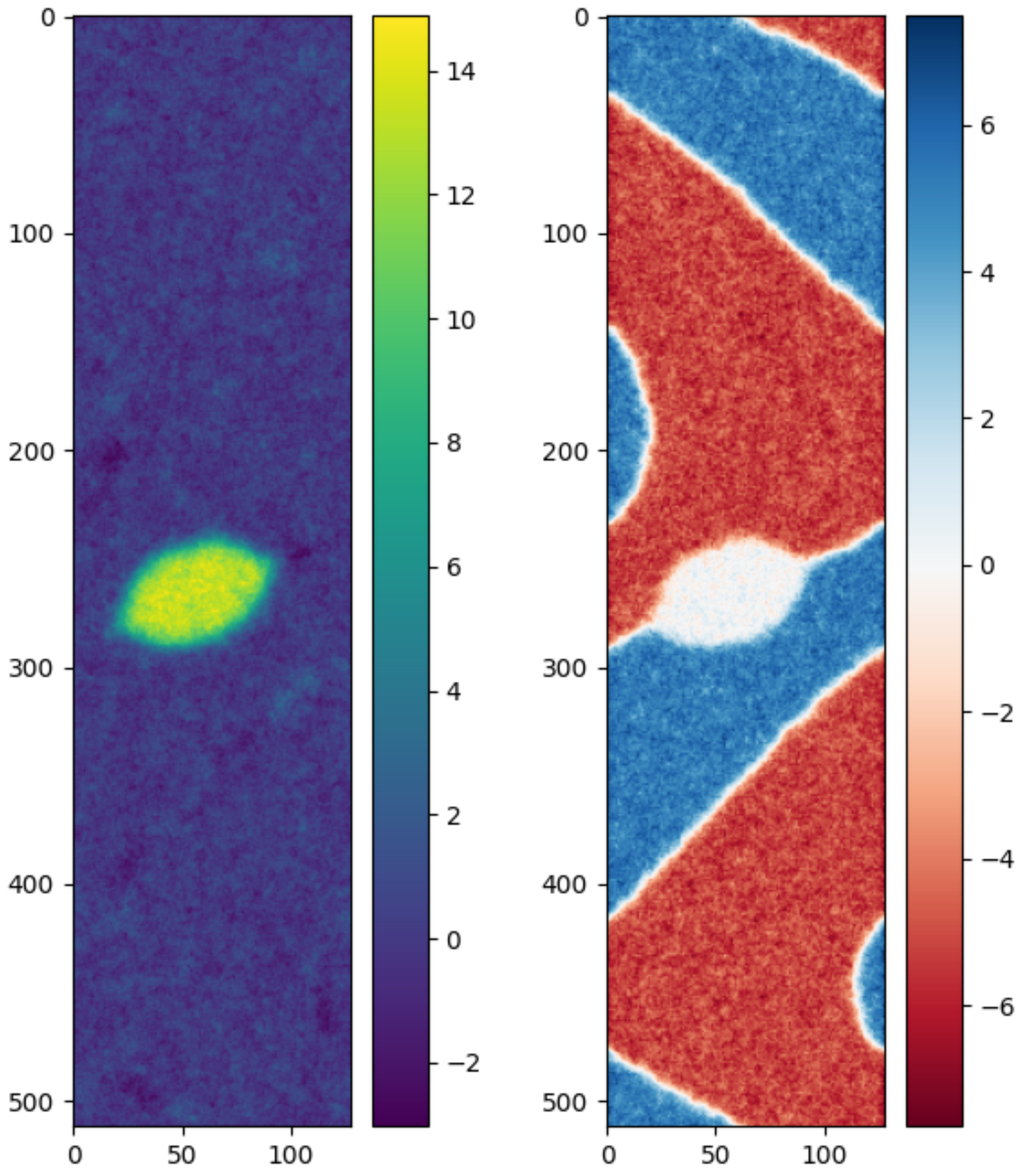
See e.g. Ekstedt, Gould, Hirvonen [2308.15652], JHEP

Theory prediction: seeded

- Toy model of thermal seeded tunneling in $d = 2 + 1$

- Nucleation rate per unit wall length estimated as:

$$\Gamma = \sigma_{\rm DW} e^{-B/T}$$



Theory prediction: seeded

- Toy model of thermal seeded tunneling in $d = 2 + 1$

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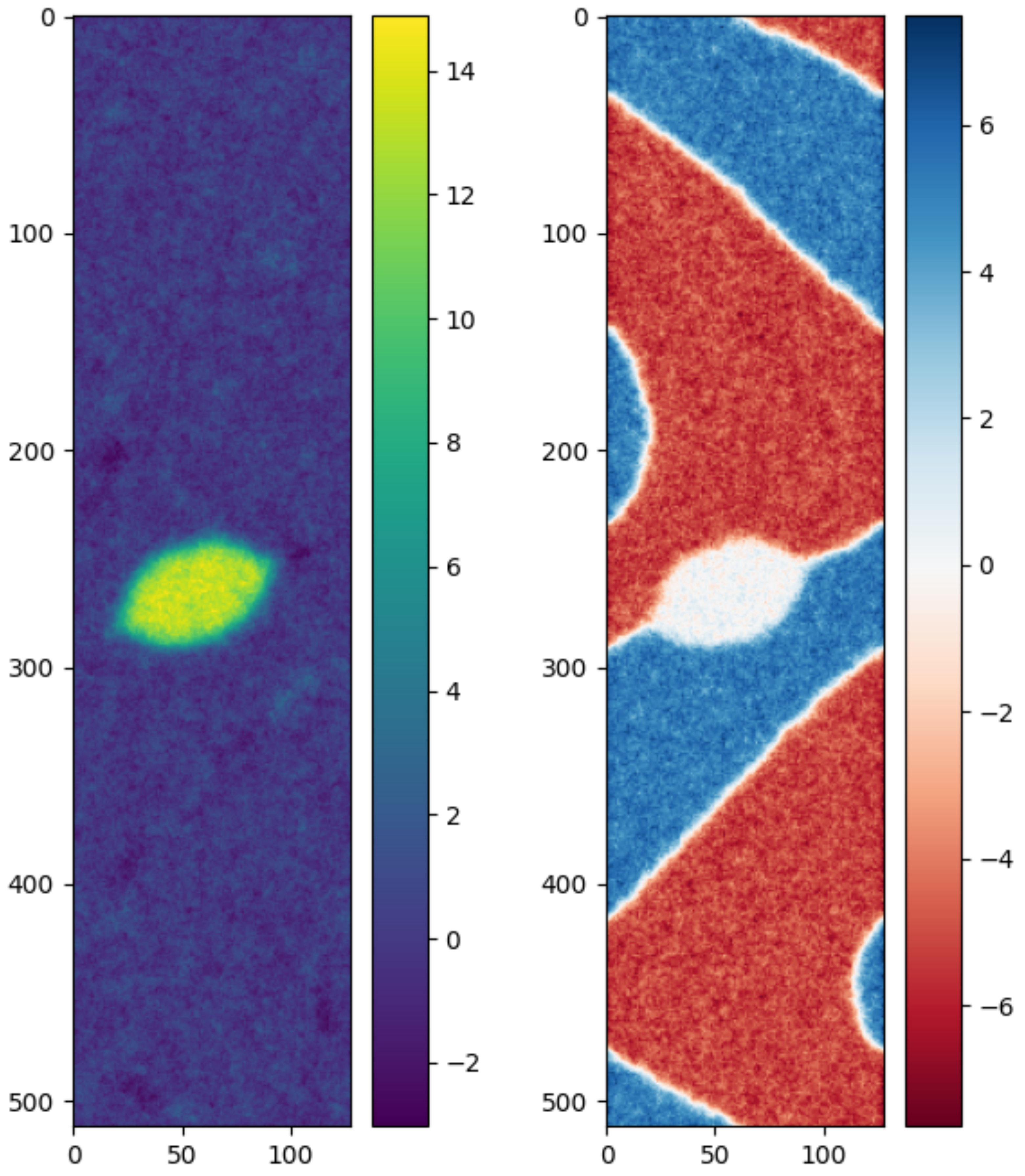


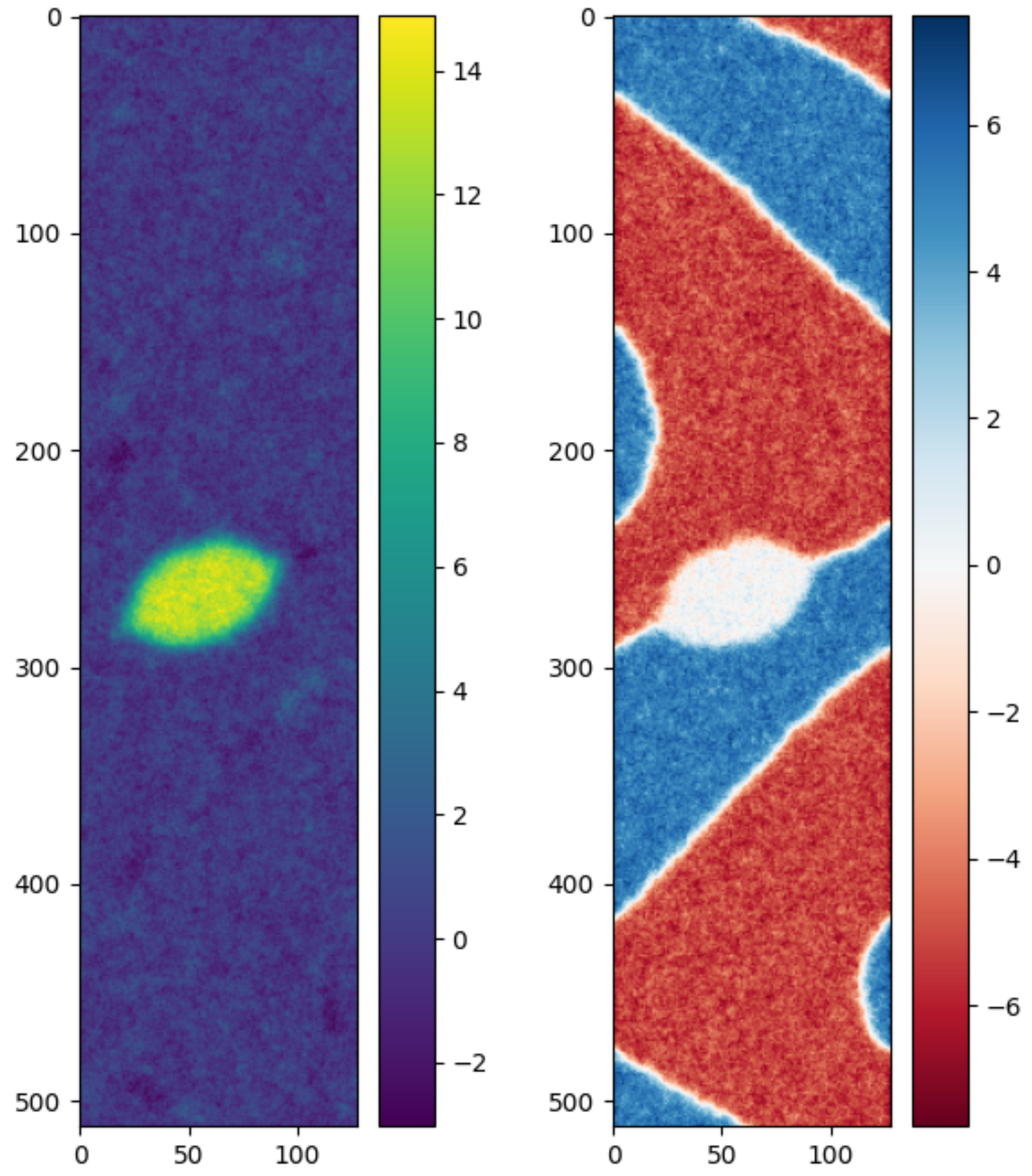
Theory prediction: seeded

- Toy model of thermal seeded tunneling in $d = 2 + 1$

- Nucleation rate per unit wall length estimated as:

$$\Gamma = ?\, e^{-B/T}$$

- How to properly calculate the rate taking into account the fluctuation determinant?



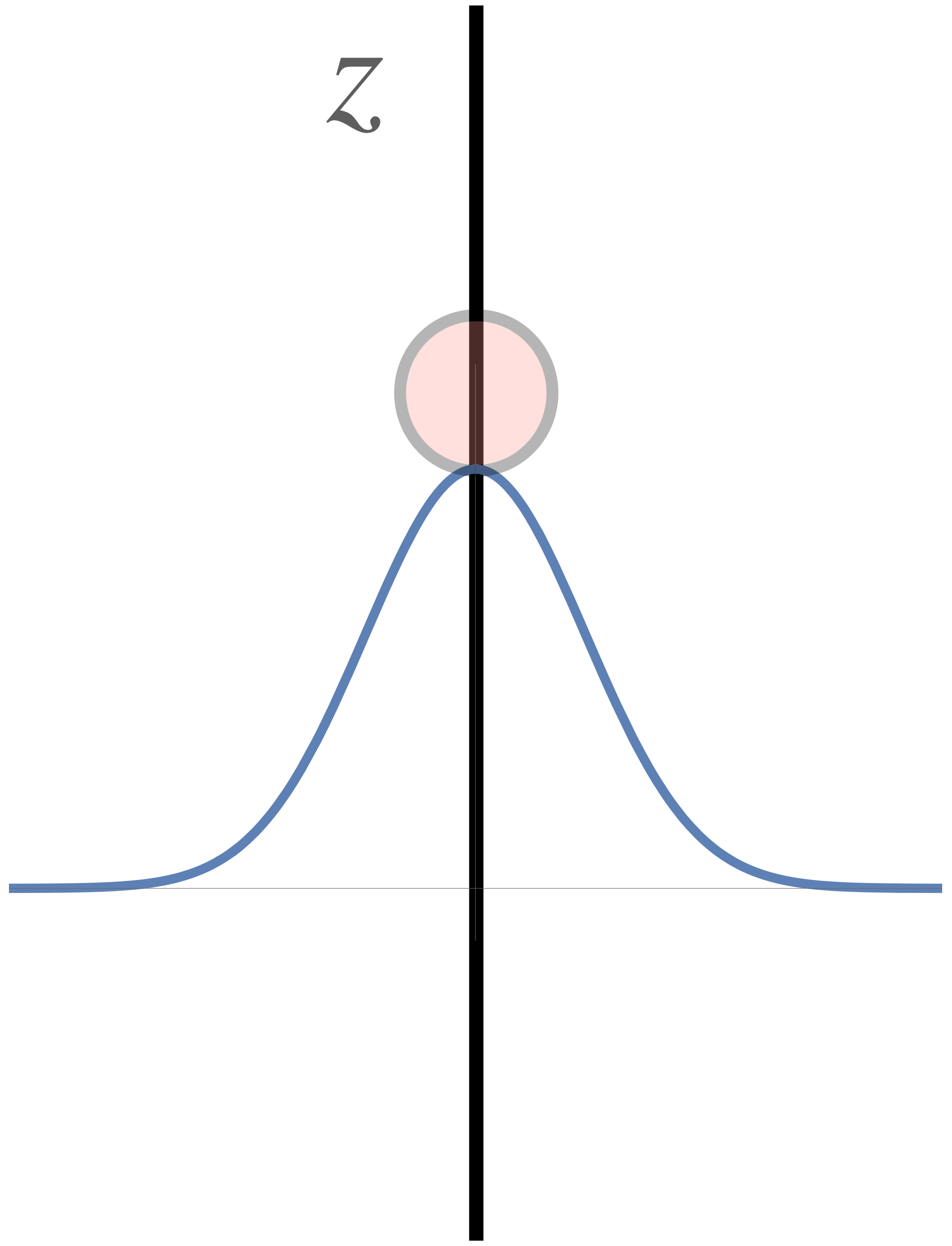
Theory prediction: seeded

- Consider the $d = 1 + 1$ dimensionally-reduced theory on the domain wall

$$S_{1+1}[\phi_0] = \int dz dt \left\{ \frac{1}{2}(\partial_\mu \phi_0)^2 - \tilde{V}(\phi_0) \right\}$$

$$\tilde{V}(\phi_0) = \frac{1}{2}\omega^2 \phi_0^2 - \frac{1}{3!}c_3 \phi_0^3 + \frac{1}{4!}\phi_0^4$$

The 1+1 theory describes homogenous nucleation: we can apply standard results!



$$h^\mu(x) = h(r)\phi_0(z,t)$$

Theory prediction: seeded

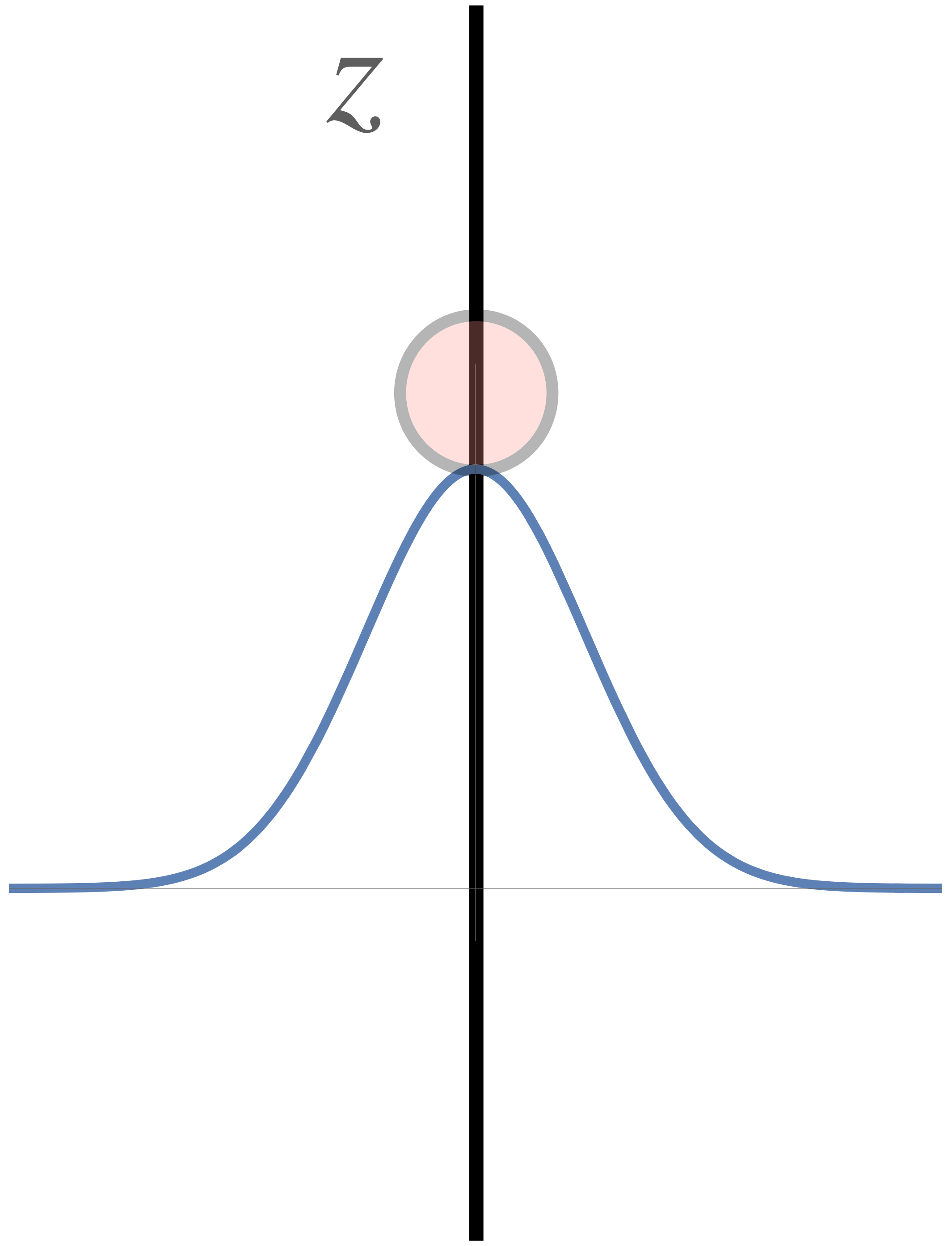
- Determinant around the bounce solution as*:

$$\left|\frac{\det'}{\det}\right|^{-1/2} = \left((2\pi)^{d/2-1}\phi_\infty \frac{1}{d}\frac{\partial V}{\partial \phi}(\phi_b(0))\right)^{d/2} \quad (d=1)$$

$$\phi \sim \phi_F + \phi_\infty K(d/2-1, m_F r)(m_F/r)^{d/2-1}$$

$$\downarrow$$

$$-u'' + m_F^2 u = 0 \longrightarrow \frac{1}{m_F} e^{-m_F r}\sqrt{\pi/2}$$



$$h^\mu(x) = h(r)\phi_0(z,t)$$

*Obtained from $l=1$ in $d=3$ result in Ekstedt, Gould, Hirvonen [2308.15652], JHEP

Theory prediction: seeded

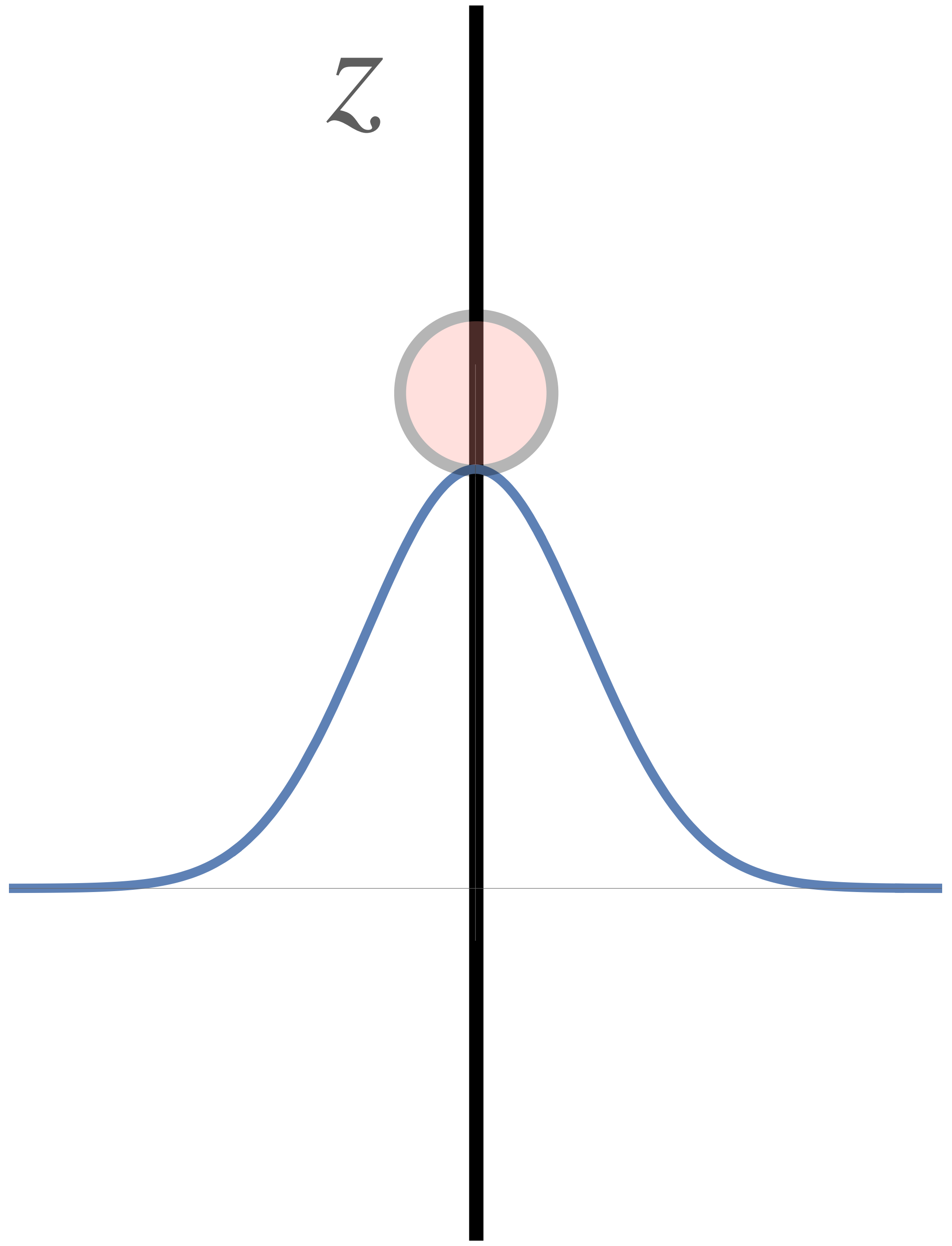
- Dynamical factor similarly obtained as

$$A_{\text{dyn}} = \frac{1}{2\pi}\left(\sqrt{|\lambda_-| + \frac{\eta^2}{4}} - \frac{\eta}{2}\right)$$

damping

Negative eigenvalue for the growth of critical bubble

$$\left(-\partial^2 - \cancel{\frac{d-1}{r}}\partial + V''(\phi_{\text{b}})\right) f(r) = \lambda_- f(r)$$



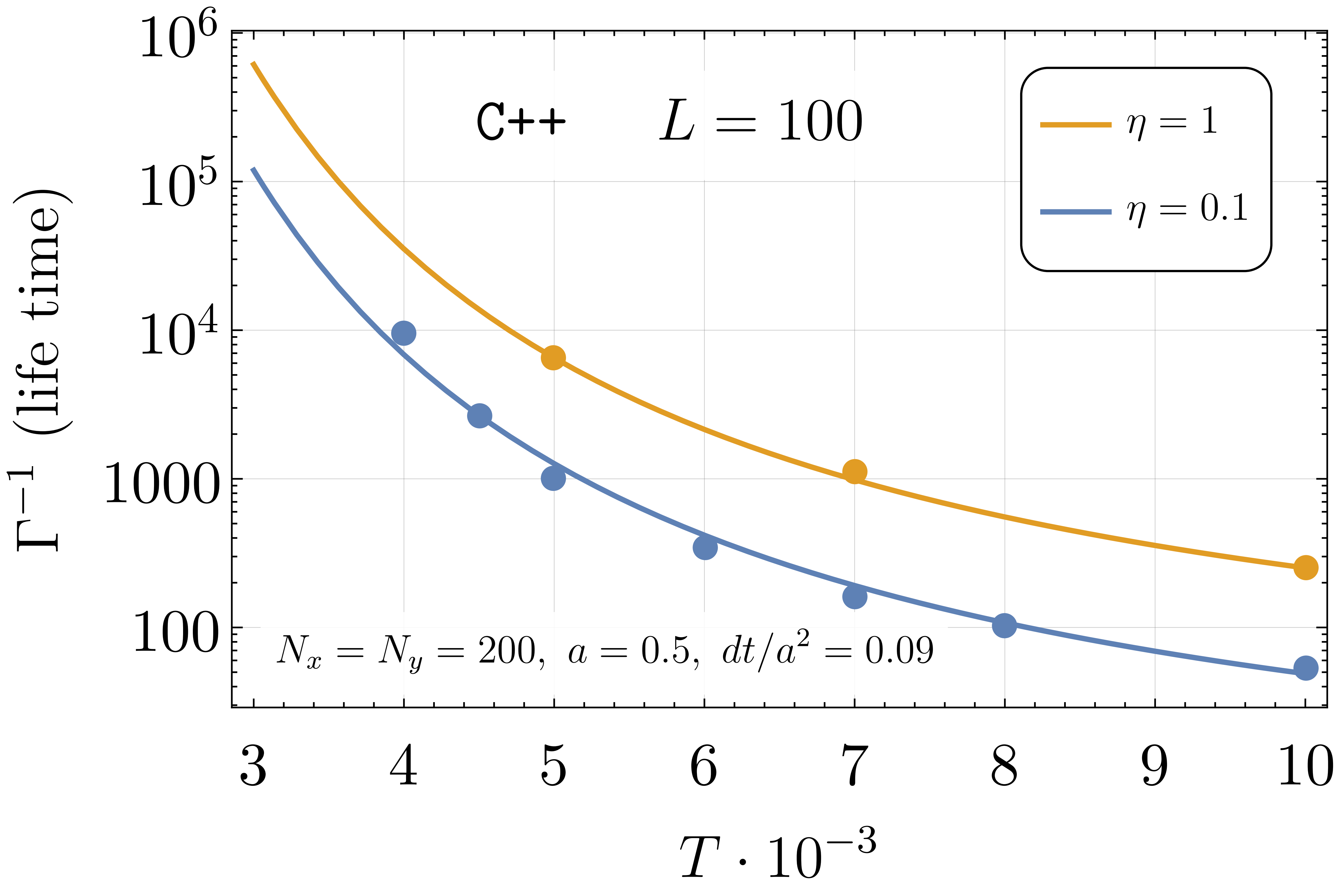
$$h^\mu(x) = h(r)\phi_0(z,t)$$

Real-time simulation vs theory

- Theory prediction:

$$\frac{\Gamma_T}{\sigma_{\rm DW} \cdot e^{-B/T}} \sim 10^{-2}$$

(preliminary)



What about other defects?

SB, Mariotti [2405.08060] SciPost Phys.

QCD axion strings

T

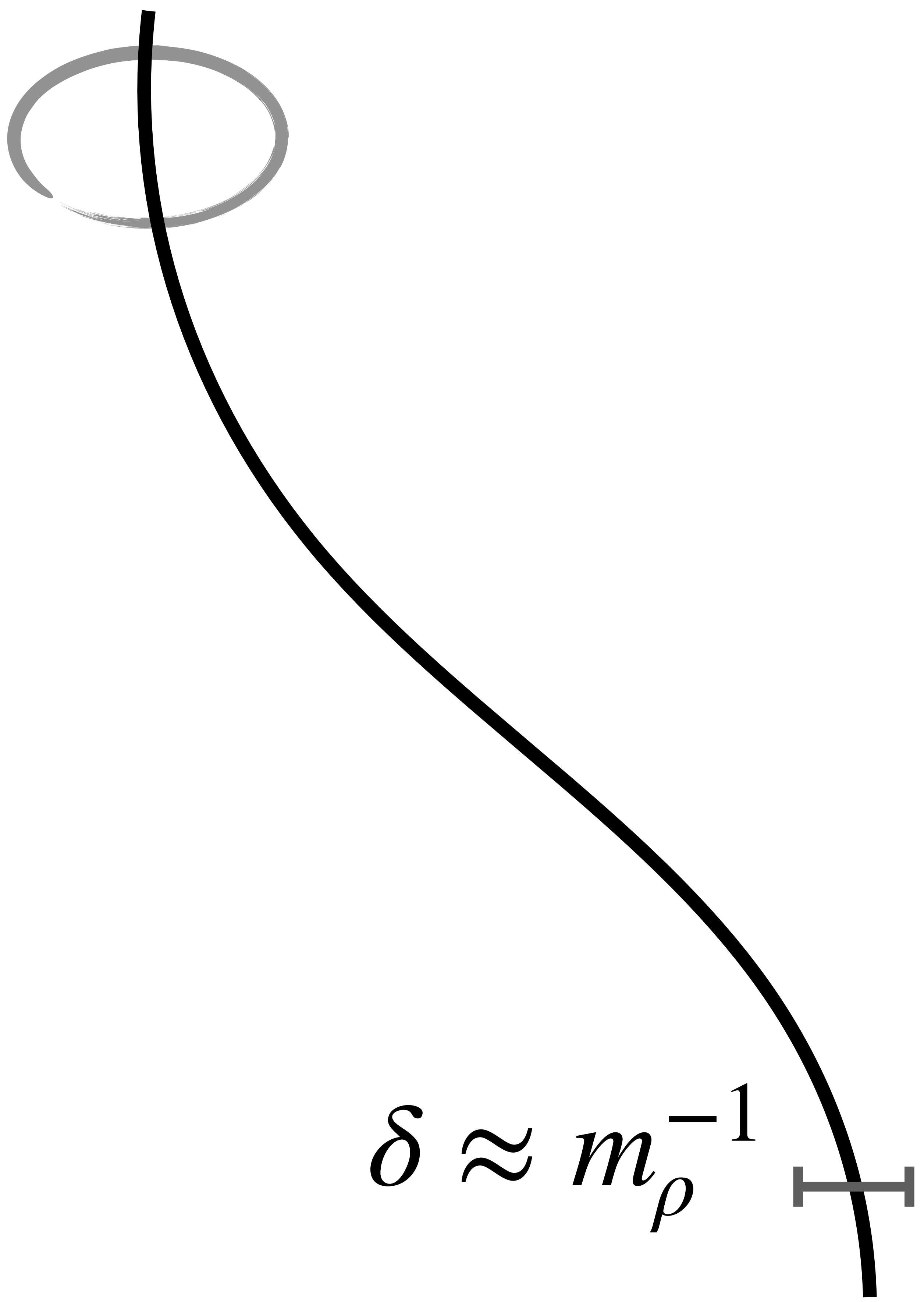
f_a

- Strings form at PQ phase transition

QCD

- Strings connected by axion domain walls
- String—wall network collapses

$\alpha(\theta) : 0 \to 2\pi$



$\delta \approx m_\rho^{-1}$

QCD axion strings

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???

QCD

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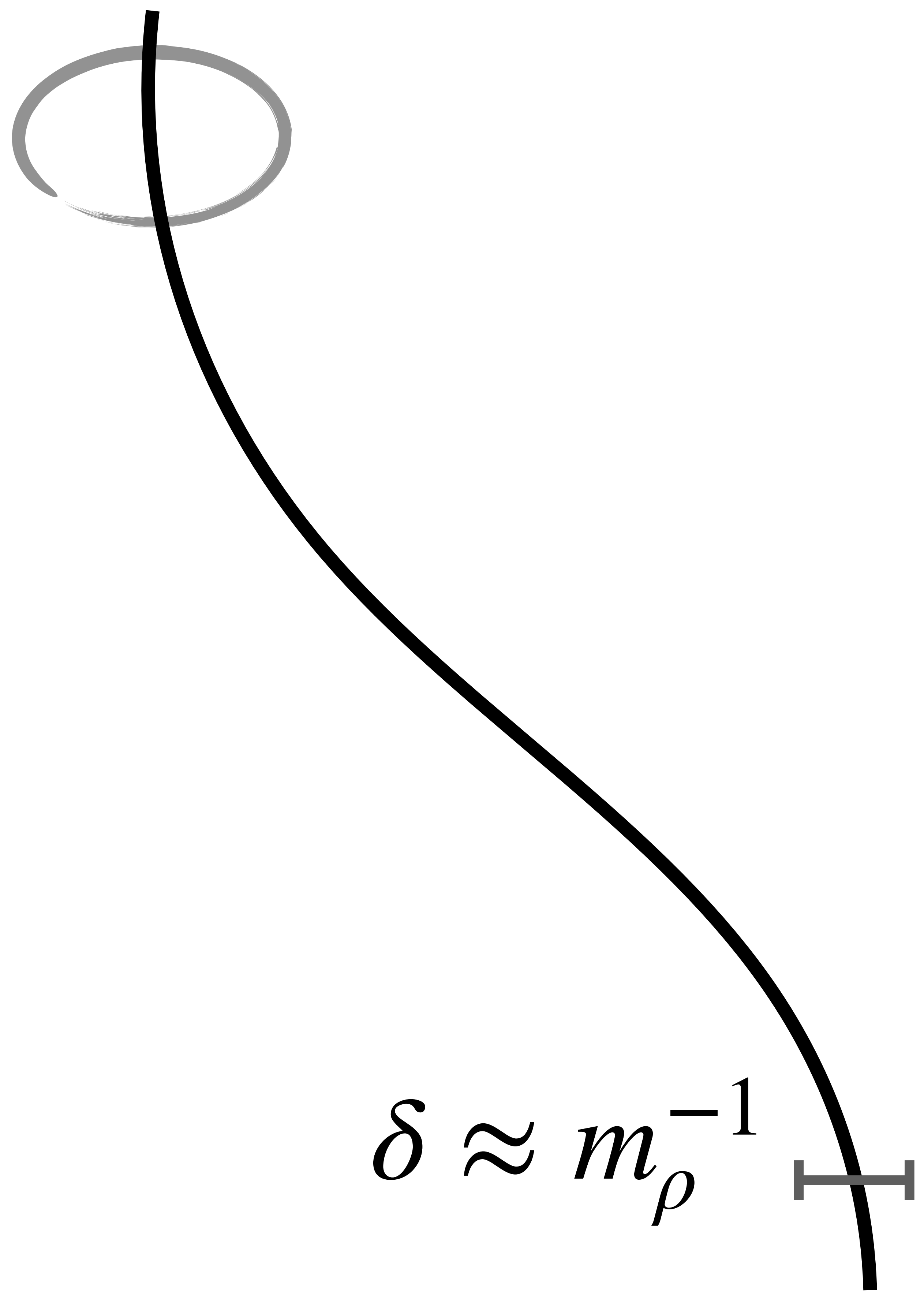
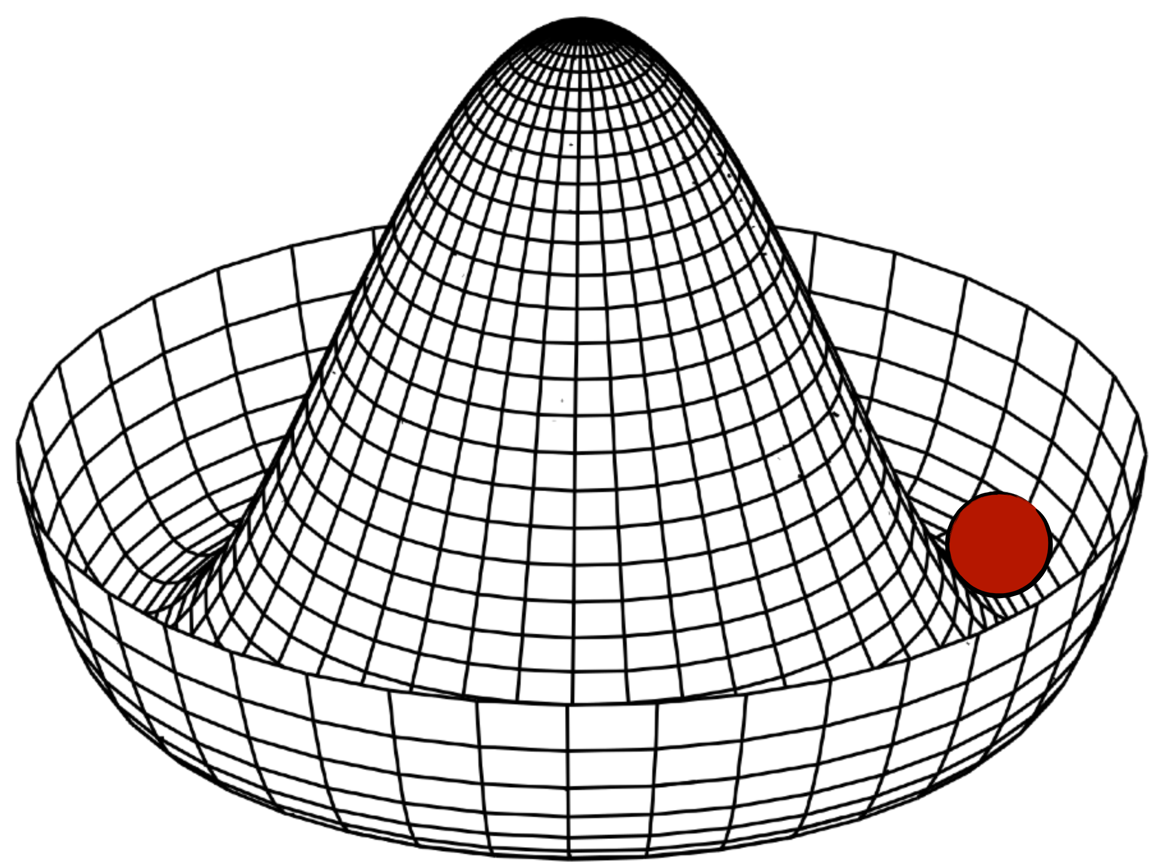


$\delta \approx m_\rho^{-1}$

QCD axion strings

- Potential for PQ field

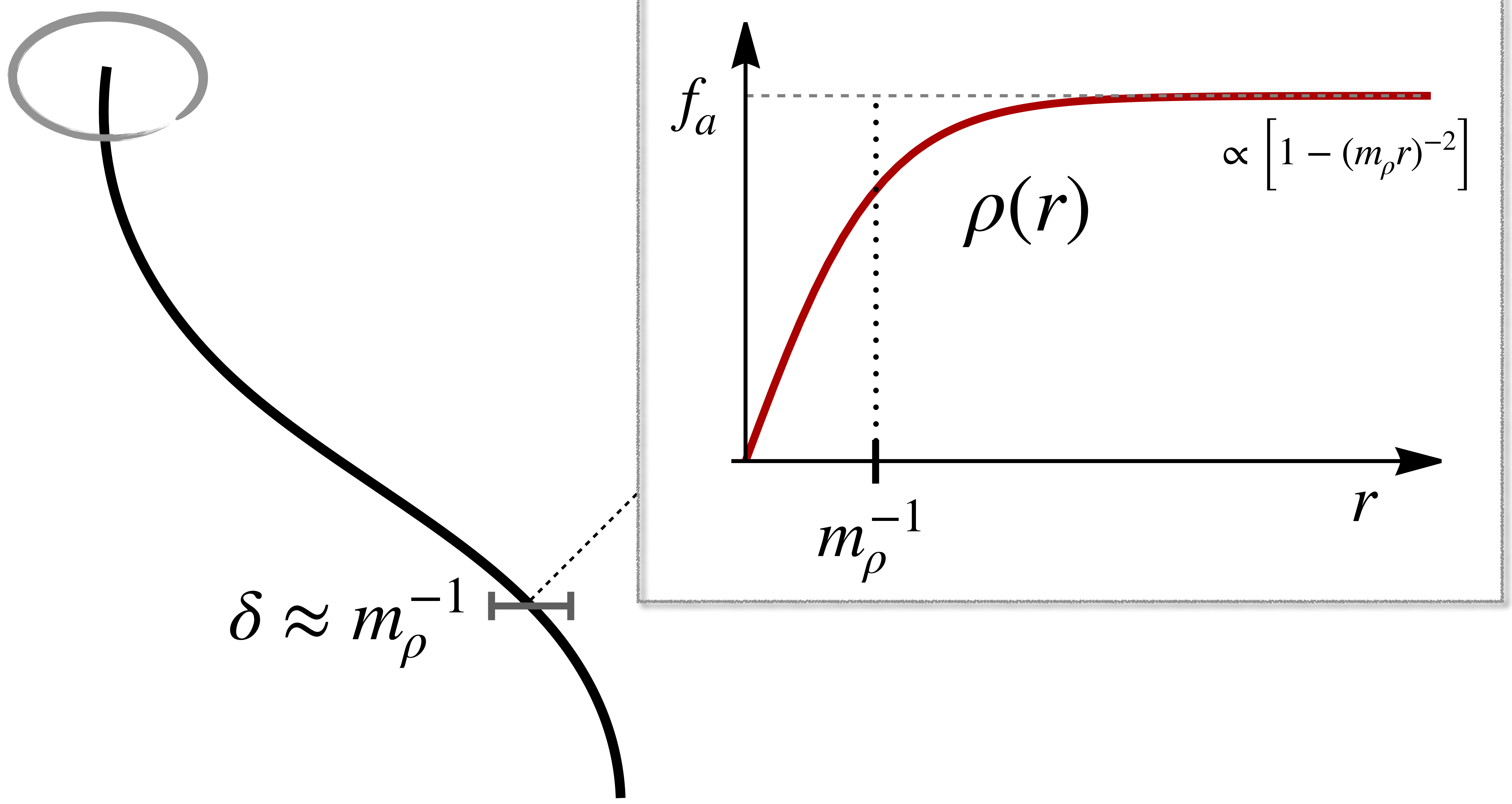
$$\Phi = \rho e^{i\alpha}$$



$$V_{\rm PQ}(\Phi)$$

- Global string solution

$$\alpha(\theta) : 0 \to 2\pi$$



QCD axion strings

- Consider the minimal KSVZ axion model with a Higgs portal:

$$\mathcal{V} = V_{\mathrm{PQ}}(|\Phi|) + V_{\mathrm{EW}}(|\mathcal{H}|;T) + \kappa \left(|\Phi|^2 - \frac{f_a^2}{2}\right)\left(|\mathcal{H}|^2 - \frac{v^2}{2}\right)$$

$$\eta \left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2$$

QCD axion strings

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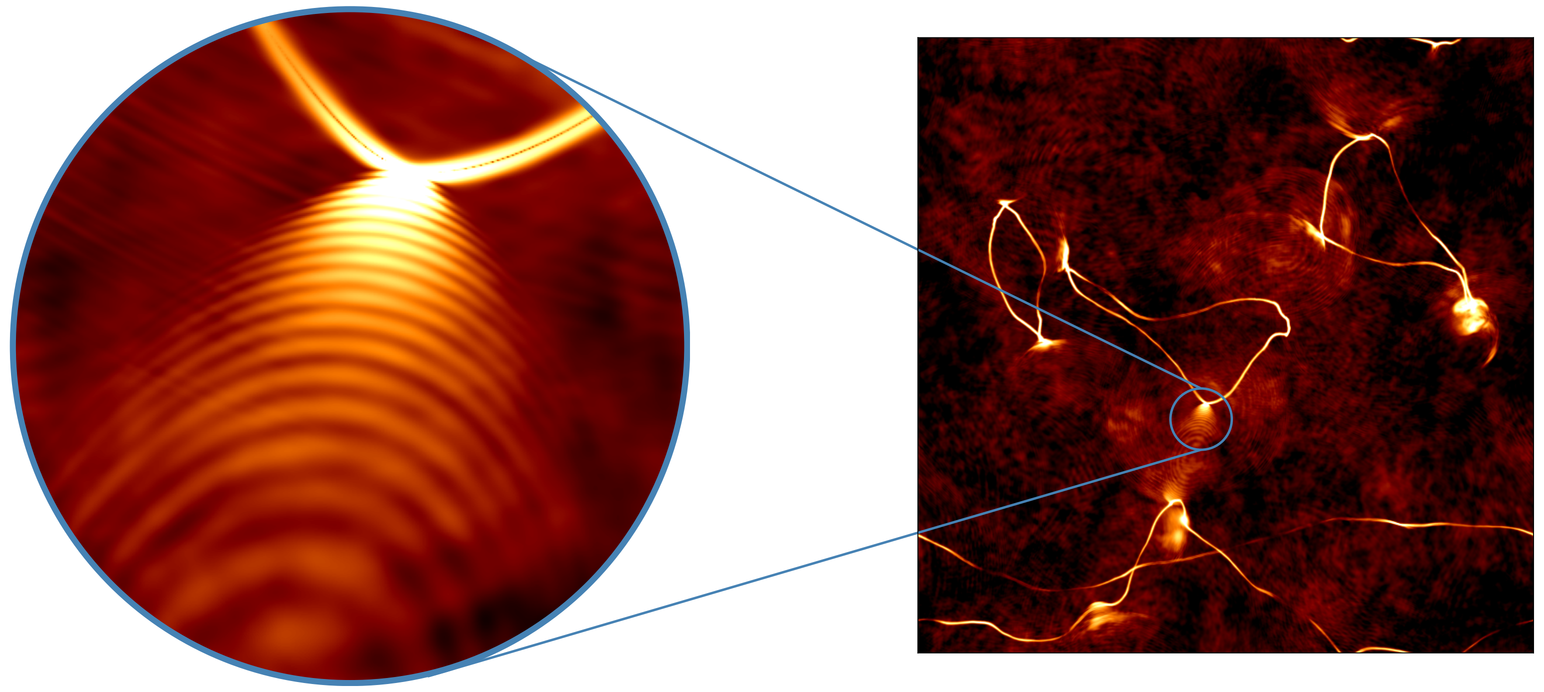

How do axion strings affect electroweak symmetry breaking?

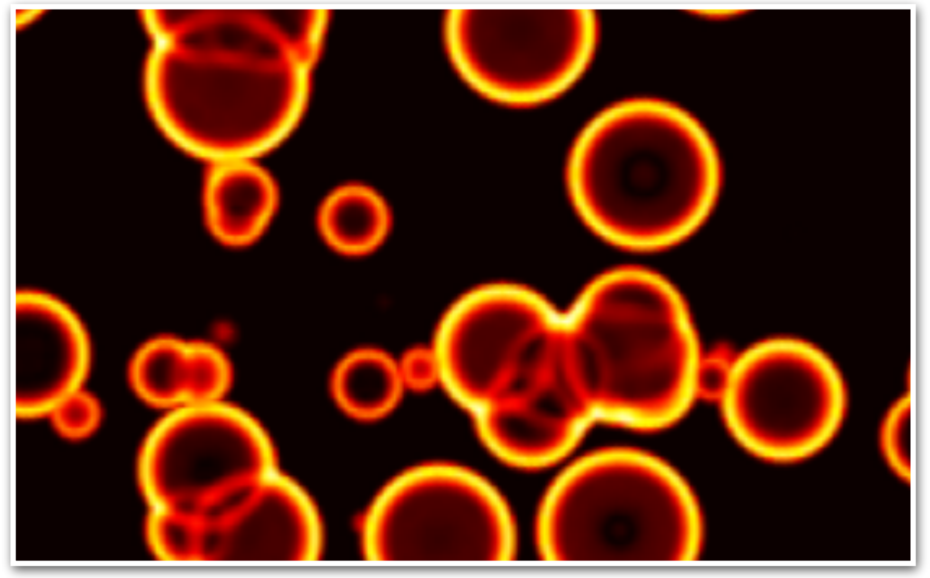

Fig. from 2308.01334

EFT with heavy defects

- Large hierarchy between the mass of the Higgs and the PQ radial mode
- Physics captured by electroweak scale EFT, SM + axion (or ALP):

$$S_{\mathrm{EFT}}[h] = \int d^4x \left\{ \frac{1}{2}(\partial_\mu h)^2 - V_{\mathrm{EW}}(h) - \frac{1}{2}\frac{\kappa}{\eta}(\partial_\mu \alpha)^2 h^2 + \pi \frac{\kappa}{\eta} C(\epsilon)\delta^{(2)}(r-\epsilon)h^2 \right\}$$

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- δ-potential imposes UV matching condition:

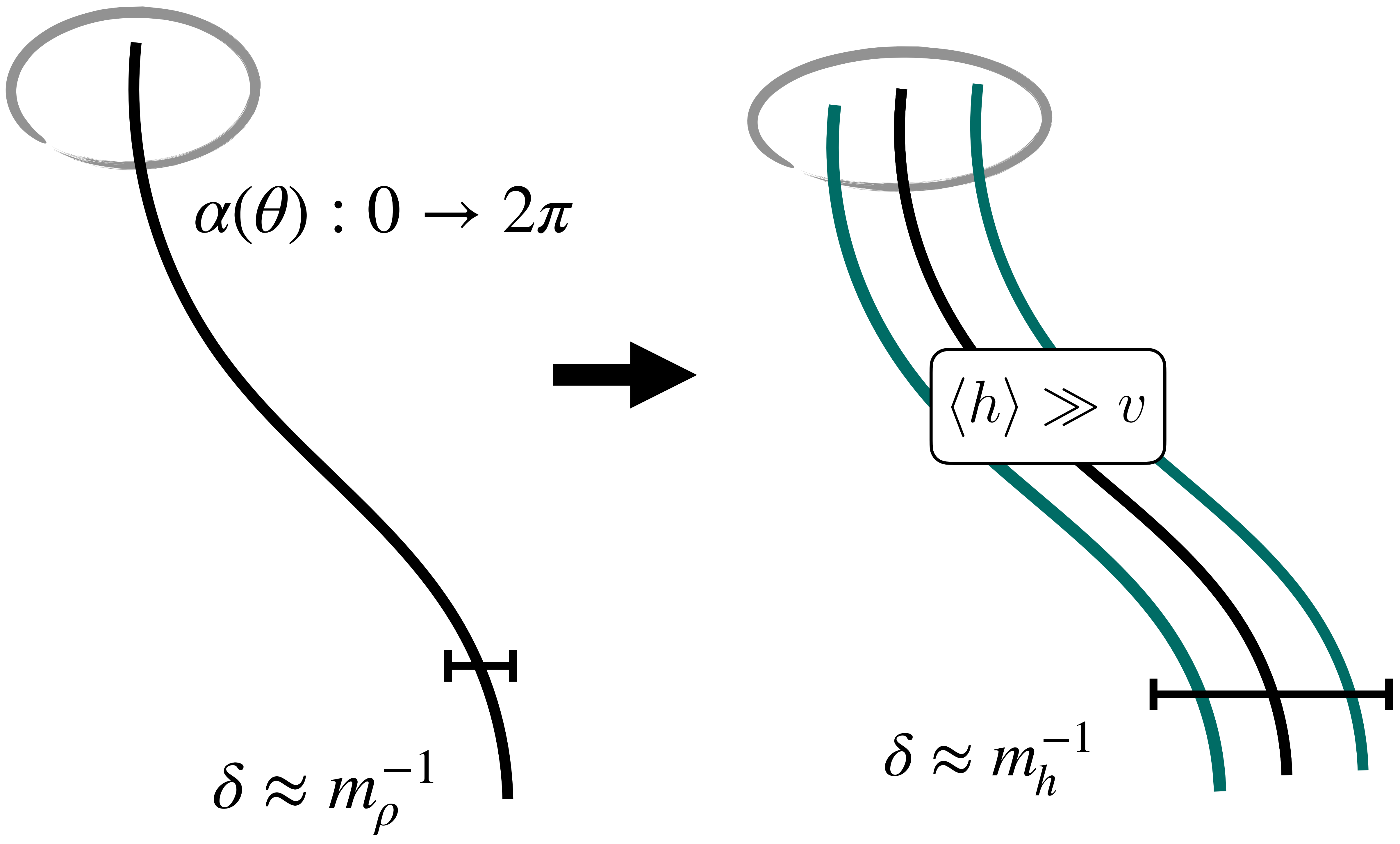
$$\epsilon\, h'(\epsilon) = -C(\epsilon)\frac{\kappa}{\eta}h(\epsilon)$$

$$\epsilon \sim 1/m_\rho$$

Two possibilities

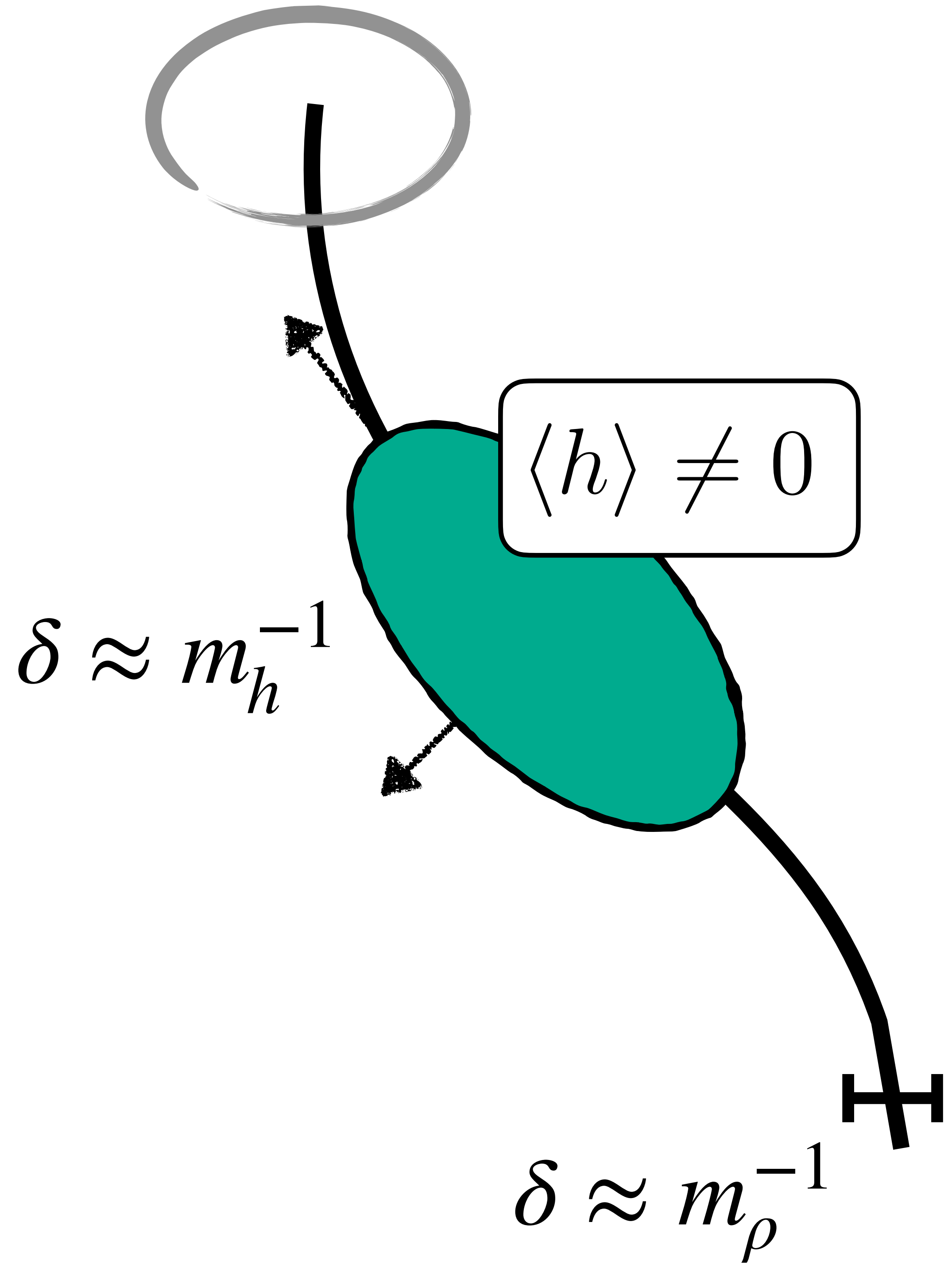
SM + PQ

Axion strings can form a Higgs core at $T \gg 10^2$ GeV



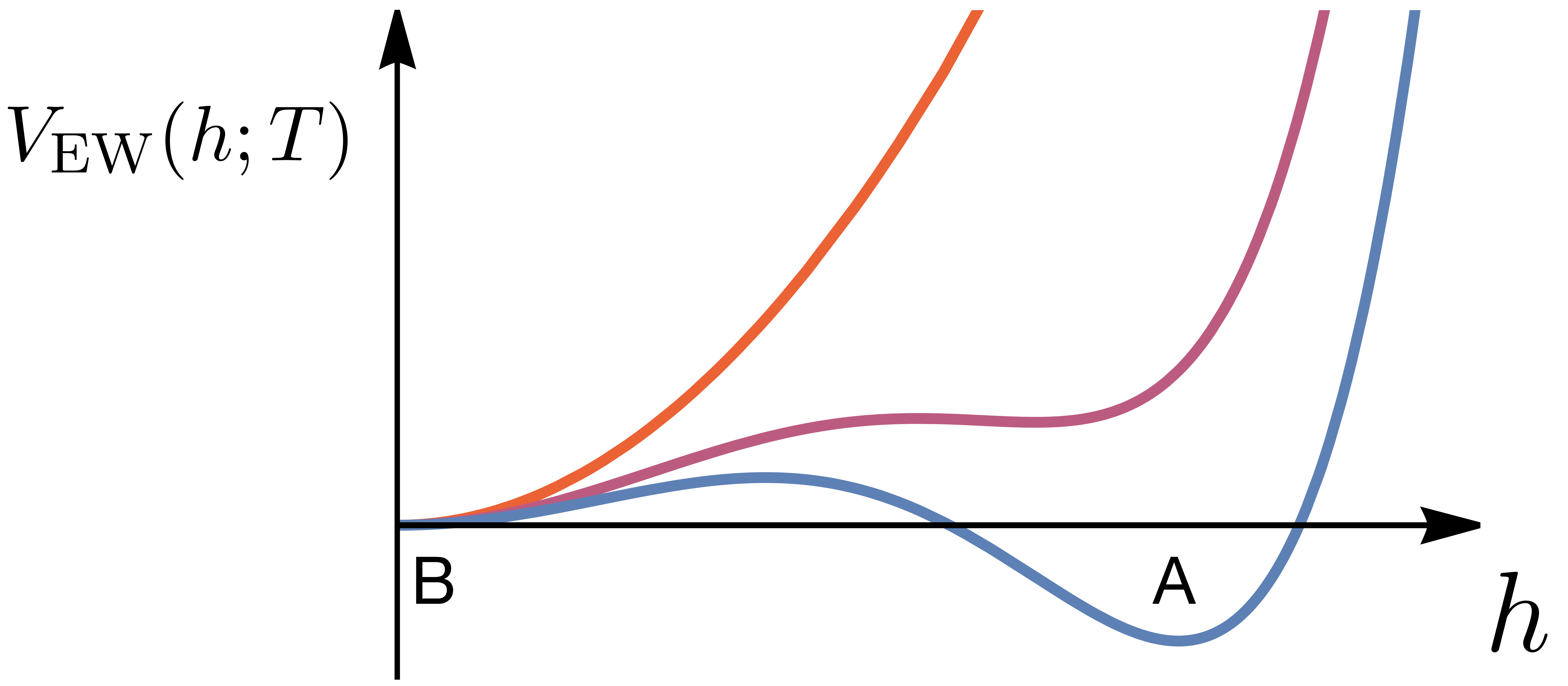
First order EWPT + PQ

Axion strings can trigger bubble nucleation



FOPT + PQ

- Consider first order EWPT with false vacuum B metastable at $T = 0$



$$V_{\rm EW}(h;T) = -\frac{1}{2}\left(\mu^2 - c_h T^2\right) h^2 + \frac{\delta}{3}\frac{m_h^2}{v^2} h^3 + \frac{1}{4}\lambda h^4$$

FOPT + PQ

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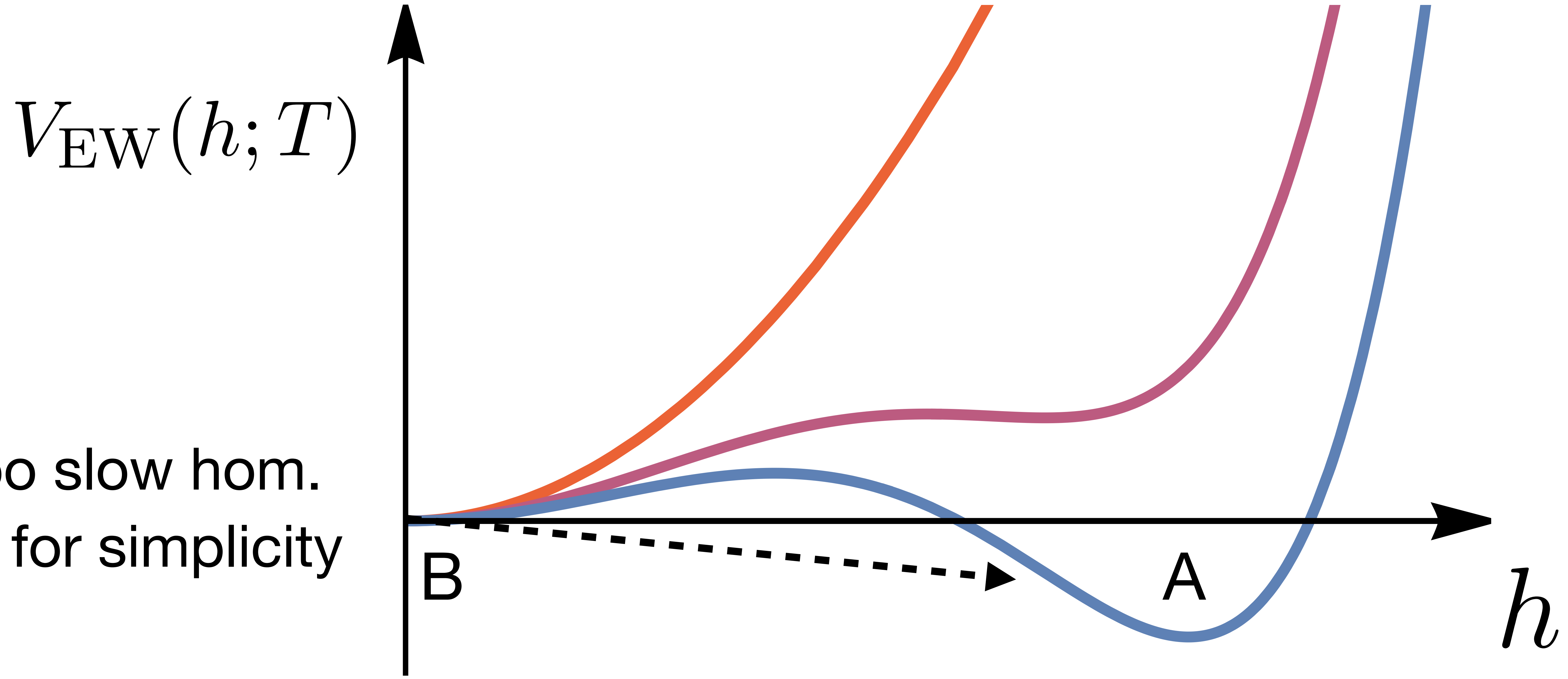
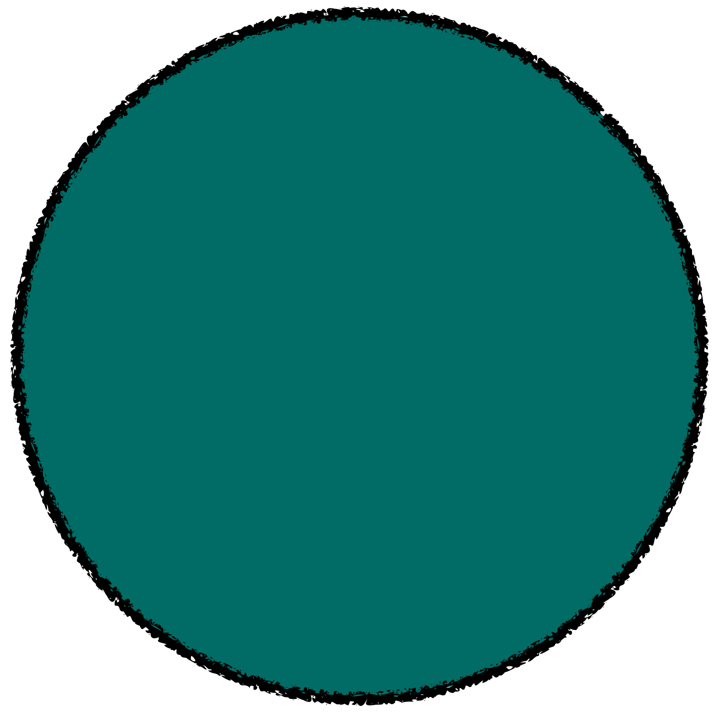


Assume too slow hom. nucleation for simplicity

$$V_{\rm EW}(h;T) = -\frac{1}{2}\left(\mu^2 - c_h T^2\right) h^2 + \frac{\delta}{3}\frac{m_h^2}{v^2} h^3 + \frac{1}{4}\lambda h^4$$

FOPT + PQ

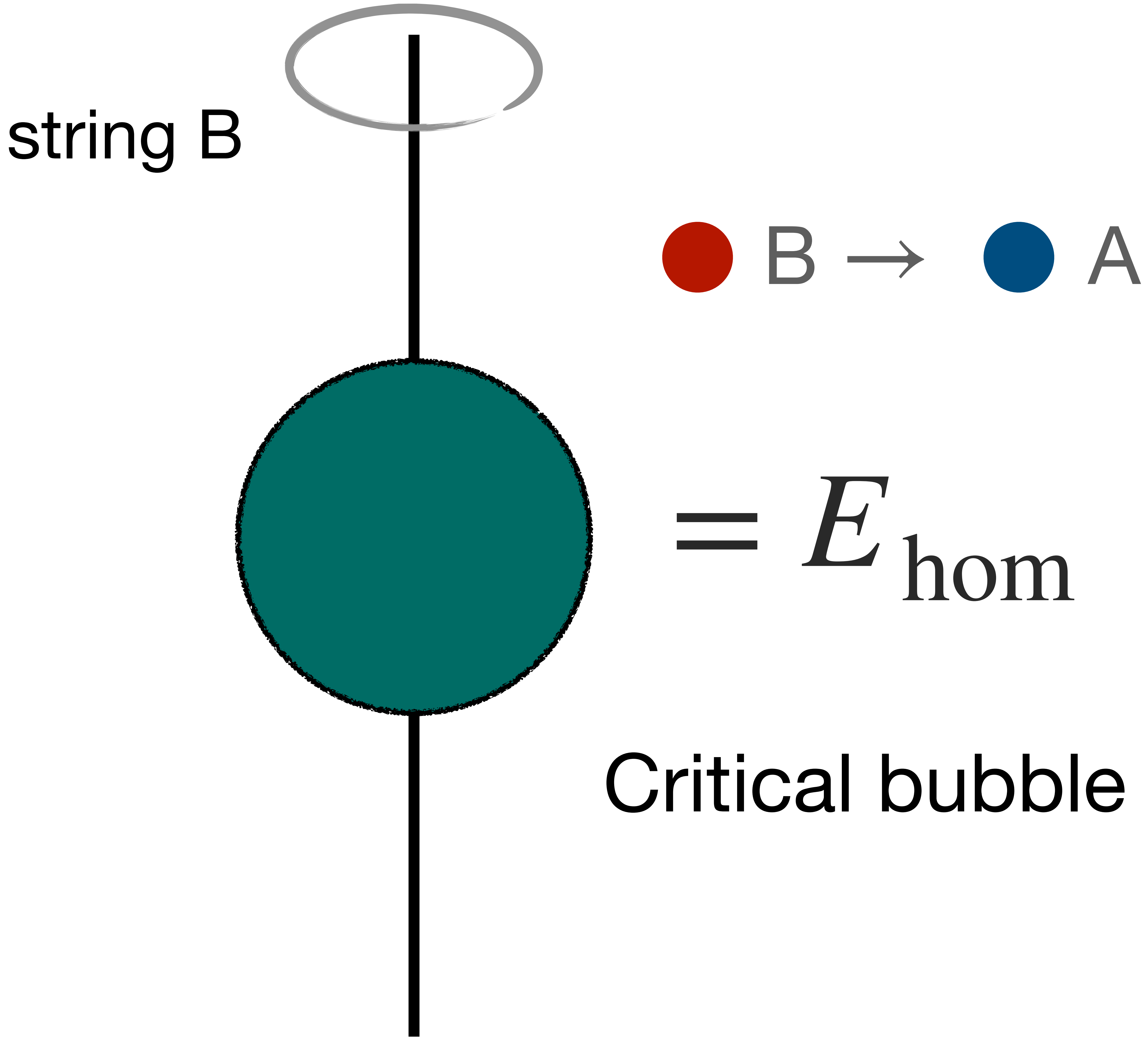
B $\rightarrow$ A



$$= E_{\text{hom}}$$

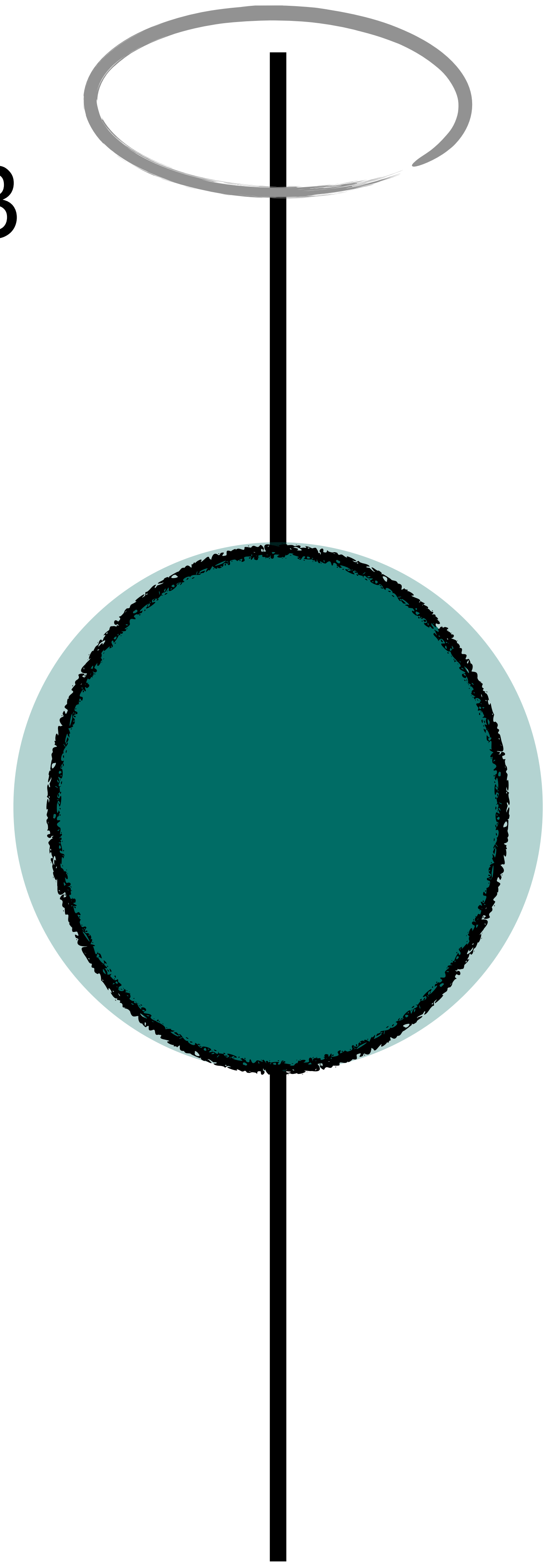
Critical bubble

FOPT + PQ



$$= E_{\text{hom}} \qquad (\kappa = 0)$$

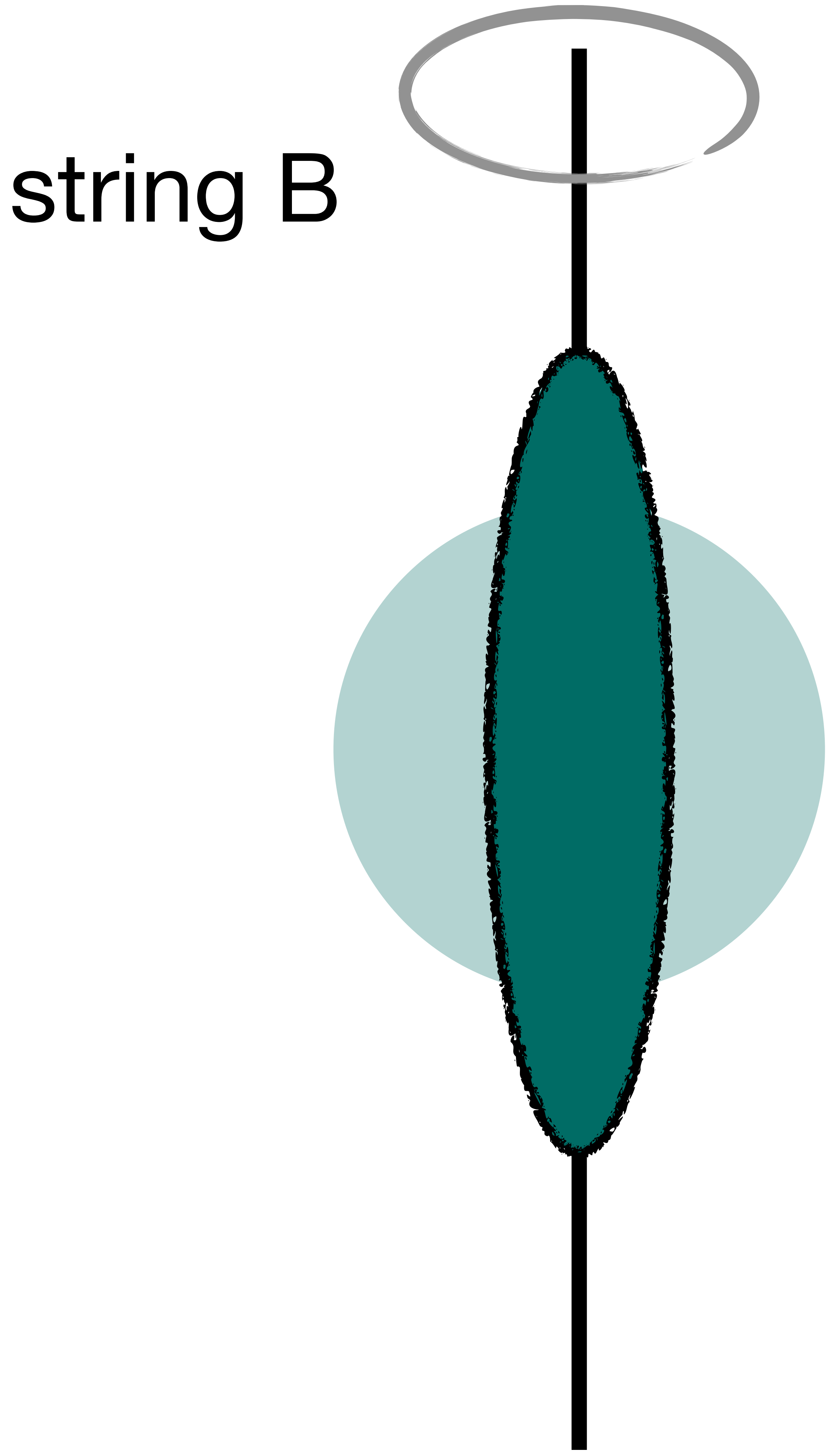
FOPT + PQ



String B → String A

$$\lesssim E_{\rm hom} \qquad (\kappa/\eta \ll 1)$$

FOPT + PQ

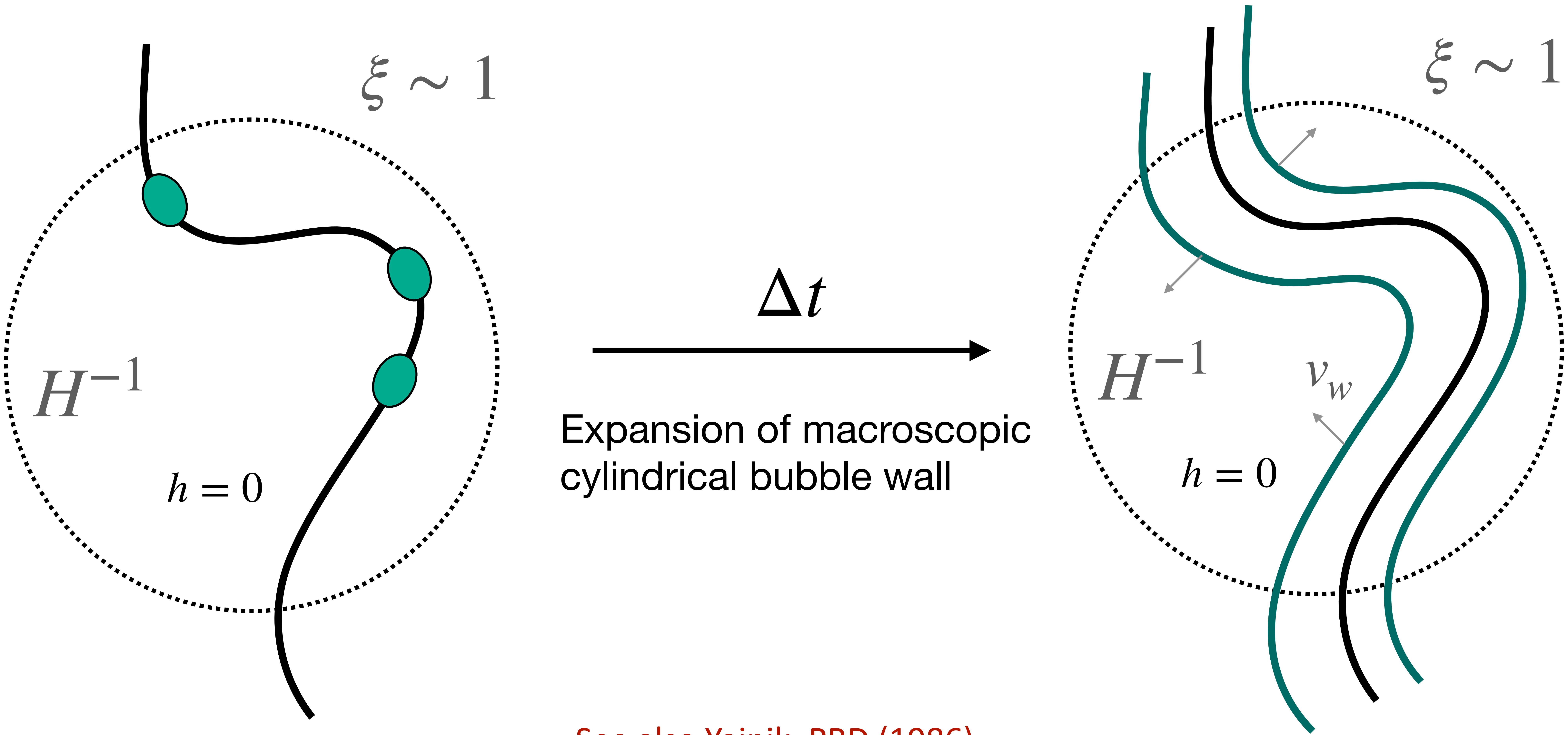


String B → String A

$$\ll E_{\text{hom}} \quad (\kappa/\eta \sim \text{crit.})$$

Critical bubble

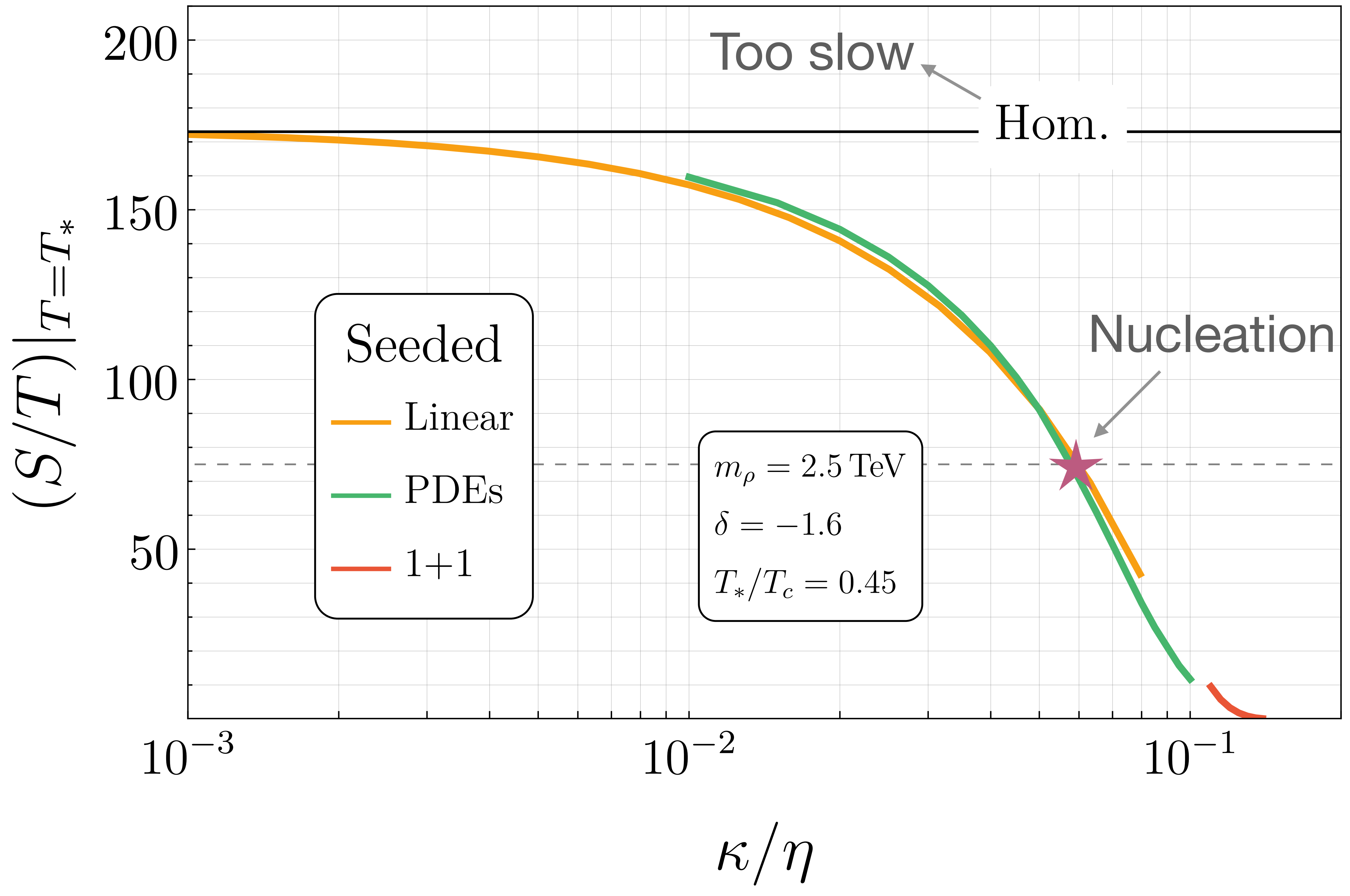
FOPT + PQ



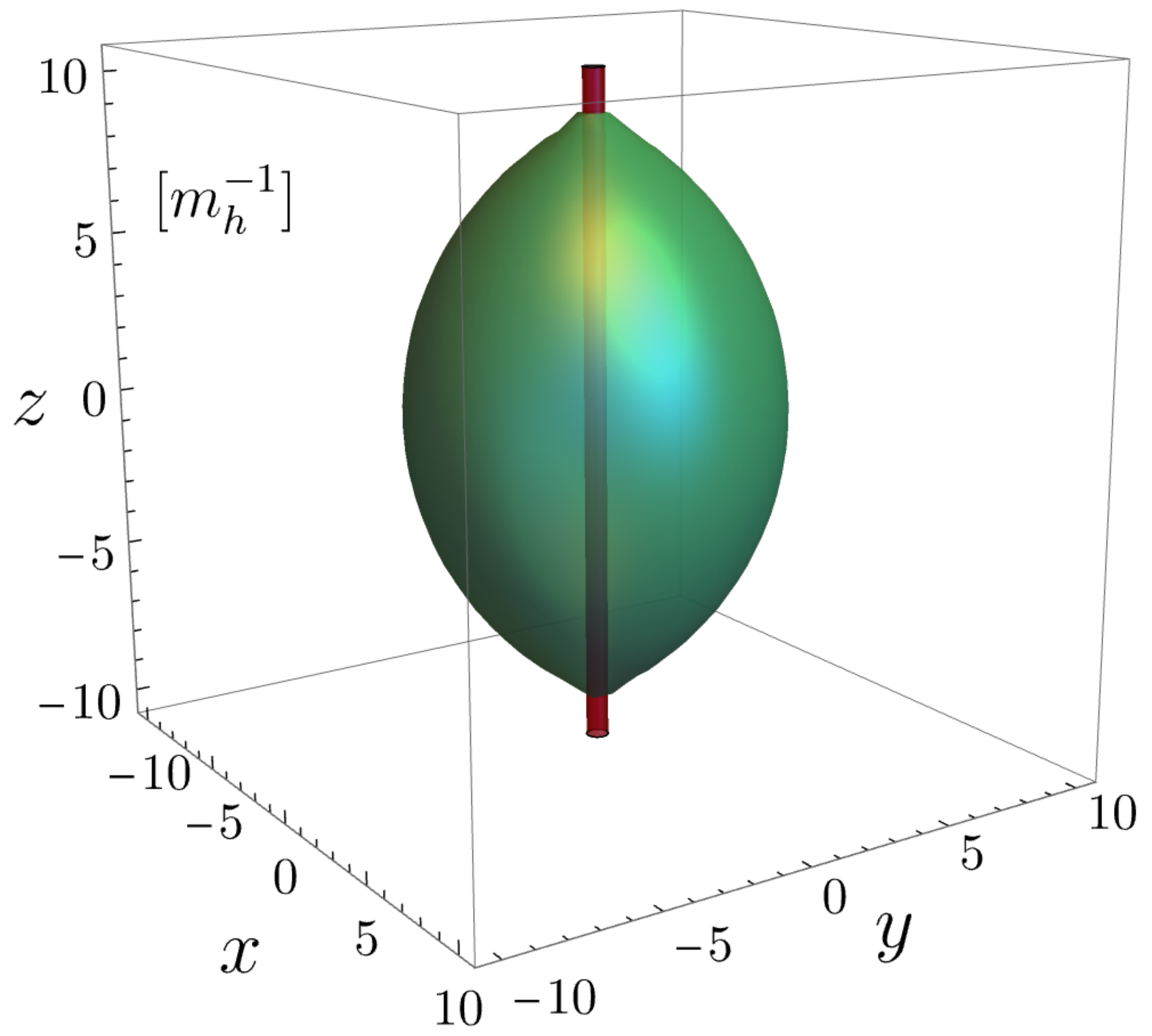
See also Yajnik, PRD (1986)

FOPT + PQ

- Comparison of homogenous nucleation probability vs string-seeded tunneling:



- Critical bubble:

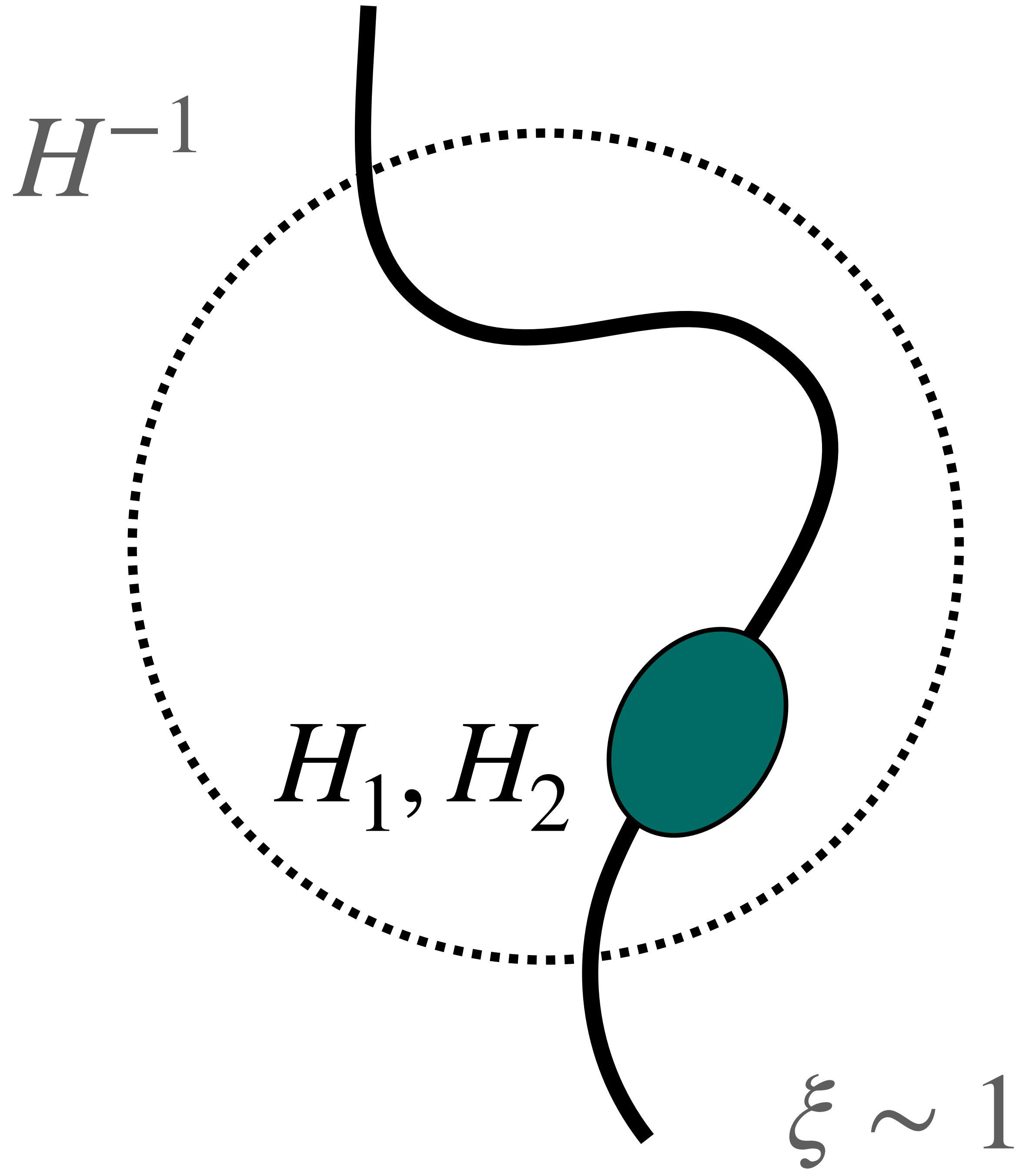


DFSZ QCD axion strings

- Consider the DFSZ model where two Higgs doublets share a PQ charge:

$$\mathcal{L} = |\partial_\mu S|^2 + |D_\mu H_1|^2 + |D_\mu H_2|^2 - V_S(|S|) - V_{\rm EW}(H_1, H_2) - \left(\kappa S^2 H_1^\dagger H_2 + \text{h.c.}\right)$$

U(1) symmetry with massless A^0



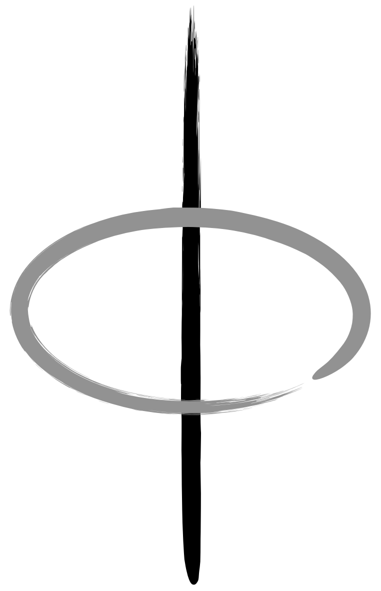
How does a non-zero κ affect EWSB?

SB, Y. Hamada, in prep.

DFSZ QCD axion strings

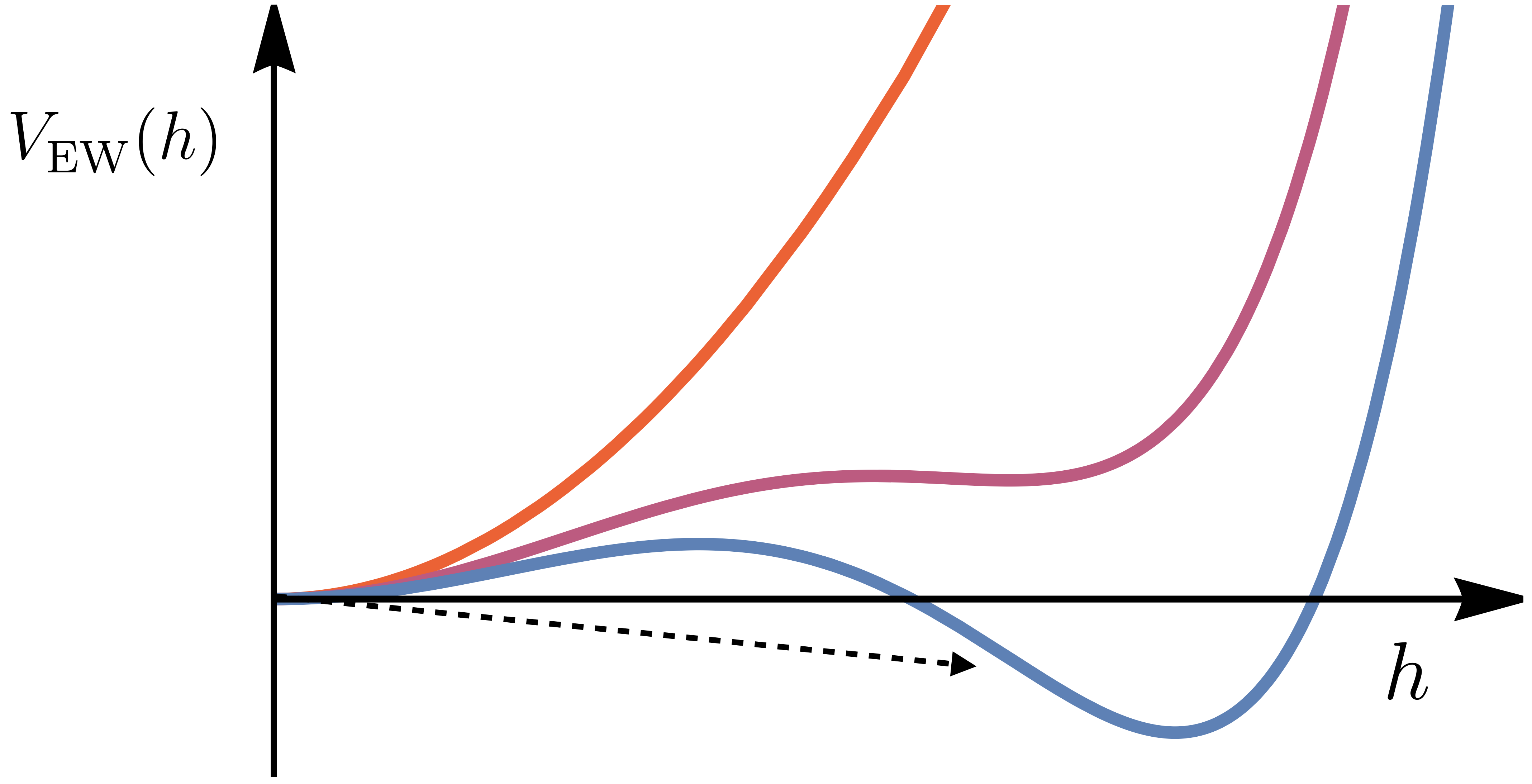
- Consider a **toy model** with PQ scalar + complex Higgs, in the string background:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}h^2(\partial_\mu \phi)^2 - V_{\mathrm{EW}}(h) + \frac{1}{2}\kappa\,\rho^2(r)h^2\cos(2\theta + 2\phi)$$



PQ: $S = \frac{1}{\sqrt{2}}\rho(r)e^{i\theta}$

Higgs: $\varphi = \frac{1}{\sqrt{2}}he^{i\phi}$



DFSZ QCD axion strings

- Consider $\kappa \ll 1$ and perturb around the homogeneous, spherical, solution:

$$h = h(R) + \kappa\,\delta h, \quad \phi = \cancel{c} + \kappa\,\delta\phi$$

- Decoupled linear equations for the fluctuations with a source term:

$$\Box \delta h + V''_{\rm EW}(h(R))\delta h = h(R)\rho^2(r)\cos(2\theta + \cancel{2c})$$ (Higgs equation)

$$\partial_\mu \left[h^2(R)\partial^\mu \delta\phi\right] = -\,h^2(R)\rho^2(r)\sin(2\theta + \cancel{2c})$$ (Goldstone equation)

DFSZ QCD axion strings

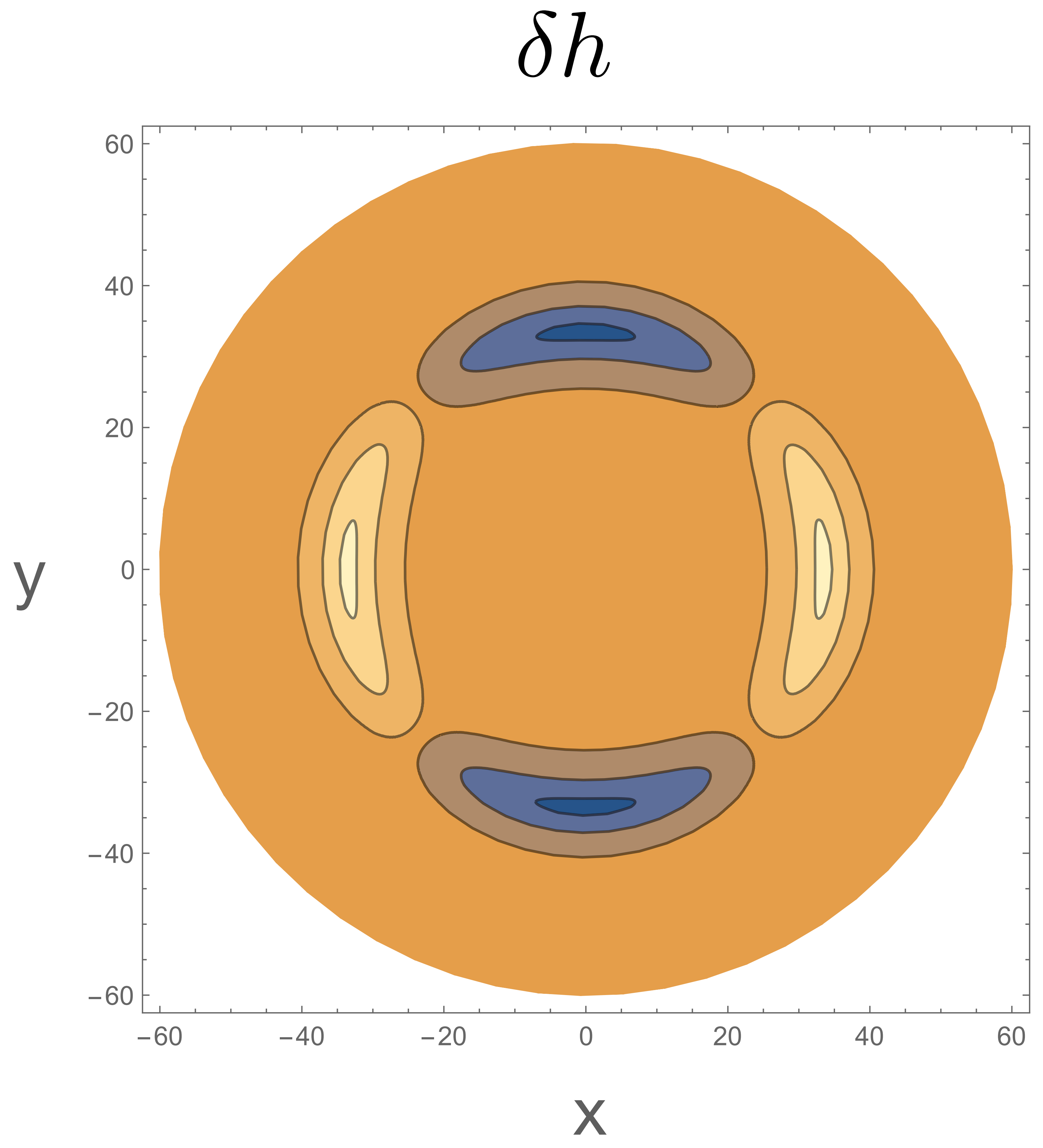
- Solution for δh from spectral decomposition:

$$\mathcal{O}\delta h = \sum_\lambda c_\lambda \phi_\lambda(x) = S(x)$$

$$\delta h = \sum_\lambda c_\lambda \frac{1}{\lambda^2}\phi_\lambda(x) \qquad c_\lambda = \int dx\, \phi^*_\lambda(x) S(x)$$

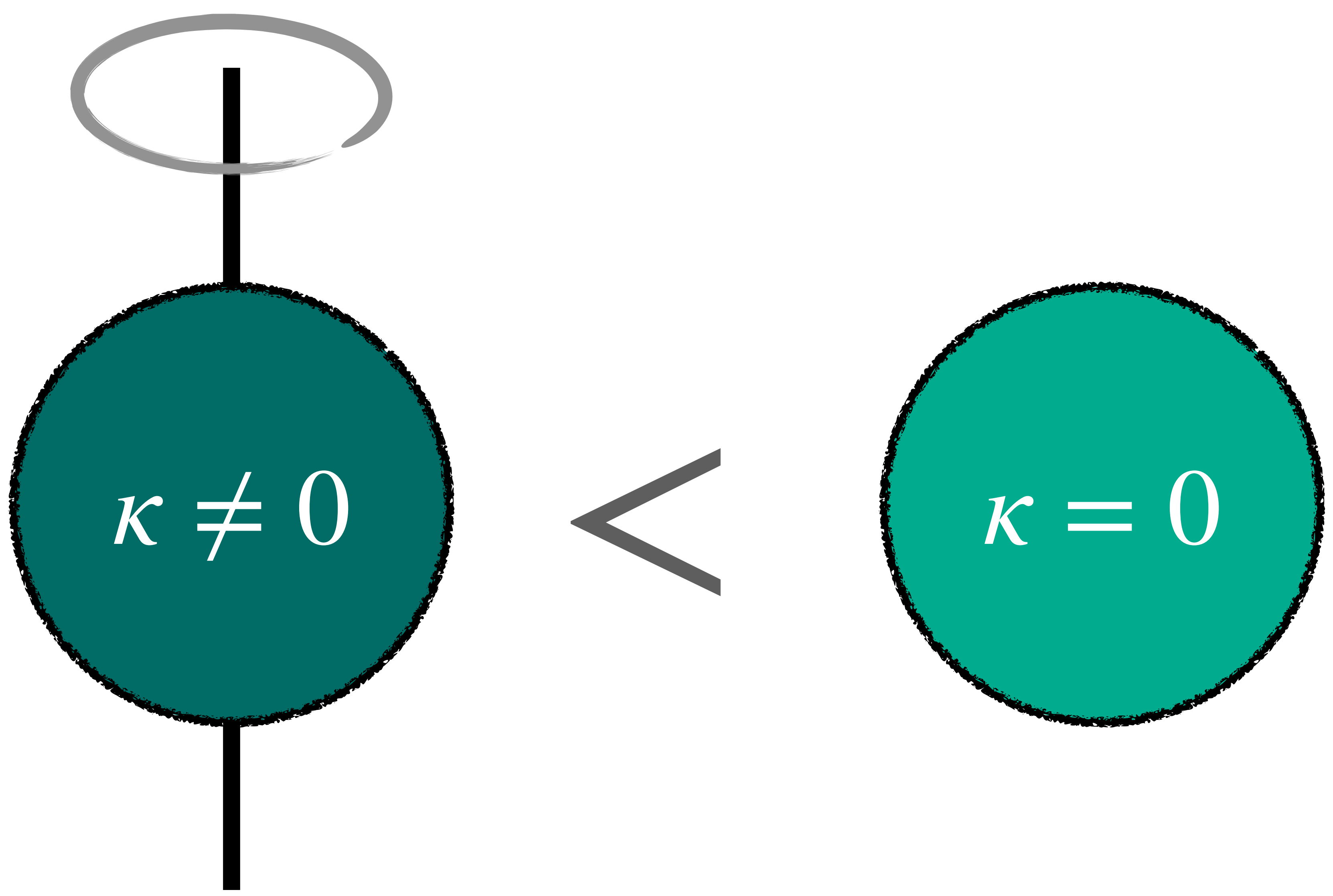
- Perturbation dominated by $\ell = 2$ and $m = 2$:

$$\delta h \sim Y_{22}(\theta,\phi) R_{22}(R) + \frac{h(R)}{m_h^2}\cos(2\phi)$$

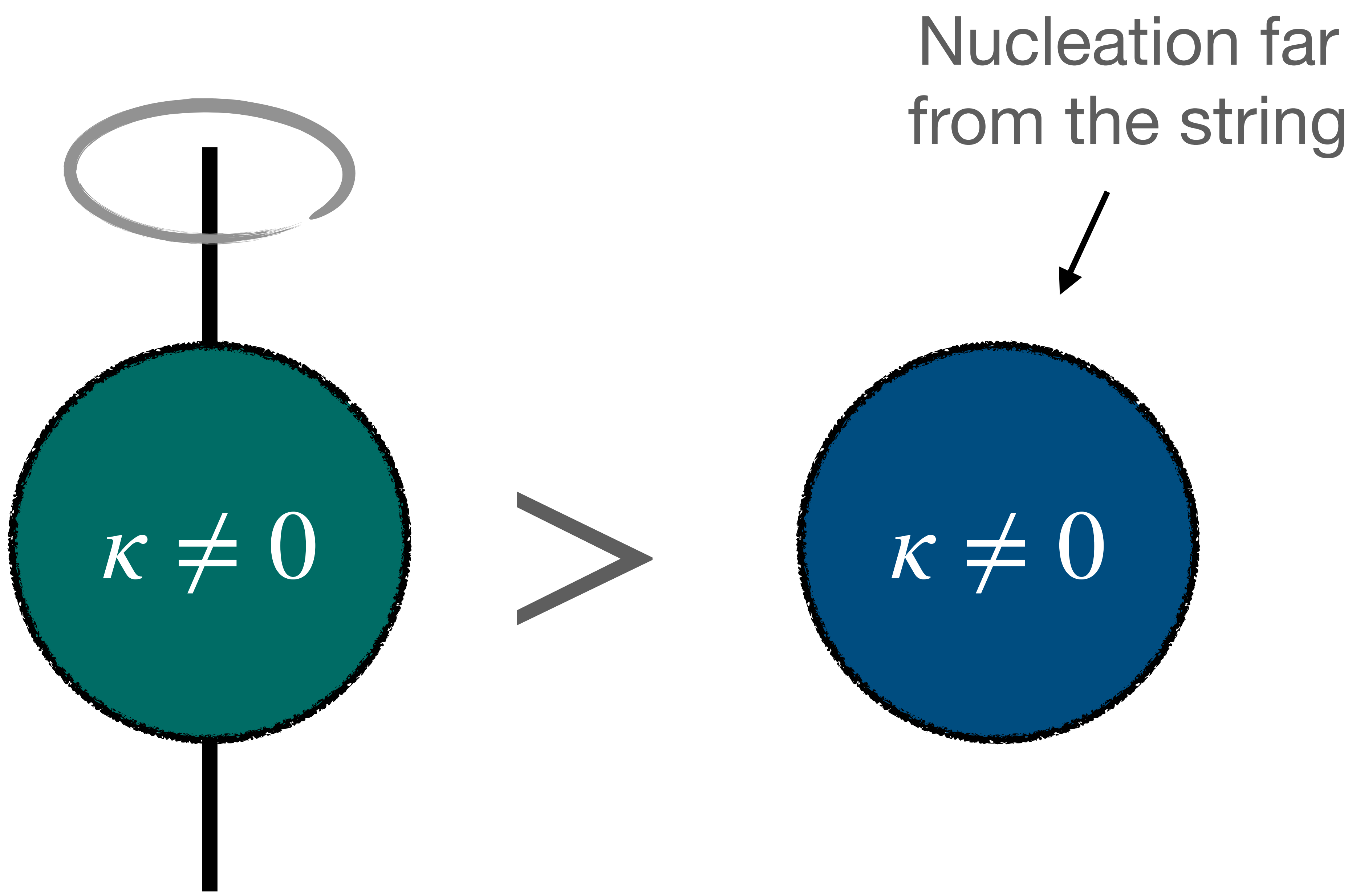


DFSZ QCD axion strings

- Seeded nucleation actually disfavored for all $\kappa \neq 0$



$$E^{\kappa\neq0}_{\text{string, c}} = E^{\kappa=0}_{\text{sph, c}} - \kappa^2 |\Delta E|$$



$$E^{\kappa\neq0}_{\text{string, c}} - E^{\kappa\neq0}_{\text{sph, c}} \sim \kappa f_a^2 v^2 R^3$$

DFSZ QCD axion strings

- Seeded nucleation actually disfavored for all $\kappa \neq 0$
- However we still need to account for KSVZ-type interactions

$$V_{\text{mix}} = \left(\kappa S^2 H_1^\dagger H_2 + \text{h.c.}\right) + \kappa_{1S}|S|^2|H_1|^2 + \kappa_{2S}|S|^2|H_2|^2$$

Summary and outlook

- The presence of impurities in the early Universe can strongly affect the way a phase transition proceeds

- The xSM with Z_2 symmetry is arguably the simplest (and complete) example for an EWPT seeded by domain walls

- Other defects can exist at the time of the EWPT: dedicated study of QCD axion strings in KSVZ model with Higgs portal, and extension to DFSZ

- Pheno aspects of seeded phase transitions: percolation, slow transitions, expansion of non—spherical bubbles, features in the GW signal?

- New opportunities to study tunneling in quantum/thermal field theory

Summary and outlook

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Thank you!