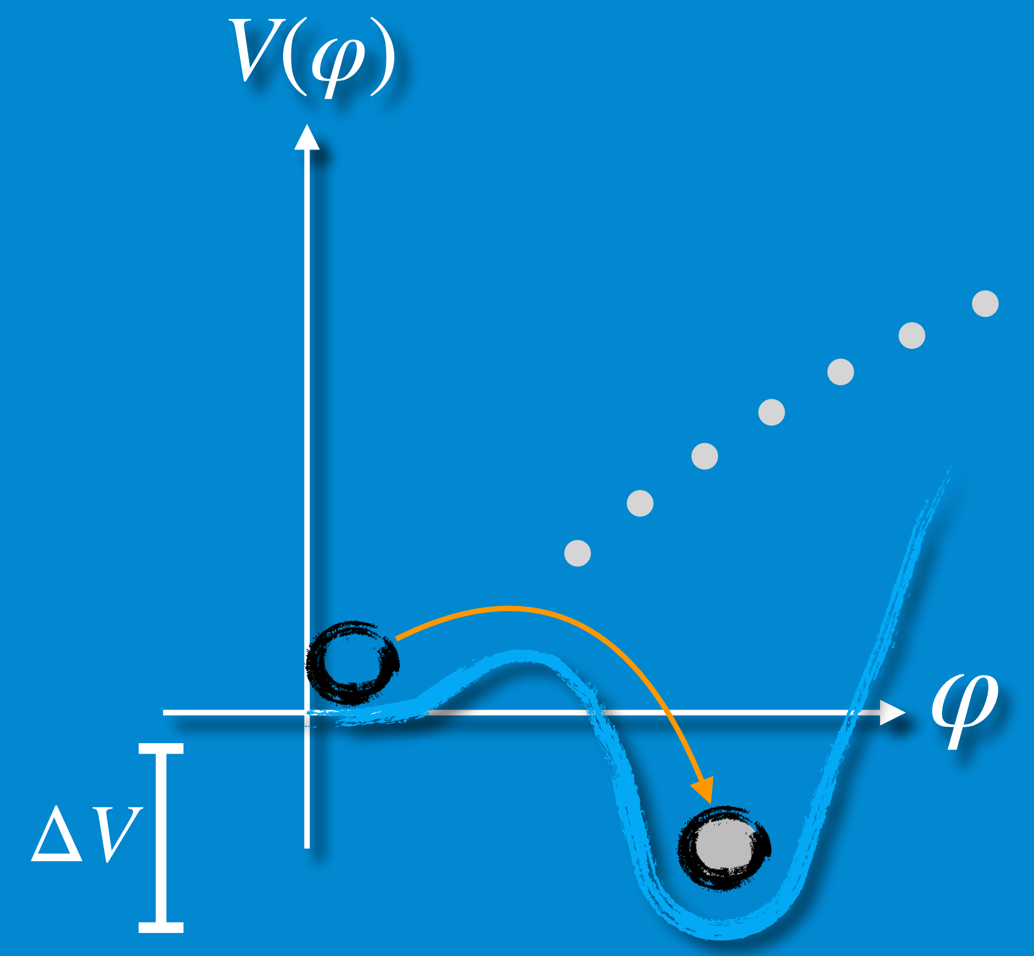
Finite-temperature (supercooled) bubble-nucleation with shifting scale hierarchies

Maciej Kierkla, University of Warsaw

Based on: 2503.13597, 2506.15496

This talk

Field theory

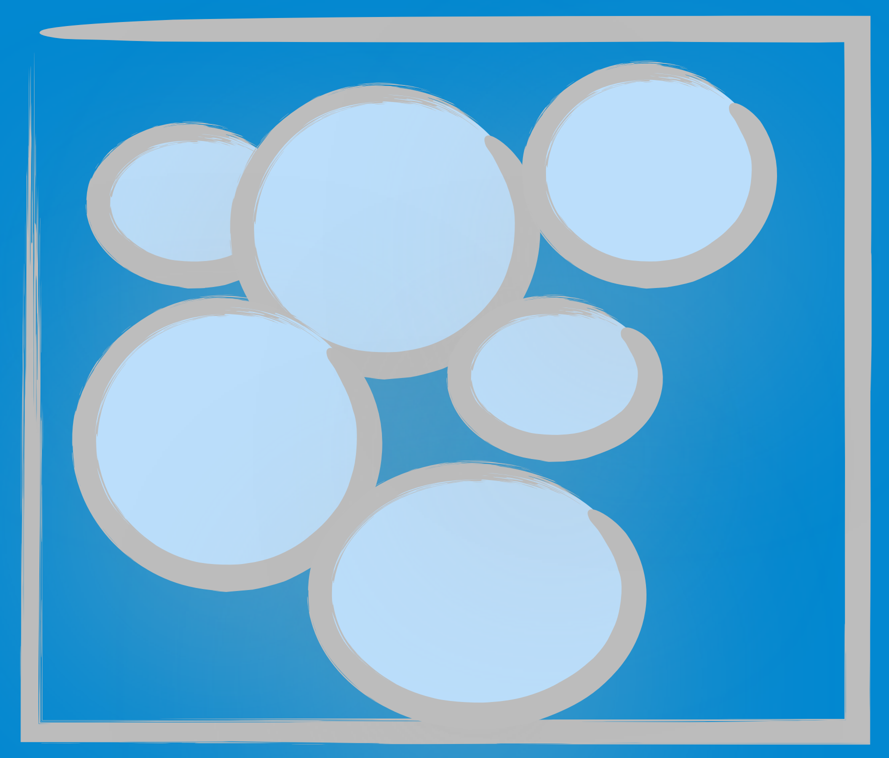


T_p

Bubble nucleation rate

$$\frac{\Gamma_T}{H^4}$$

Cosmology



Bubble nucleation rate

$$\Gamma(T) = A_{\rm dyn} \times A_{\rm stat}$$

Non-equilibrium effects

Dimension $[T]$

Equilibrium effects

Dimension $[T^3]$

See also: J. Hirvonen, 2403.07987, A. Ekstedt 2201.07331

Statistical part

In the saddlepoint approximation

$$A_{\text{stat}} = A e^{-(S_{\text{eff}}[\phi_b] - S_{\text{eff}}[\phi_F])}$$

ϕ_b *is the solution to Euclidean EOMs,*

ϕ_F *is the false vacuum*

See also: J. Hirvonen, 2403.07987, A. Ekstedt 2201.07331

Why study supercooled PTs?

- Strong GW signal!
- PTA data (?)
- DM/pBHs production [*]
- LISA is coming
- Needs robust framework

. .

4d theory

SU(2) parameters

g_X, M_X

$$V_{\text{tree}} = \frac{1}{4}\left(\lambda_1 h^4 + \lambda_2 h^2 \phi^2 + \lambda_3 \phi^4\right)$$

See also: C. D. Carone et al, 1307.8428, T. Hambye et al, 1306.2329, D. Marfatia et al, 2006.07313, I. Baldes et al, 1809.01198

4d theory

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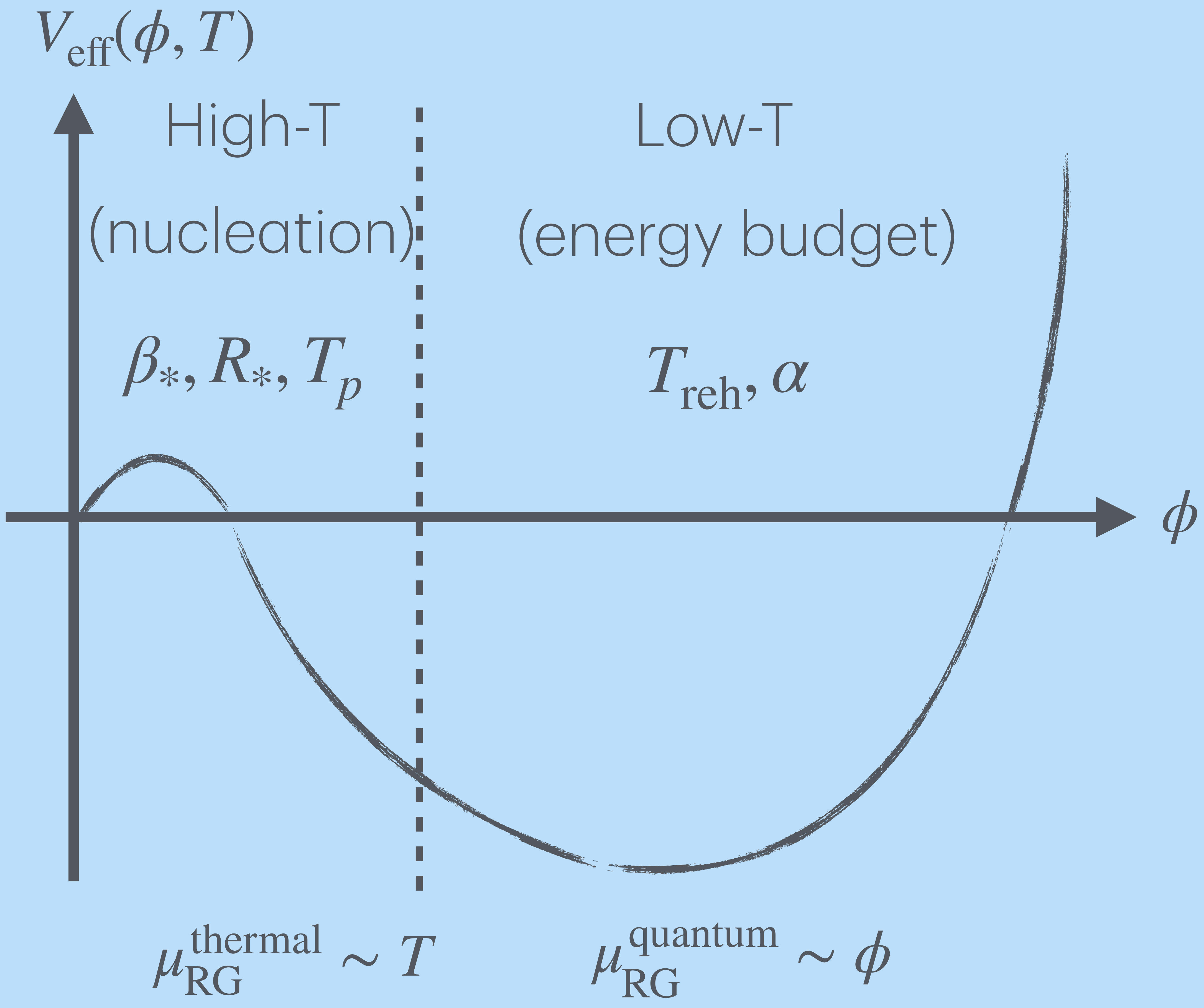
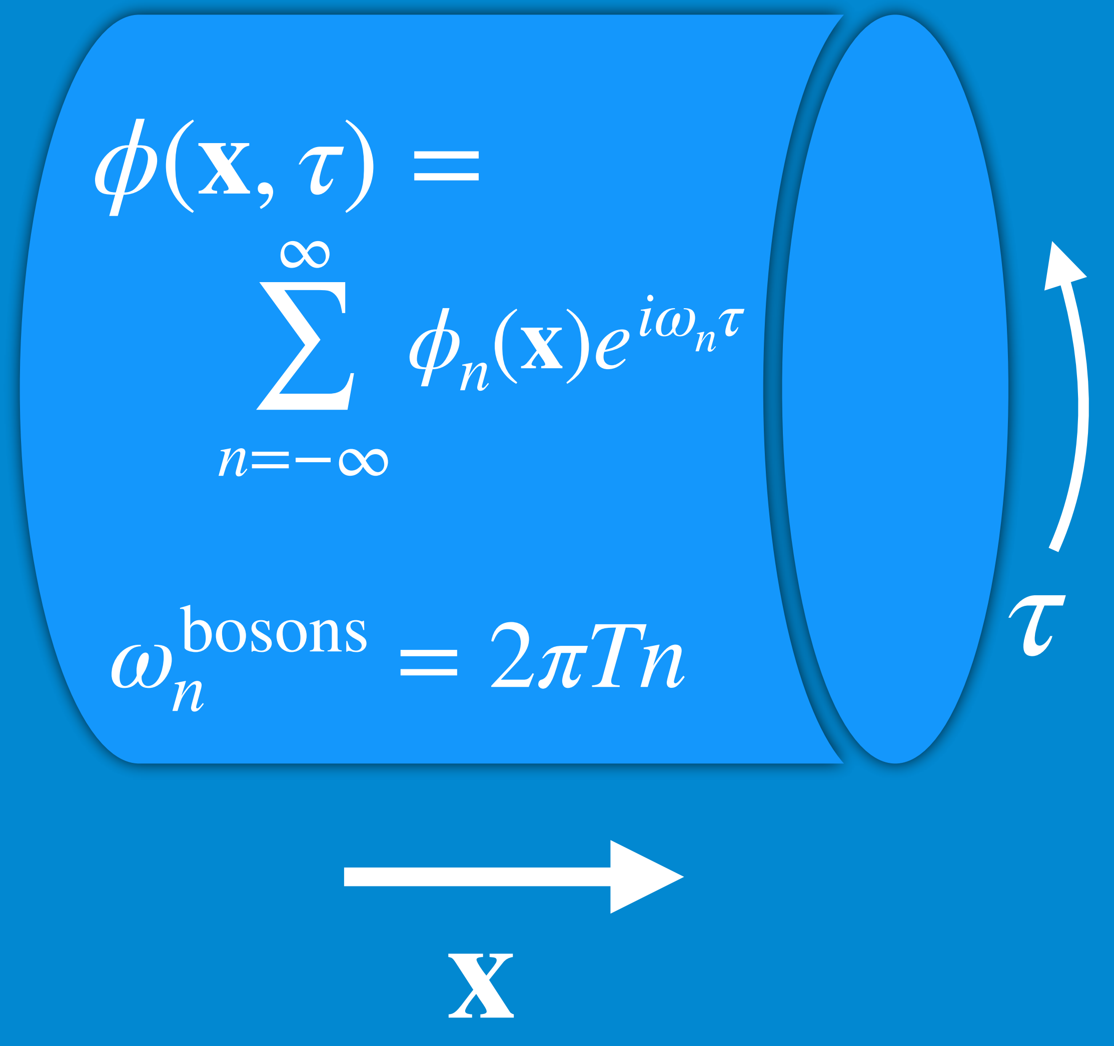
HT vs LT regimes

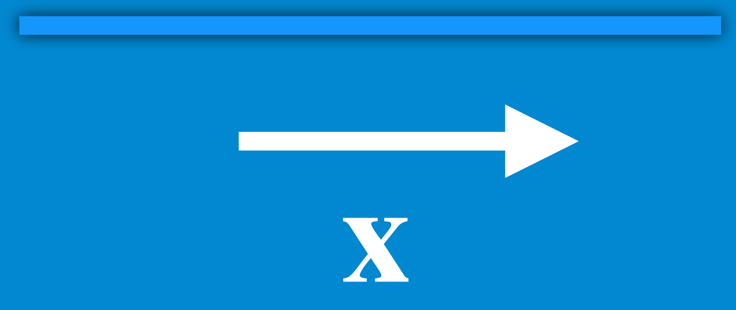


See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

High-temperature Dimensional Reduction (DR)



$\longrightarrow$ $\phi_3(\mathbf{x})$



Integrating out $\omega_{n \neq 0}$

$$Z = \int D\phi \exp\left(-\int_0^{\frac{1}{T}} \mathrm{d}\tau \int \mathrm{d}^3\mathbf{r}\ \mathscr{L}_E \right) \longrightarrow Z_3 = \int D\phi_{n=0} \exp\left(-\frac{1}{T} \int \mathrm{d}^3\mathbf{r}\ \mathscr{L}_3 \right)$$

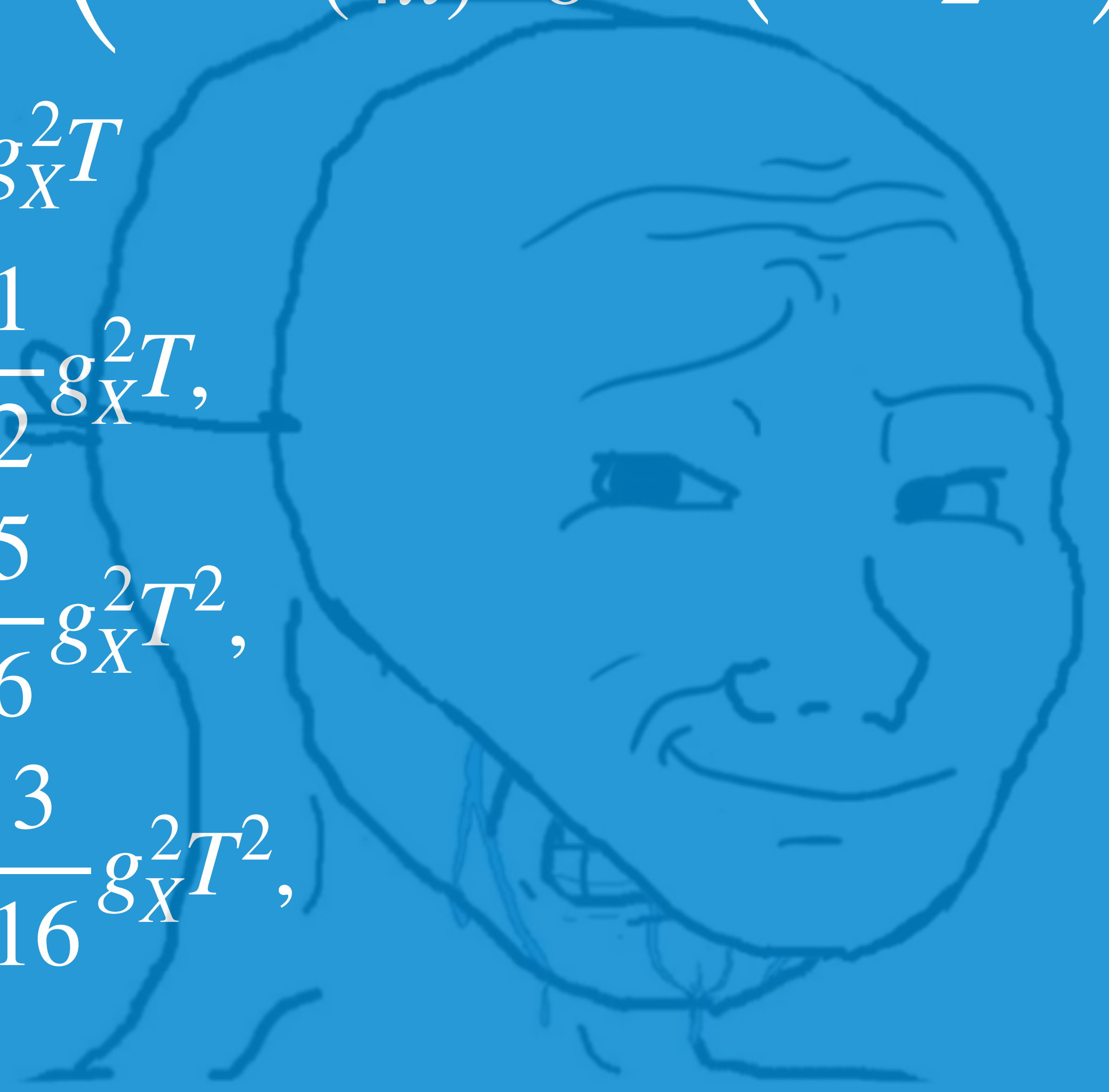
Fields: ϕ, X

Couplings: g, λ

Fields: $\phi_3, X_3, X_{0,3}$

Couplings: $m_3(T), g_3(T), \lambda_3(T)$

Daisy resummation = DR at LO matching



$$\lambda_3 = T\left(\lambda_\varphi + \frac{1}{(4\pi)^2}\frac{3}{8}g_X^4\left(1 - \frac{3}{2}L_b\right)\right)$$

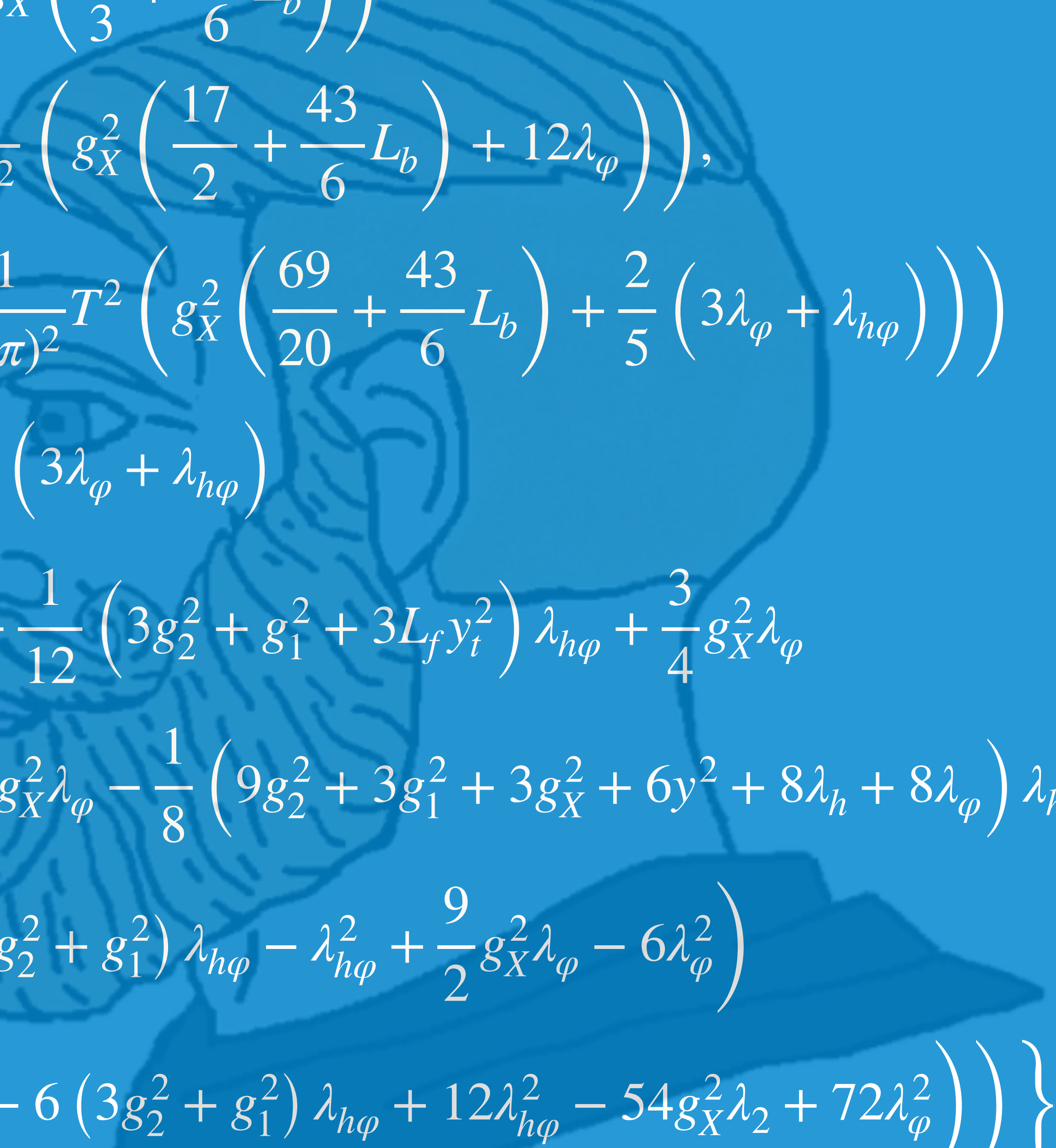
$$g_{X,3}^2 = g_X^2 T$$

$$h_3 = \frac{1}{2}g_X^2 T,$$

$$m_{D,X}^2 = \frac{5}{6}g_X^2 T^2,$$

$$m_3^2 = \frac{3}{16}g_X^2 T^2,$$

VS



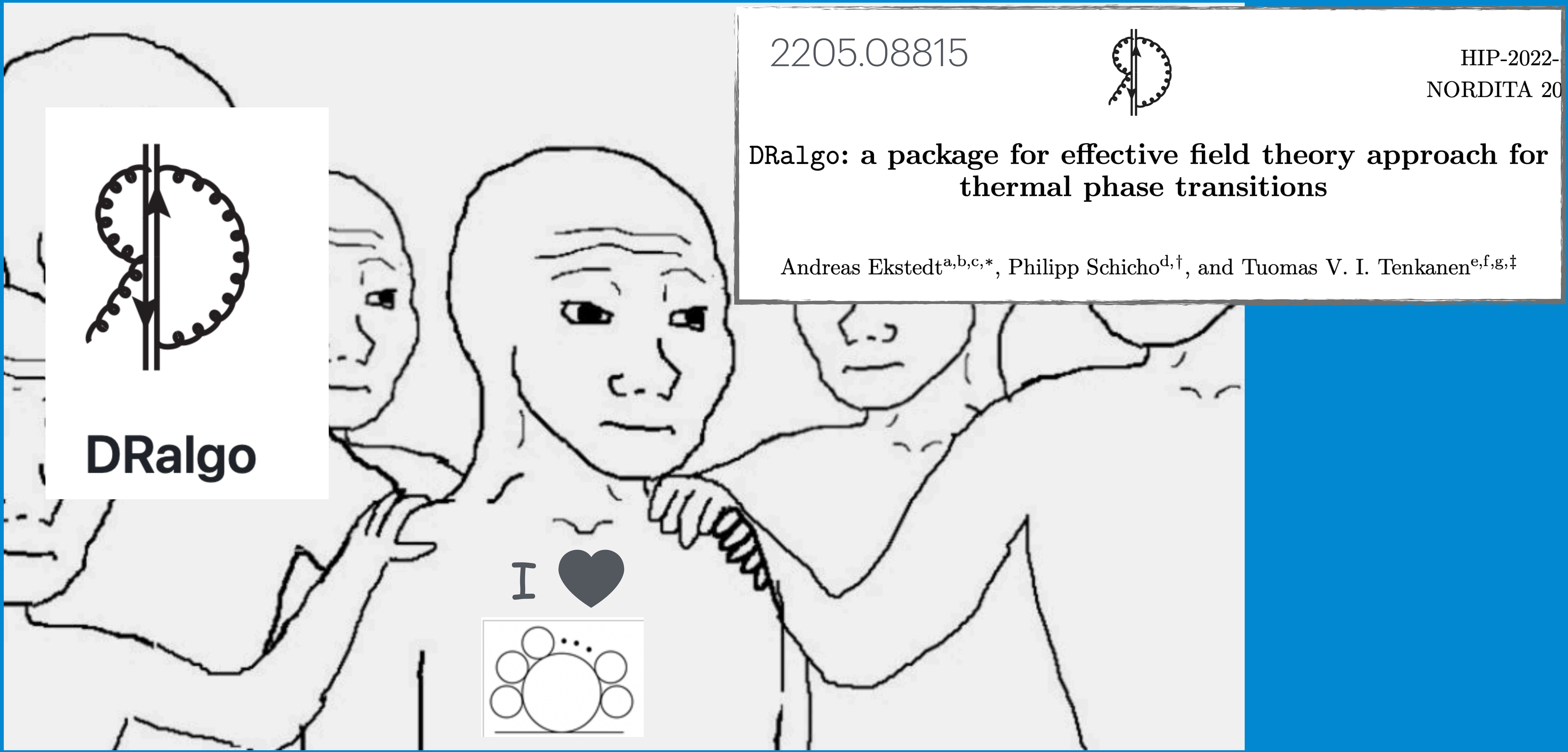
$$\lambda_3 = T\left(\lambda_\varphi + \frac{1}{(4\pi)^2}\left(\frac{3}{8}g_X^4 + L_b\left(-\frac{9}{16}g_X^4 - 12\lambda_\varphi + \frac{9}{2}g_X^2\lambda_\varphi - \lambda_{h\varphi}^2\right)\right)\right)$$

$$g_{X,3}^2 = g_X^2 T\left(1 + \frac{1}{(4\pi)^2}g_X^2\left(\frac{2}{3} + \frac{43}{6}L_b\right)\right)$$

$$h_3 = \frac{1}{2}g_X^2 T\left(1 + \frac{1}{(4\pi)^2}\left(g_X^2\left(\frac{17}{2} + \frac{43}{6}L_b\right) + 12\lambda_\varphi\right)\right),$$

$$m_{D,X}^3 = \frac{5}{6}g_X^2 T^2\left(1 + \frac{1}{(4\pi)^2}T^2\left(g_X^2\left(\frac{69}{20} + \frac{43}{6}L_b\right) + \frac{2}{5}\left(3\lambda_\varphi + \lambda_{h\varphi}\right)\right)\right)$$

$$\begin{aligned} m_3^2 = T^2 \Bigg\{ & \frac{3}{16}g_X^2 + \frac{1}{6}\left(3\lambda_\varphi + \lambda_{h\varphi}\right) \\ & + \frac{1}{(4\pi)^2}\Bigg(\frac{167}{96}g_X^4 + \frac{1}{12}\left(3g_2^2 + g_1^2 + 3L_f y_t^2\right)\lambda_{h\varphi} + \frac{3}{4}g_X^2\lambda_\varphi \\ & + L_b\left(-\frac{47}{32}g_X^4 - \frac{9}{4}g_X^2\lambda_\varphi - \frac{1}{8}\left(9g_2^2 + 3g_1^2 + 3g_X^2 + 6y^2 + 8\lambda_h + 8\lambda_\varphi\right)\lambda_{h\varphi} + \frac{1}{6}\lambda_{h\varphi}^2\right) \\ & + \gamma\left(\frac{81}{32}g_X^4 + \frac{1}{2}\left(3g_2^2 + g_1^2\right)\lambda_{h\varphi} - \lambda_{h\varphi}^2 + \frac{9}{2}g_X^2\lambda_\varphi - 6\lambda_\varphi^2\right) \\ & + \ln(A)\left(-\frac{243}{2}g_X^4 - 6\left(3g_2^2 + g_1^2\right)\lambda_{h\varphi} + 12\lambda_{h\varphi}^2 - 54g_X^2\lambda_2 + 72\lambda_\varphi^2\right)\Bigg)\Bigg\} \\ & + \frac{1}{(4\pi)^2}\ln\left(\frac{\mu_3}{\mu_4}\right)\left(-\frac{36}{16}g_{X,3}^4 + \frac{3}{2}\lambda_{VL8}^2 - 6g_{X,3}^2\lambda_{VL8}\right) \end{aligned}$$


DRalgo
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DRalgo: a package for effective field theory approach for thermal phase transitions
Andreas Ekstedt[a,b,c,*], Philipp Schicho[d,†], and Tuomas V. I. Tenkanen[e,f,g,‡]

(Soft scale) 3d EFT

$$S_3 = \int \mathrm{d}^3\mathbf{x} \left\{ \frac{1}{4} F^a_{ij} F^a_{ij} + \left(D_i\phi\right)^\dagger \left(D_i\phi\right) + \frac{1}{2}\left(D_i X^a_0\right)^2 + V_3\left(\phi, X^a_0\right) \right\}$$

$$V_3 = \frac{1}{2} m_3^2 \phi_3^2 + \frac{1}{4} \lambda_3 \phi_3^4$$

$$m_3 \sim m_D = gT \neq 0$$

$$m^2_{X,3} = \frac{1}{4} g^2_{X,3} \phi_3^2, \quad m^2_{X_0,3} = m^2_{D,X} + \frac{1}{2} h_3 \phi_3^2 .$$

See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

(Soft scale) 3d EFT

$$S_3 = \int d^3\mathbf{x} \left\{ \frac{1}{4} F^a_{ij} F^a_{ij} + (D_i\phi)^\dagger (D_i\phi) + \frac{1}{2} (D_i X^a_0)^2 + V_3(\phi, X^a_0) \right\}$$

$$V_3 = \frac{1}{2} m_3^2 \phi_3^2 + \frac{1}{4} \lambda_3 \phi_3^4$$

Thermal mass

$$m_3 \sim m_D = gT \neq 0$$

$$m^2_{X,3} = \frac{1}{4} g^2_{X,3} \phi_3^2, \quad m^2_{X_0,3} = m^2_{D,X} + \frac{1}{2} h_3 \phi_3^2 .$$

See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

Effective action perturbatively

$$S_{\text{eff}}[\phi_b] = \underbrace{S^{\text{LO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^3)} + \underbrace{S^{\text{NLO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^4)} + \ldots$$

One can get to NLO just with the LO bounce solution!

=

See also: A. Ekstedt 2201.07331, 2104.11804

Nucleation rate in 3D SU(2)cSM at 1-loop

$$\Gamma = A_{\text{dyn}}\,\text{Det}_S\, e^{-\left(S_3[\phi_\text{b}] - S_3[\phi_\text{F}]\right) - \log \text{Det}_{X_0} - \log \text{Det}_{XG}}$$

overall prefactor
$\sim T^4$

(Effective) action in
the exponential

See also: A. Ekstedt, O. Gould, J. Hirvonen, 2308.15652; A. Ekstedt 2104.11804

Nucleation rate in 3D SU(2)cSM at 1-loop

$$\Gamma = A_{\rm dyn}\, \mathrm{Det}_S\, e^{-\left(S_3\left[\phi_{\rm b}\right] - S_3\left[\phi_{\rm F}\right]\right) - \log \mathrm{Det}_{X_0} - \log \mathrm{Det}_{XG}}$$

Can be easily computed using

BubbleDet: A Python package to compute functional determinants for bubble nucleation

Andreas Ekstedt*[1], Oliver Gould†[2], and Joonas Hirvonen‡[3]

Vector-Goldstone determinant

(Not so easy to compute...)

See also: A. Ekstedt, O. Gould, J. Hirvonen, 2308.15652; A. Ekstedt 2104.11804

Vector-Goldstone determinant in derivative expansion

$$\frac{m_3}{m_X} \ll 1$$

$$\delta Z = -\frac{11}{16\pi}\frac{g_3}{\phi_b}$$

$$-\log \mathrm{Det}_{VG} \simeq \underbrace{\int_x \frac{-3}{48\pi} g^3 \phi_b^3}_{\text{LO}} + \underbrace{\int_x \frac{1}{2} \delta Z[\phi_b] (\partial \phi_b)^2}_{\text{NLO}} + \ldots$$

Barrier generating term

Terms with higher powers in derivatives

See also: A. Ekstedt, O. Gould, J. Hirvonen, 2308.15652; A. Ekstedt 2104.11804, J. Hirvonen et al 2112.08912, MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

$S_{\rm eff}$ at LO and NLO in derivative expansion

$$S^{\rm LO}[\phi_b^{\rm LO}] = 4\pi \int {\rm d}r\ r^2 \left[\frac{1}{2}\left(\partial_i \phi_b^{\rm LO}\right)^2 + V^{\rm LO}[\phi_b^{\rm LO}]\right]$$

$$S^{\rm NLO}[\phi_b^{\rm LO}] = 4\pi \int {\rm d}r\ r^2 \left[\frac{1}{2}\delta Z[\phi_b](\partial\phi_b)^2 + V^{\rm NLO}[\phi_b^{\rm LO}]\right]$$

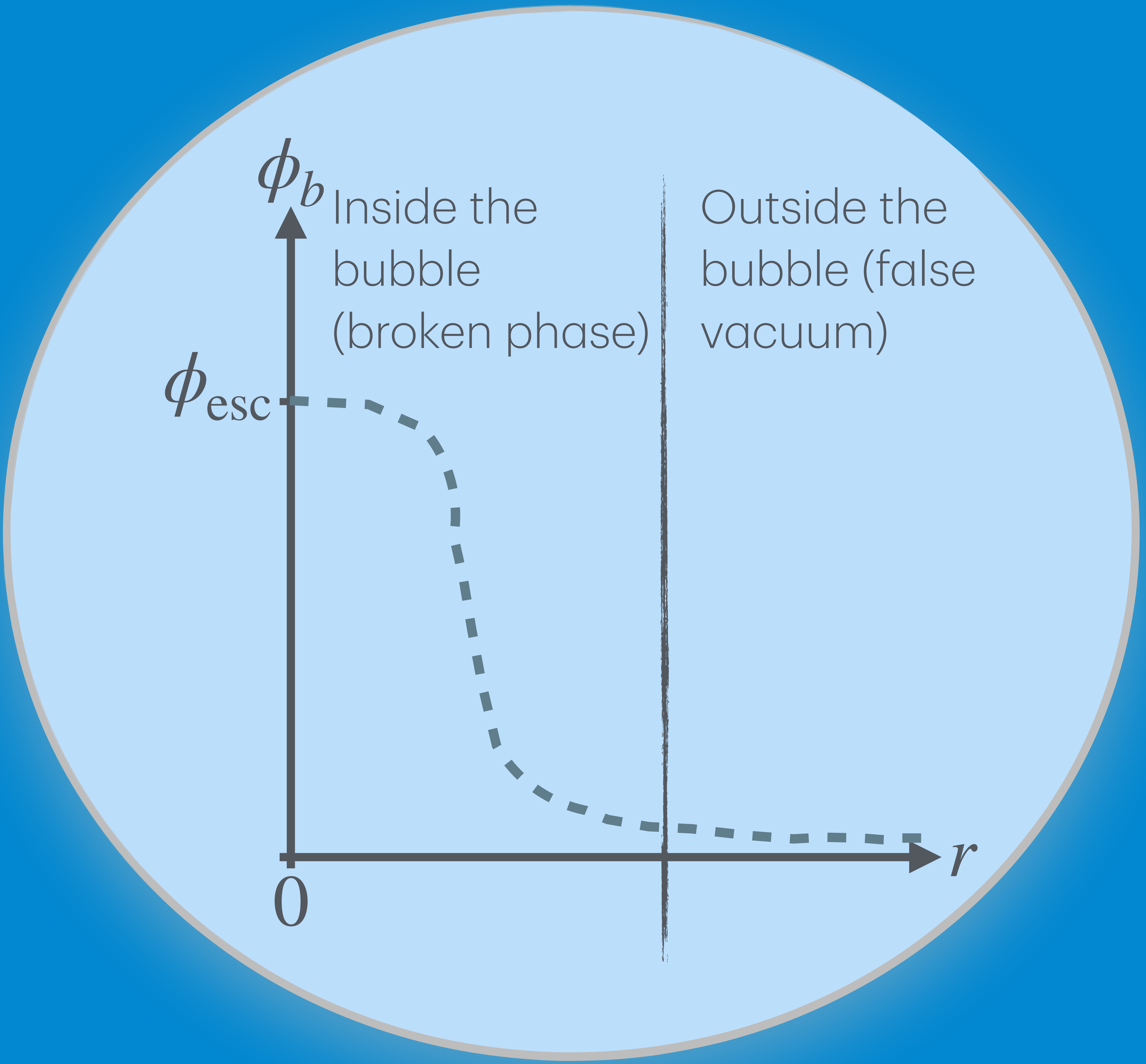
$$V^{\rm LO}[\phi_b^{\rm LO}] = \frac{1}{2}m_3^2\phi_3^2 + \frac{1}{4}\lambda_3\phi_3^4 - \frac{1}{12\pi}\left(6(m_{X,3}^2)^{\frac{3}{2}} + 3(m_{X_0,3}^2)^{\frac{3}{2}}\right)$$

$$\begin{aligned}V^{\rm NLO} = \frac{1}{(4\pi)^2}\Bigg\{ &\frac{3}{64}g_{X,3}^2\Big(56m_{X,3}^2(1-3\ln(3)) + g_{X,3}^2v_3^2(2-\ln(256)) \\ &+2\left(80m_{X,3}^2 - 3g_{X,3}^2v_3^2\right)\ln\left(\frac{\mu_3}{2m_{X,3}}\right)\Big)\Bigg\} \\ &+\frac{1}{(4\pi)^2}\Bigg\{\frac{3}{4}g_{X,3}^2\left(6m_{X_0,3}^2 + 4m_{X,3}m_{X_0,3} - m_{X,3}^2\right) + \frac{15}{4}\kappa_3 m_{X_0,3}^2 \\ &-\frac{3}{8}h_3^2v_3^2\left(1 + 2\ln\left(\frac{\mu_3}{2m_{X_0,3}}\right)\right) - \frac{3}{2}g_{X,3}^2\left(m_{X,3}^2 - 4m_{X_0,3}^2\right)\ln\left(\frac{\mu_3}{2m_{X_0,3} + m_{X,3}}\right)\Bigg\}\end{aligned}$$

See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

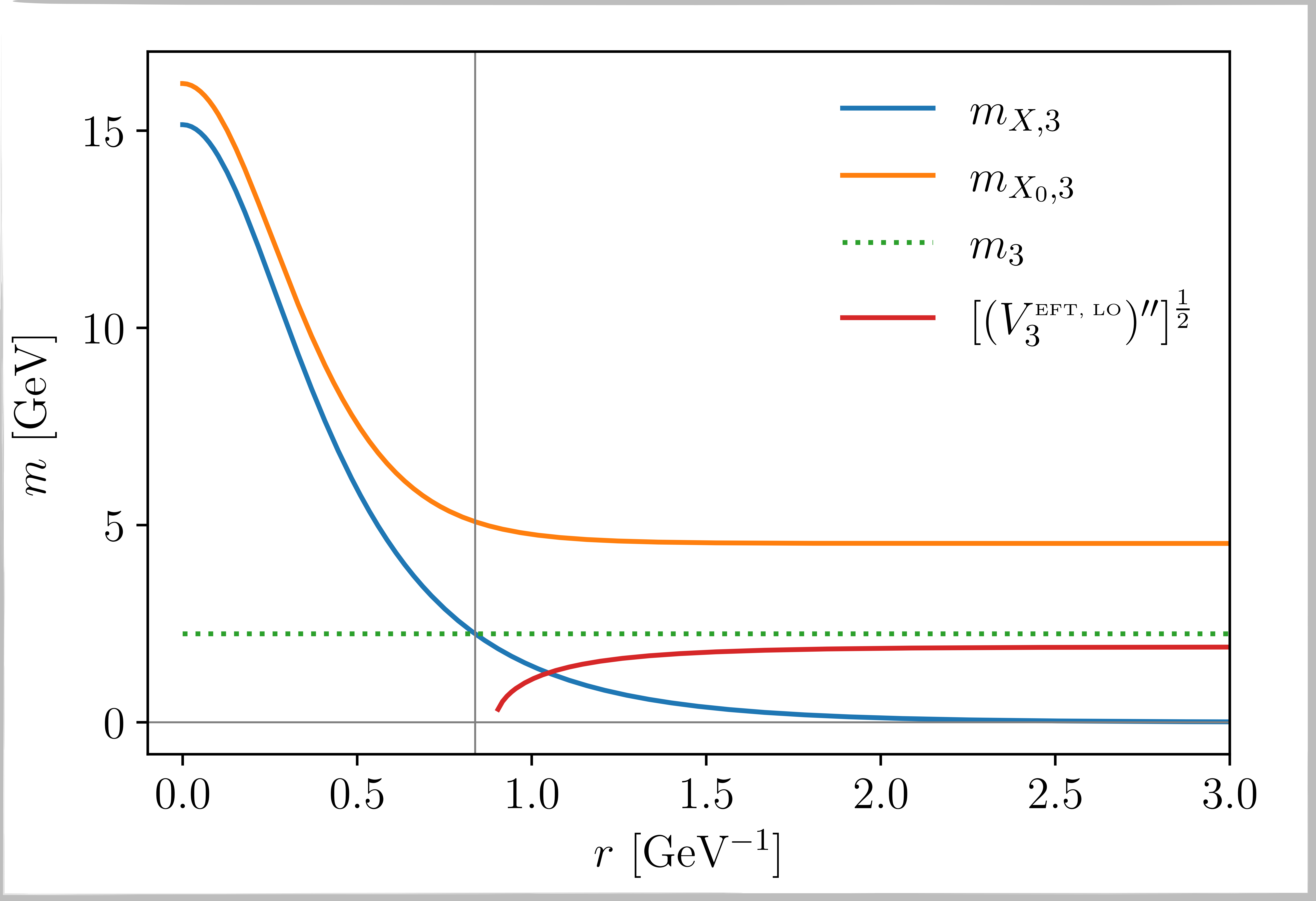
Q: Is derivative expansion always valid? 🤔

LO bounce solution



See also: S. Coleman Phys.Rev.D 15 (1977), A. Ekstedt 2201.07331, 2104.11804

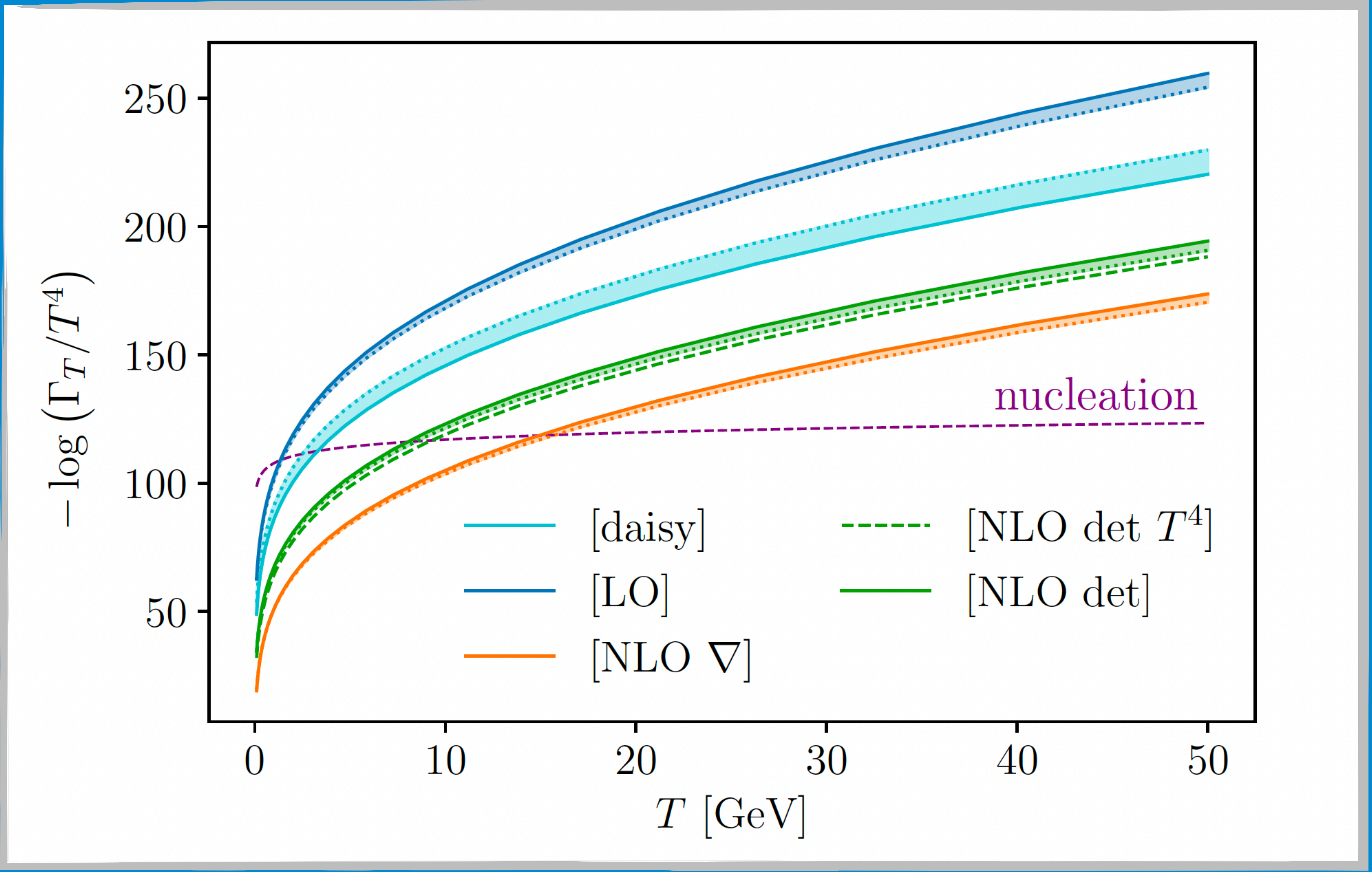
Scale-shifters in SU(2)cSM



Q: Is derivative expansion always valid?

A: Nope 😢

Results: Nucleation rate from different “recipes”



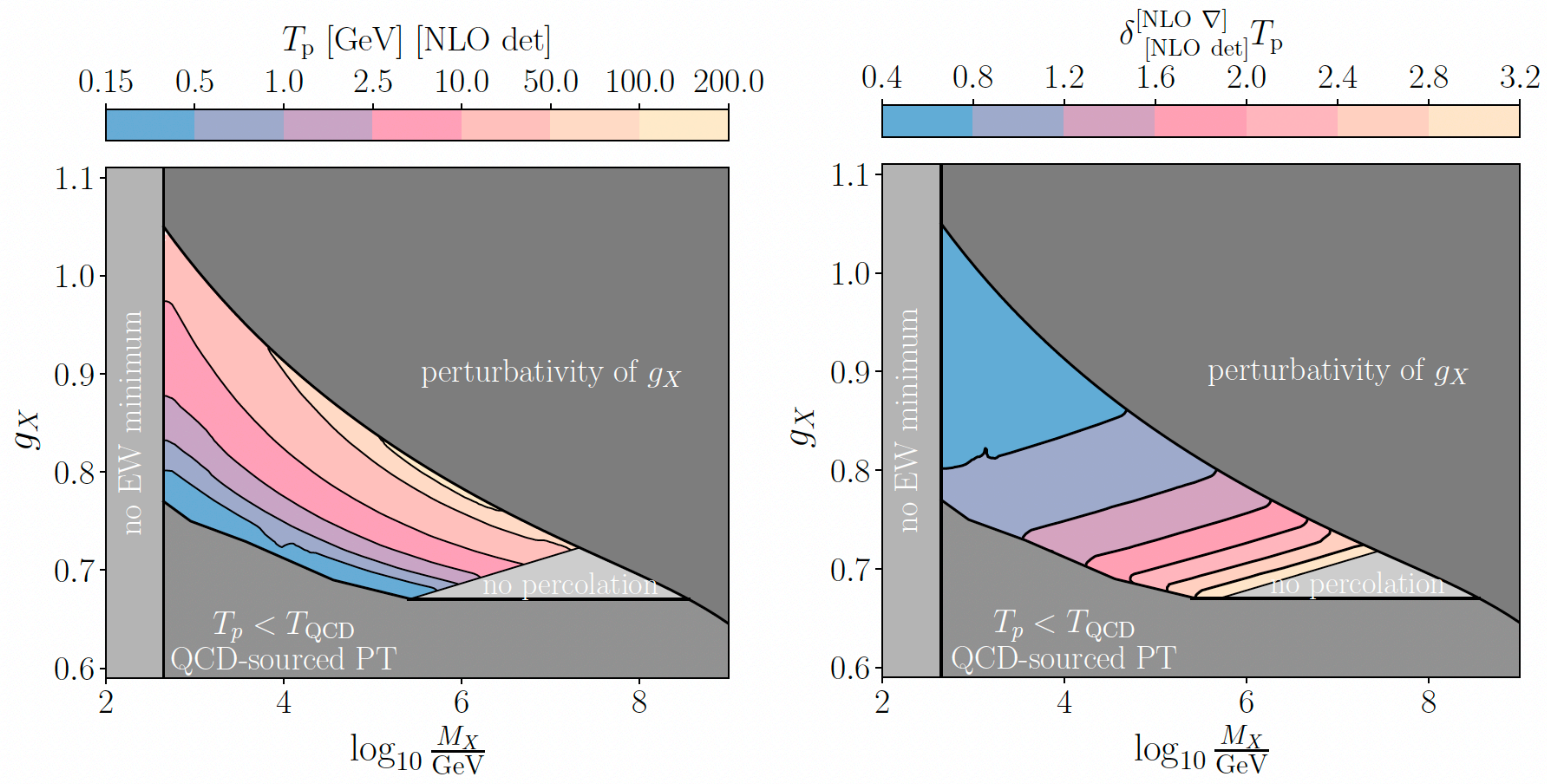
Note:

$A_{\rm dyn} \det_S \simeq T^4$

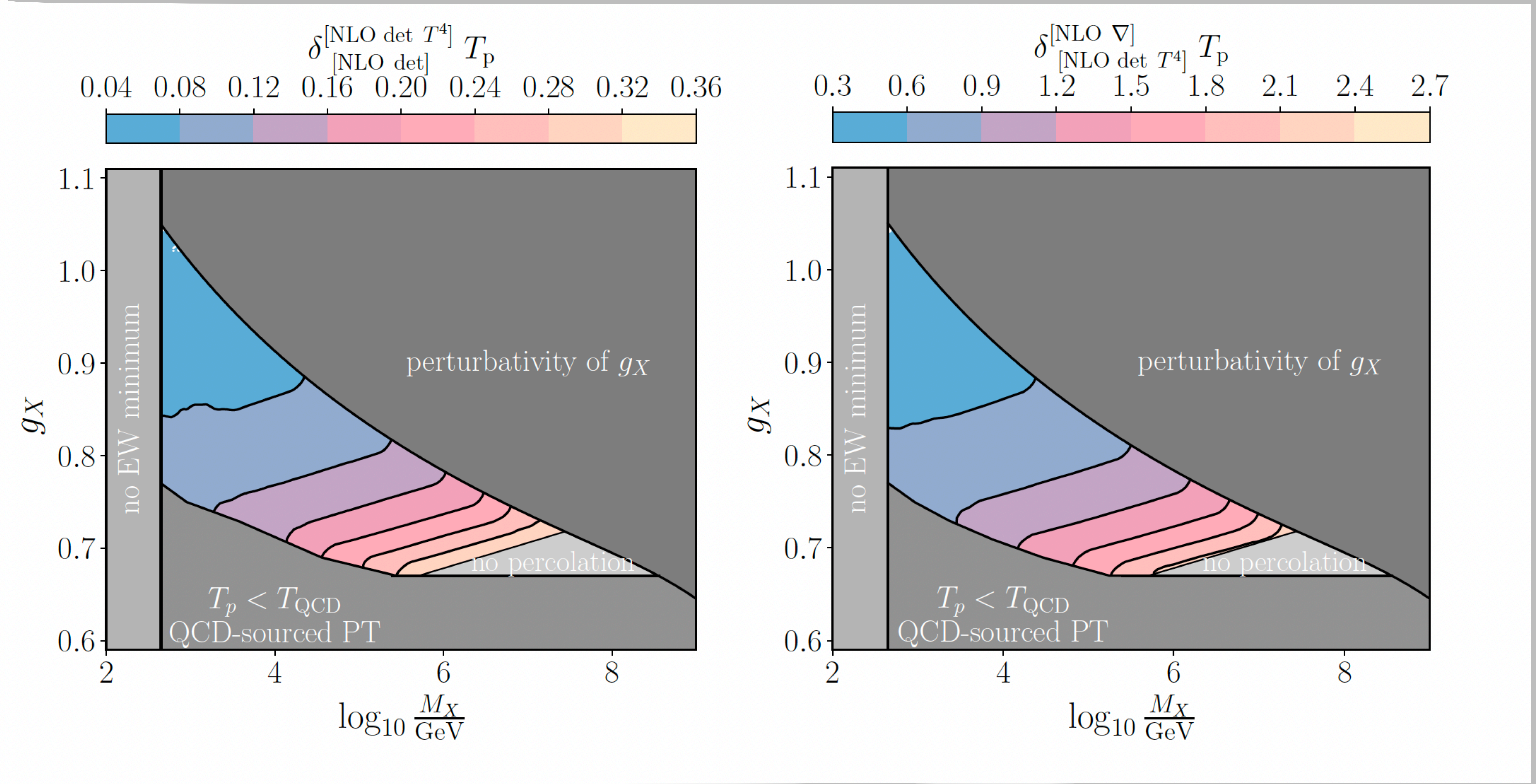
everywhere

except [NLO det]

NLO nucleation rate results: percolation temperature



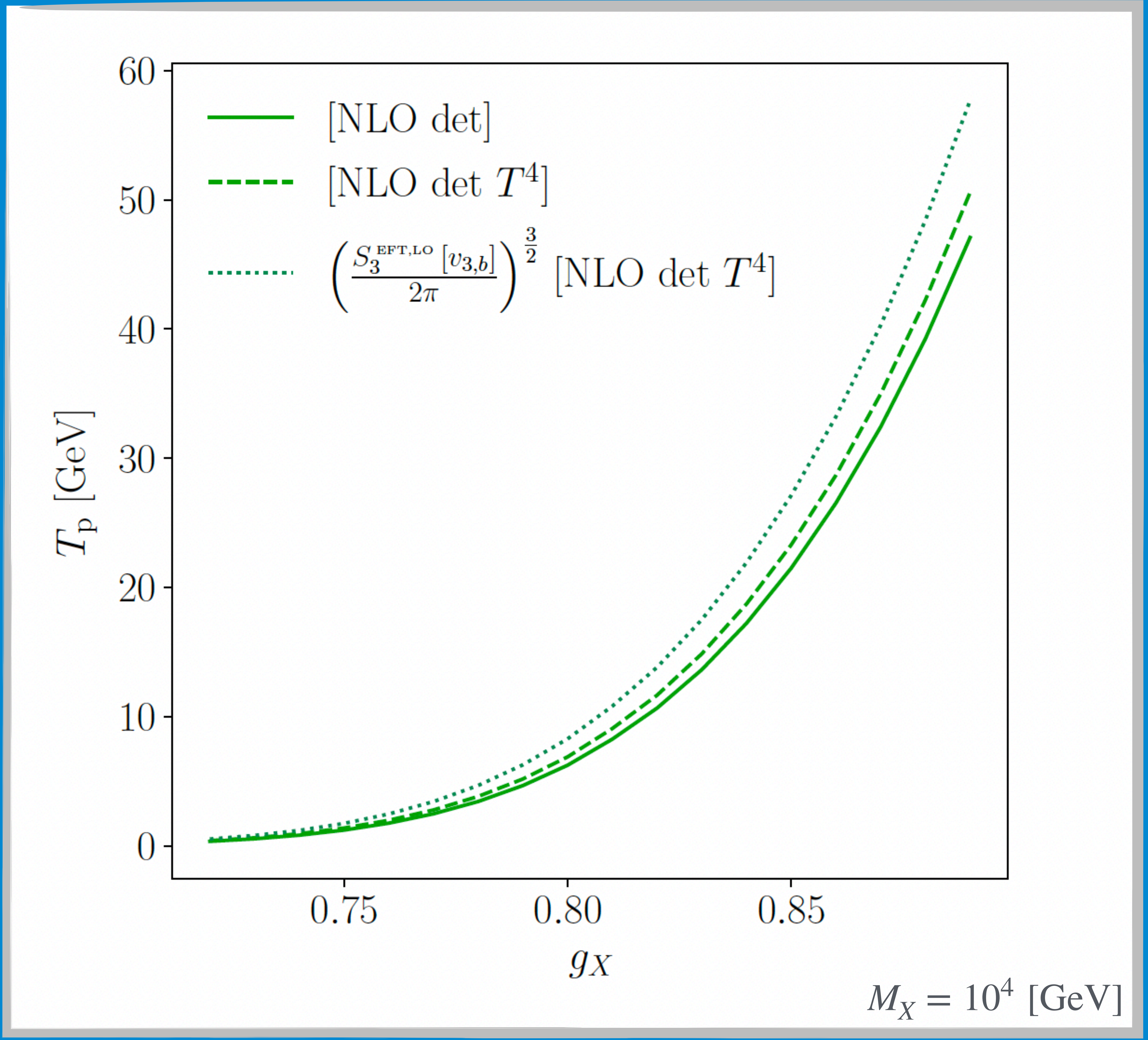
Direct influence of scale shifters: percolation temperature



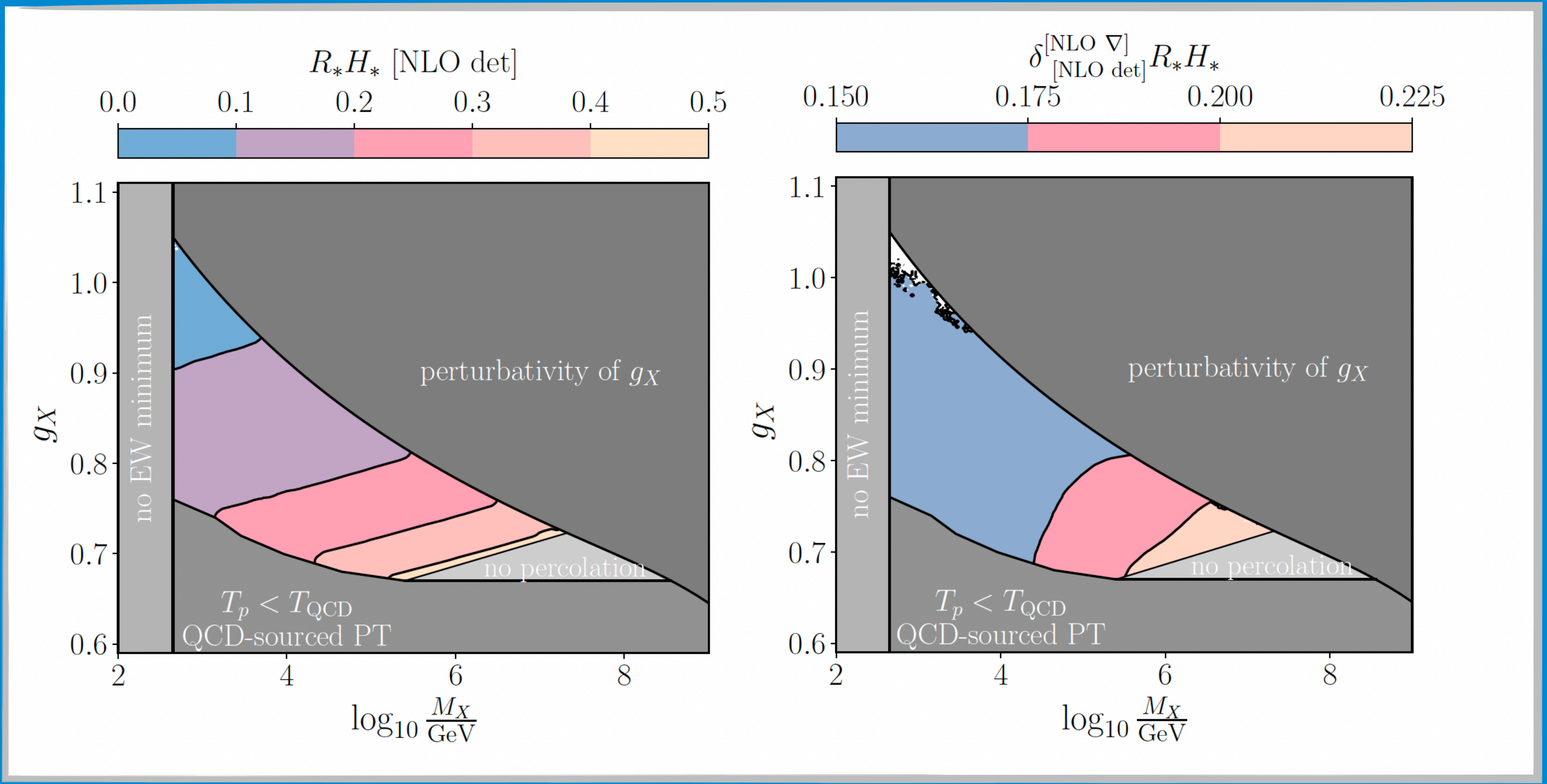
How to approximate the scalar prefactor?

Reminder:

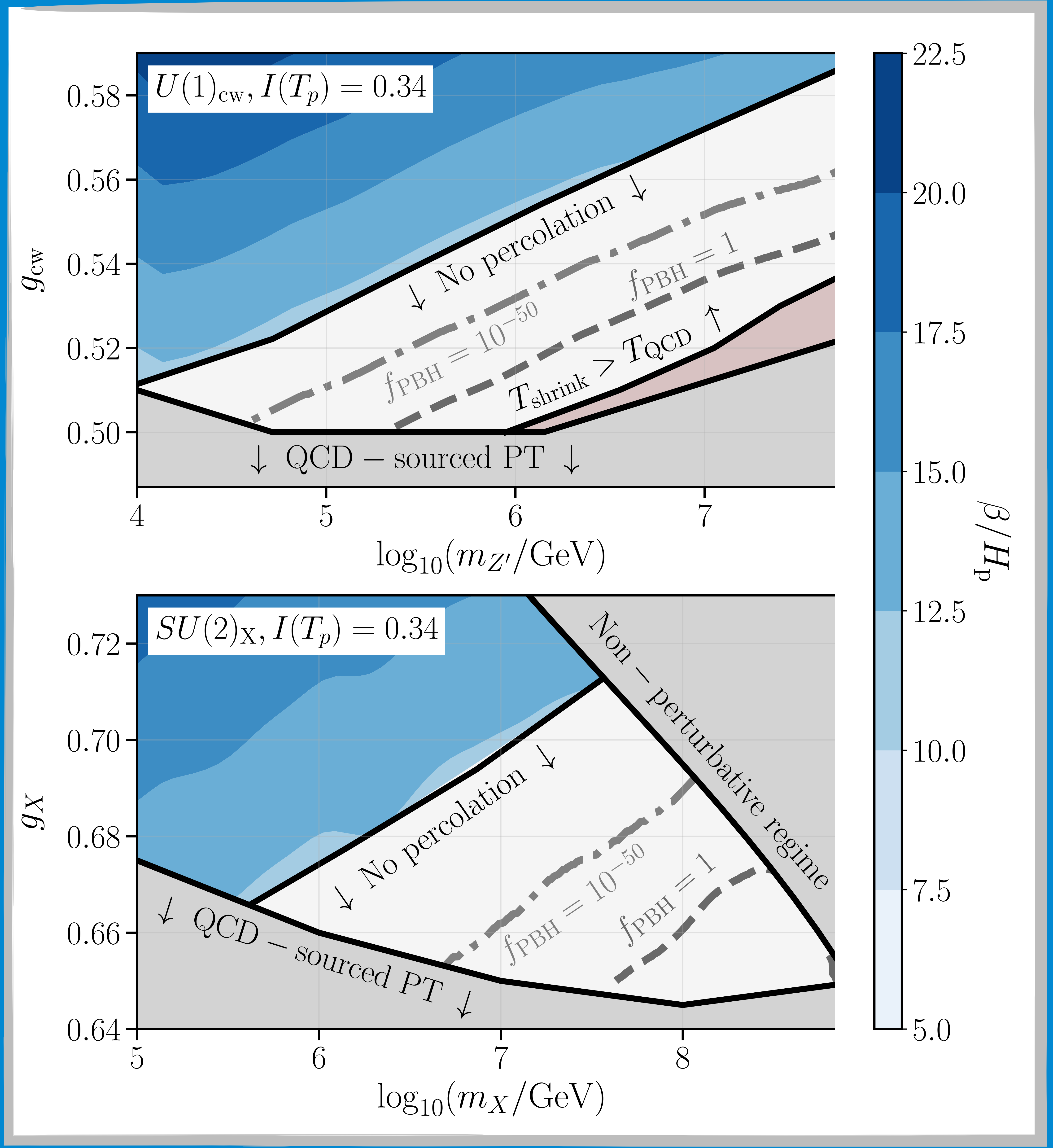
$$\Gamma_T = Ae^{-S}$$



NLO nucleation rate results: bubble radius



U(1) model results & pBHs



See also talks by Yann and Piotr

MK, et al 2506.15496,

Summary

Derivative expansion for **gauge modes** introduces significant **errors**.

Including scalar determinant helps, but it's not the main source of errors

T^4 can work better than $T^4\left(\frac{S}{2\pi}\right)^{\frac{3}{2}}$

To do: Compute the dynamical part more carefully

NLO corrections are **mandatory** for reliable pheno predictions

To do: impact of NLO corrections on model reconstruction from GW

It'd be nice to have a comparison to lattice!

Thank you!

Feel free to ask questions: maciej.kierkla@fuw.edu.pl

What happens at high-temperature?

We are interested in dark sector only i.e. scalar + SU(2) gauge boson

$$V_0 + V_{CW} + V_T^{HT} = \frac{9M_X(\varphi,t)^4}{64\pi^2}\left(\log\frac{M_X(\varphi,t)^2}{\mu(\varphi)^2} - \frac{5}{6}\right) - \frac{9M_X(\varphi)^4}{64\pi^2}\left(\log\frac{M_X^2(\varphi)}{16\pi^2T^2} - \frac{3}{2} + 2\gamma_E\right) + \dots$$

$$\log\frac{\mu^2}{16\pi T^2}$$

Computing effective action - 3D EFT toy model

Consider a simple theory involving two fields Φ, χ

$$\mathscr{L}_3 = \frac{1}{2}(\partial_i \Phi)^2 + \frac{1}{2}(\partial_i \chi)^2 + \frac{1}{2}m^2\Phi^2 + \frac{\lambda}{4}\Phi^4 + \kappa\chi^2\Phi^2 + \frac{1}{2}M^2\chi^2$$

In order to find nucleation rate we want to
compute effective action using the background field method

$$\Gamma \sim e^{-S_{\text{eff}}[\phi_b]} = \int \mathscr{D}\chi e^{-S_3[\phi_b,\chi]}$$

Integrate out fluctuations
on the background

Computing effective action - 3D toy model

$$\mathscr{L}_3 = \frac{1}{2}(\partial_i\Phi)^2 + \frac{1}{2}(\partial_i\chi)^2 + \frac{1}{2}m^2\Phi^2 + \frac{\lambda}{4}\Phi^4 + \kappa\chi^2\Phi^2 + \frac{1}{2}M^2\chi^2$$

Expand field on its background

$$\Phi \to \phi_b(r) + H(x)$$

At 1-loop we need quadratic terms only

$$\mathscr{L} \supset \chi \underbrace{\left(-\nabla^2 + M^2 + \kappa\phi_b^2\right)}_{\mathscr{M}_\chi} \chi$$

Performing the $\int \mathscr{D}\chi$

$$e^{iS_{\text{eff}}[\phi_b]} \sim \left[\det\left(-\nabla^2 + M^2 + \kappa\phi_b^2\right)\right]^{-\frac{1}{2}} e^{-S_3[\phi_b]}$$

Z-factor

$$Z_3^{\mathrm{NLO}}(\phi_3) = -\frac{11}{16\pi}\frac{g_3}{\phi_3} + \frac{1}{64\pi}\frac{h_3^2\phi_3^2}{m_{X_0,3}^2}$$

Spatial
gauge modes

Temporal
gauge modes

Higher-order momentum terms

$$S_3^{\mathrm{EFT}}[v_{3,b}] \supset \int_{\mathbf{x}} \Big[V^{\mathrm{EFT}}(v_3) + \frac{1}{2} Z_{2,3} (\partial_i v_3)^2 + \frac{1}{2} Z_{4,3} (\partial^2 v_3)^2 \\ + \frac{1}{2} Y_{3,3} (\partial_i v_3)^2 \partial^2 v_3 + \frac{1}{8} Y_{4,3} (\partial_i v_3)^2 (\partial_j v_3)^2 + \mathcal{O}(\partial^6) \Big] ,$$

See also: MK, P. Schicho B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2503.13597

Higher-order momentum terms

At 2-loop

$$S_3^{\mathrm{EFT}}[v_{3,b}] \supset \int_{\mathbf{x}} \Big[V^{\mathrm{EFT}}(v_3) + \frac{1}{2} Z_{2,3} (\partial_i v_3)^2 + \frac{1}{2} Z_{4,3} (\partial^2 v_3)^2 + \frac{1}{2} Y_{3,3} (\partial_i v_3)^2 \partial^2 v_3 + \frac{1}{8} Y_{4,3} (\partial_i v_3)^2 (\partial_j v_3)^2 + \mathcal{O}(\partial^6) \Big] ,$$

All of the red terms are divergent at large radius

See also: MK, P. Schicho B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2503.13597

Taming scale-shifters

Step 1. Obtain LO bounce solution ϕ_b^{LO} using S^{LO}

Step 2. Evaluate determinants numerically $\mathrm{Det}_{VG}[\phi_b^{\mathrm{LO}}]$

Step 2. Get the NLO bosons contribution $S^{\mathrm{NLO}} \sim V_3^{\mathrm{NLO}}$ on ϕ_b

Step 3. Obtain the action

$$S_{\mathrm{eff}} = (S^{\mathrm{LO}} - S^{\mathrm{LO}}_{\mathrm{bosons}}) - \log \mathrm{Det}_{VG} - S^{\mathrm{pot}}_{\mathrm{NLO}}$$

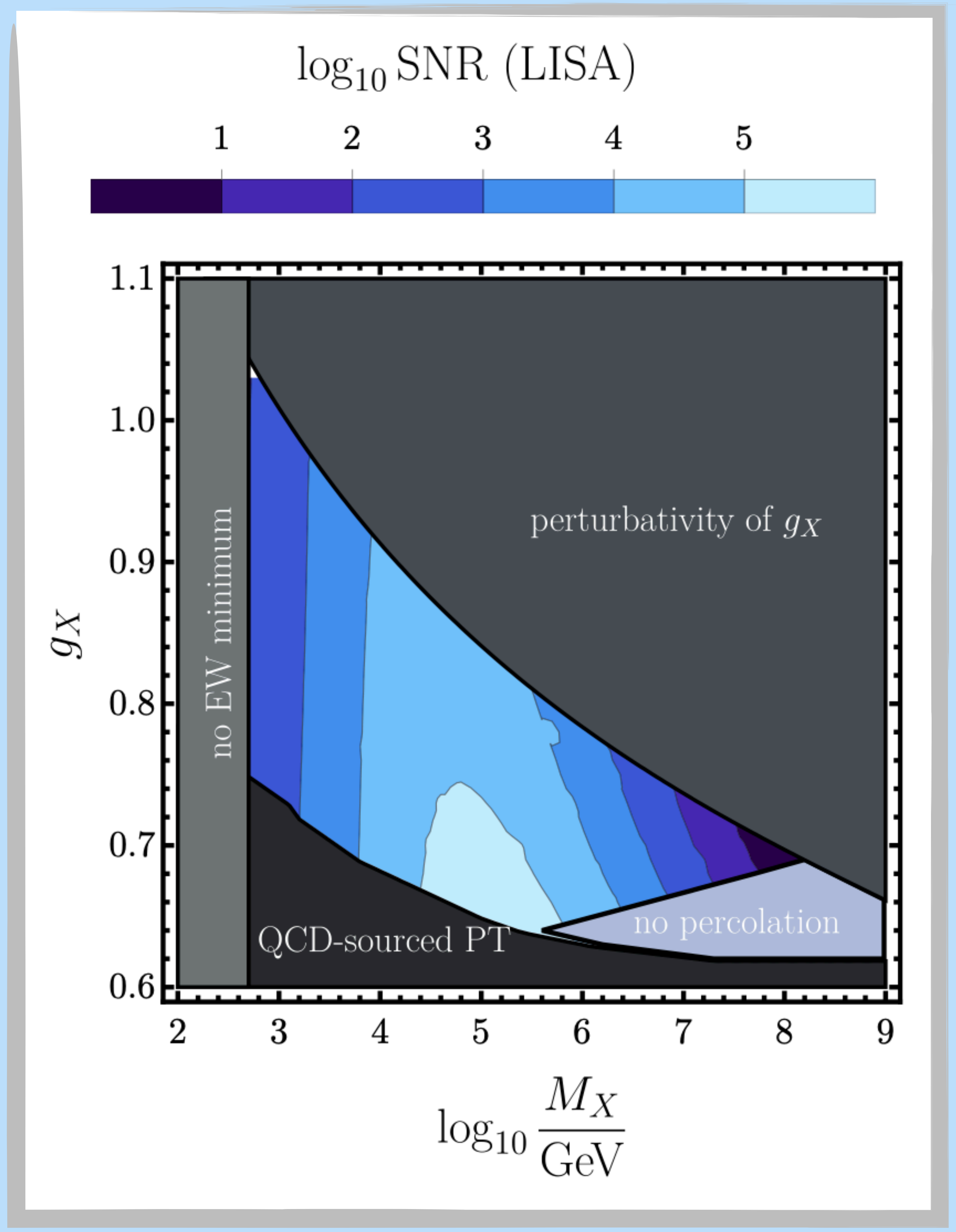
Step 4. Calculate nucleation rate

$$\Gamma_T = A_{\mathrm{dyn}}\, \mathrm{Det}_H\, e^{-S_{\mathrm{eff}}[\phi_b]}$$

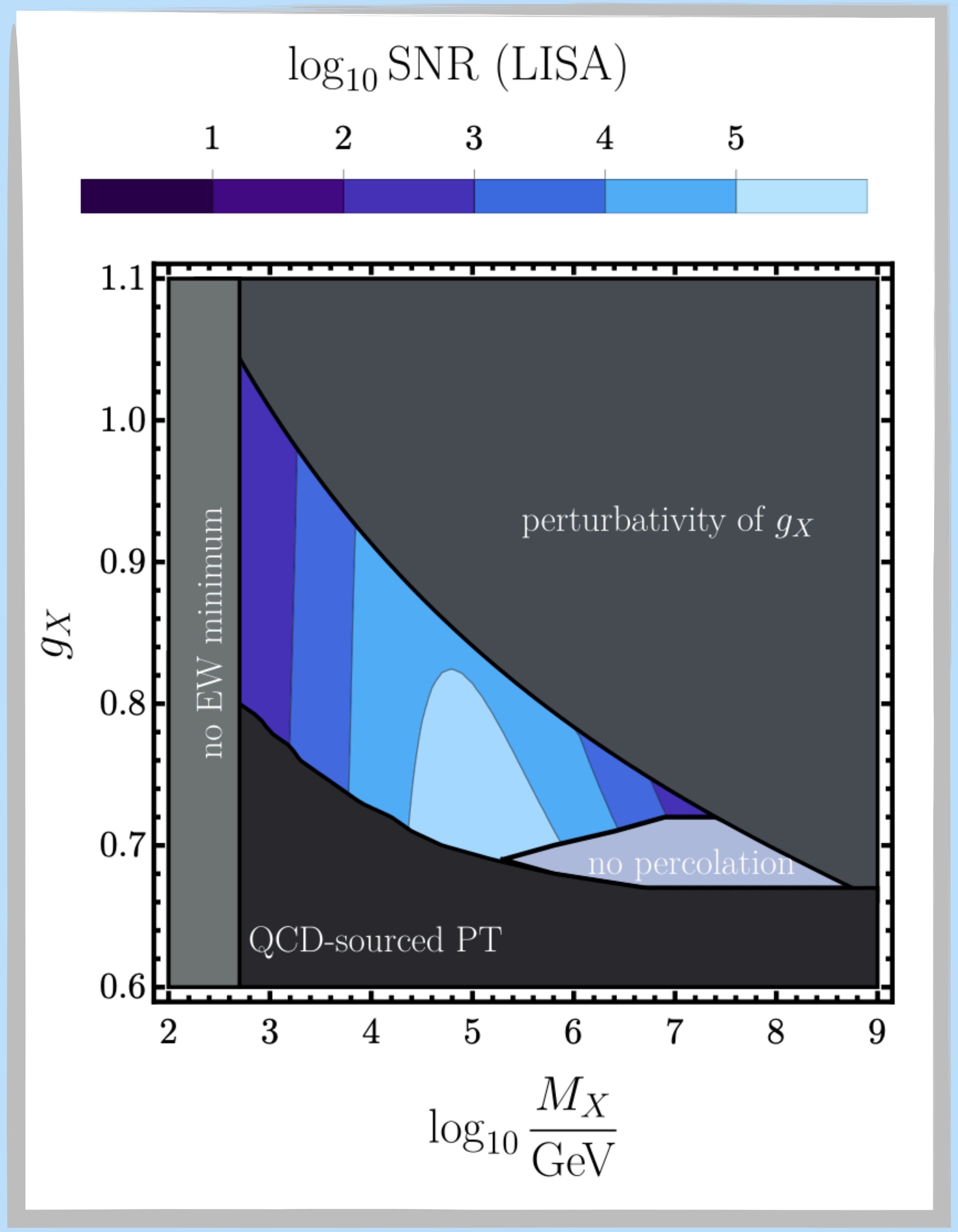
See also: O. Gould, J. Hirvonen, 2108.04377, MK, P. Schicho, B. Świeżewska, T. Tenkanen, J. Van de Vis 2502.xxxxx

LISA SNR ([NLO grad] vs [daisy])

[NLO grad]



[daisy]



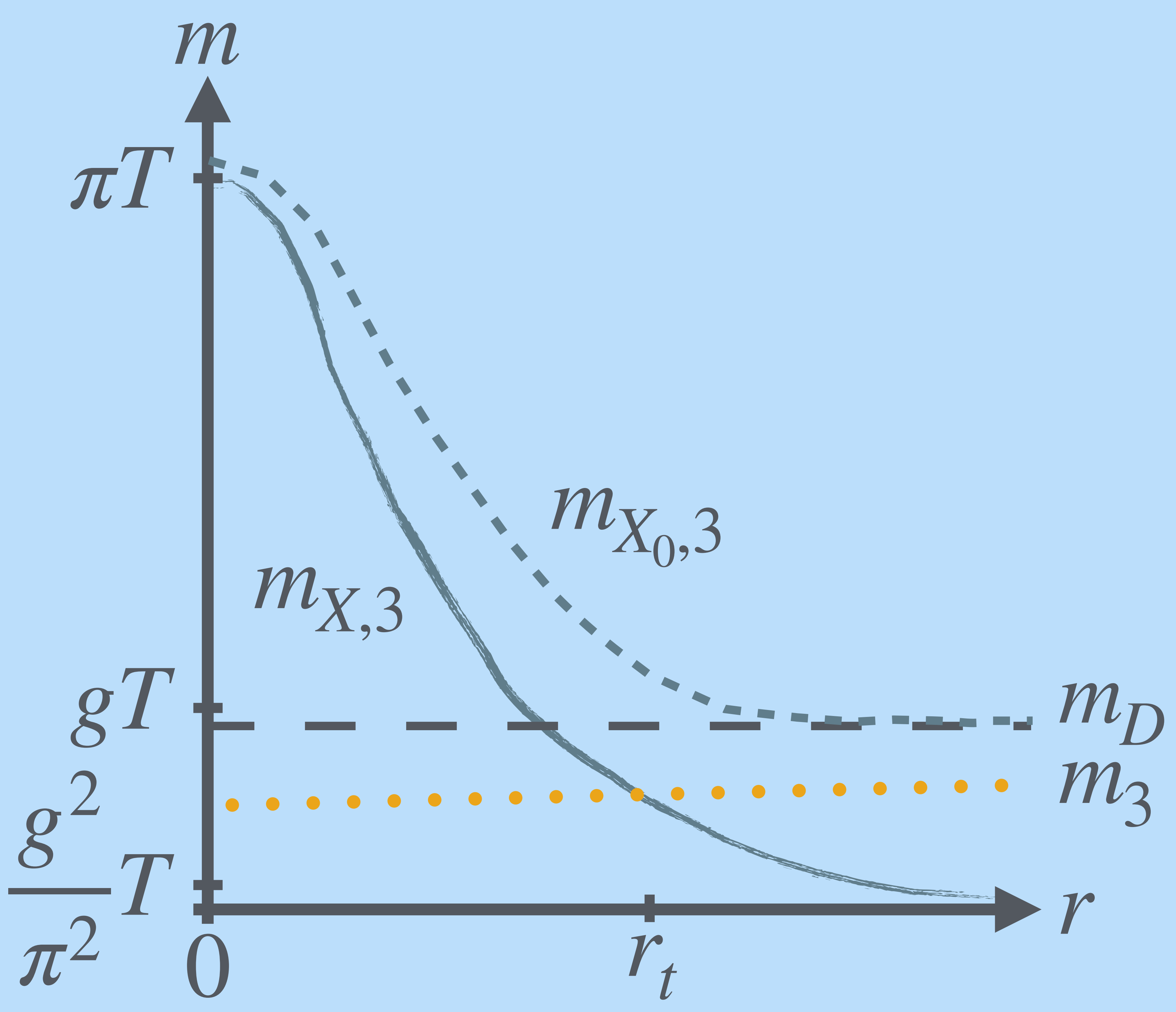
See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

Scale-shifters

Reminder:

$$m_{X,3}^2 = \frac{1}{4} g_{X,3}^2 \phi_3^2,$$

$$m_{X_0,3}^2 = m_{D,X}^2 + \frac{1}{2} h_3 \phi_3^2\,.$$



See also: O. Gould, J. Hirvonen, 2108.04377

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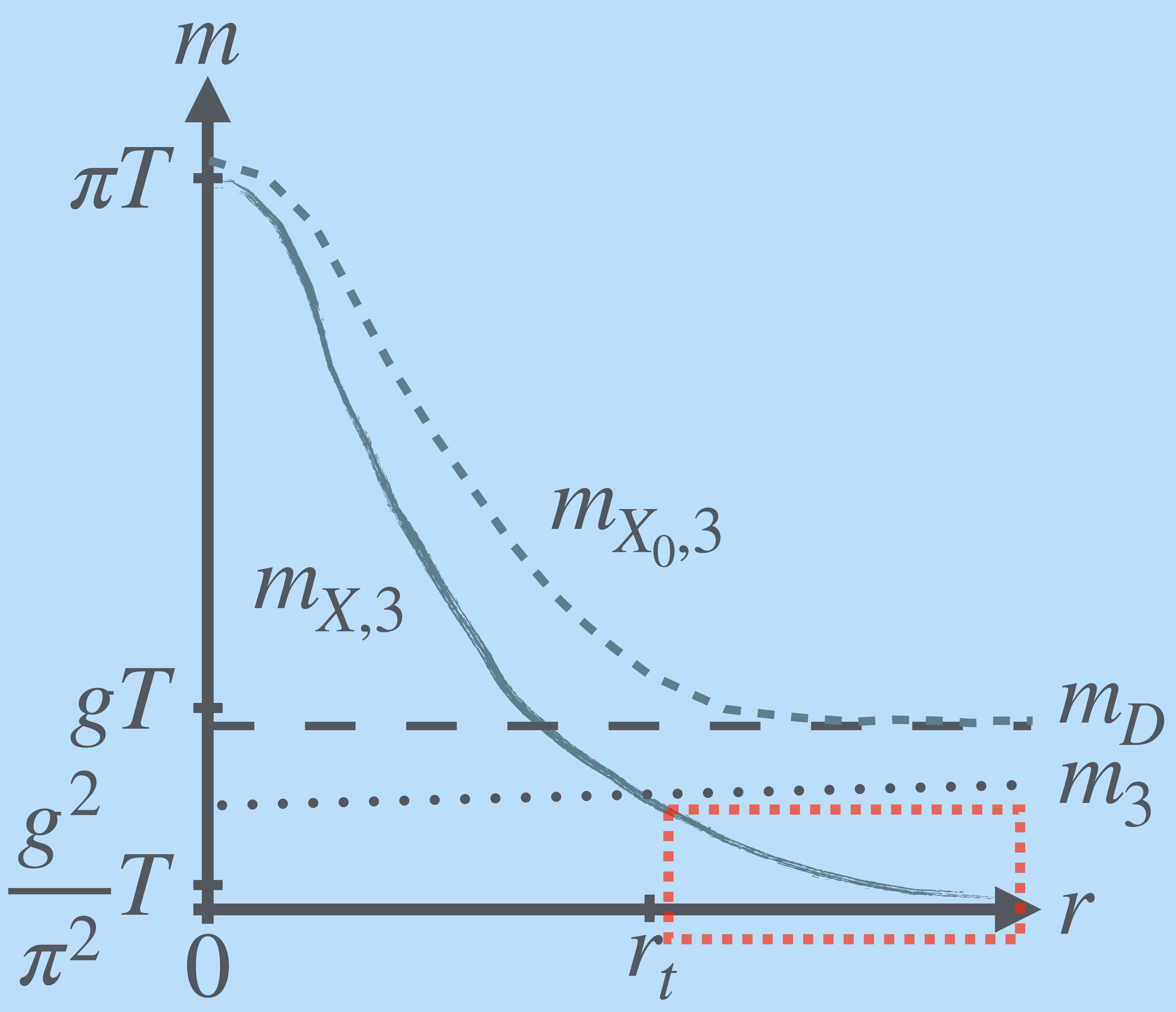

Trouble!

See also: O. Gould, J. Hirvonen, 2108.04377